On the Symmetry Breaking Structure of Maximal Supergravities

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SUGRA Symmetry Breaking

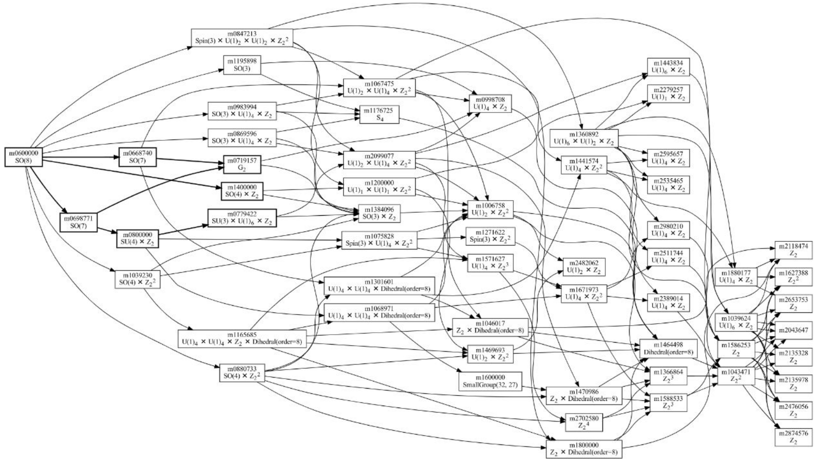
Objective of this talk

- As a non-academic actively publishing quantum gravity researcher, I am a rather unusual member of the community. ("I represent computing").
- Thanks to in particular Hermann, I have the keys to tackling some of the hard problems in the field.
- My biggest problem is one of "impedance matching":
  Enabling others in the community to utilize some of the powerful (often computational) techniques.
- We now probably mostly know the full symmetry breaking structure of de Wit-Nicolai supergravity, D=4 N=8 SO(8) SUGRA, and the methods used to obtain it have produced partial results for other models.
- Two main reasons why we have this result now:
  - Hermann and I met and started working together.
  - Having left quantum gravity research twice(!), this problem (and Hermann) brought me back.
- I would like to tell this story (without getting too technical).

SUGRA Symmetry Breaking

The Symmetry Breaking Structure of SO(8) Supergravity
(as it was understood in 2022)

$$
\begin{aligned}
\mathcal{L}/e \quad = \quad & -\tfrac{1}{2}\,R(e,\omega) - \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\left(\bar\psi^i_\mu\gamma_\nu D_\rho\psi_{\sigma i} - \bar\psi^i_\mu\overleftarrow{D}_\rho\gamma_\nu\psi_{\sigma i}\right)\\
& -\tfrac{1}{12}\left(\bar\chi^{ijk}\gamma^\mu D_\mu\chi_{ijk} - \bar\chi^{ijk}\overleftarrow{D}_\mu\gamma^\mu\chi_{ijk}\right) - \tfrac{1}{96}\,\mathcal{A}^{ijk\ell}_\mu\mathcal{A}^\mu_{ijk\ell}\\
& -\tfrac{1}{8}\left(F^+_{\mu\nu\,IJ}\left(2S^{IJ,KL} - \delta^{IJ}_{KL}\right)F^{+\,\mu\nu}{}_{KL} + \text{h.c.}\right)\\
& -\tfrac{1}{2}\left(F^+_{\mu\nu\,IJ}\left(S^{IJ,KL}O^{+\,\mu\nu\,KL}\right) + \text{h.c.}\right)\\
& -\tfrac{1}{4}\left(O^+_{\mu\nu}{}^{IJ}\left(S^{IJ,KL} + u^{ij}{}_{IJ}v_{ijKL}\right)O^{+\,\mu\nu\,KL} + \text{h.c.}\right)\\
& -\tfrac{1}{24}\left(\bar\chi_{ijk}\gamma^\nu\gamma^\mu\psi_{\nu\ell}\left(\hat{\mathcal{A}}^{ijk\ell}_\mu + \mathcal{A}^{ijk\ell}_\mu\right) + \text{h.c.}\right)\\
& -\tfrac{1}{2}\,\delta^{ij}_{i'j'}\bar\psi^{i'}_\mu\psi^{j'}_\nu\bar\psi^\mu_i\psi^\nu_j\\
& +\tfrac{\sqrt{2}}{4}\left(\bar\psi^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi_{ijk}\bar\psi^j_\mu\psi^k_\nu + \text{h.c.}\right)\\
& +\Big(\tfrac{1}{144}\,\eta\epsilon_{ijk\ell mnpq}\bar\chi^{ijk}\sigma^{\mu\nu}\chi^{\ell mn}\bar\psi^p_\mu\psi^q_\nu +\\
& \qquad \tfrac{1}{8}\,\bar\psi^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi_{ik\ell}\bar\psi_{\mu j}\gamma_\nu\chi^{jk\ell} + \text{h.c.}\Big)\\
& +\tfrac{\sqrt{2}\eta}{6\cdot 144}\left(\epsilon^{ijk\ell mnpq}\bar\chi_{ijk}\sigma^{\mu\nu}\chi_{\ell mn}\bar\psi^r_\mu\gamma_\nu\chi_{pqr} + \text{h.c.}\right)\\
& +\tfrac{1}{32}\,\bar\chi^{ik\ell}\gamma^\mu\chi_{jk\ell}\bar\chi^{jmn}\gamma_\mu\chi_{imn}\\
& -\tfrac{1}{96}\,\bar\chi^{ijk}\gamma^\mu\chi_{ijk}\bar\chi^{\ell mn}\gamma_\mu\chi_{\ell mn}\\
& +\sqrt{2}g\,A_1^{ij}\bar\psi_{\mu\,i}\sigma^{\mu\nu}\psi_{\nu\,j} + \tfrac{1}{6}\,g\,A_2{}^i{}_{jk\ell}\bar\psi_{\mu\,i}\gamma^\mu\chi^{jk\ell}\\
& +g\,A_3{}^{ijk\,\ell mn}\bar\chi_{ijk}\chi_{\ell mn} + \text{h.c.}\\
& +g^2\left(\tfrac{3}{4}\,A_1^{ij}A_{1\,ij} - \tfrac{1}{24}\,A_2{}^i{}_{jk\ell}A_{2\,i}{}^{jk\ell}\right).
\end{aligned}
$$



SUGRA Symmetry Breaking

Hermann and Me

- First met Hermann in January 2001.
- Just had finished my German Diploma in Physics (~"Master's equivalent") at TU Munich;
  my advisor Manfred Lindner established the contact.
- Hermann had a very specific research project in mind he proposed to me as a PhD project:
  Investigating the symmetry breaking structure of the recently constructed maximal (32 supercharges) gauged supergravities in D=2+1.
- Had learned GR and some QFT; did know a tiny bit about SUSY beyond D=4 and strings, but really not much.
- Had excellent grades in Physics, but *generally tended to not merely follow the established curriculum*.
- *I am a problem solver*: Always working on adding to the toolbox that allows me to tackle hard problems.
- I accepted Hermann's offer to become his PhD student partly because I wanted to master group theory.

SUGRA Symmetry Breaking

Hermann and Me

- Started as a PhD student at AEI on 2. April 2001. On 2. April 2003, handed Hermann my dissertation.
- Had first results in September 2001 - thanks to a software engineering *tour de force*:
  - Hand-crafted problem-tailored symbolic algebra + algorithms from relational databases for sparse tensor operations.
  - Total ~40 000 lines of Common Lisp code.
- Overall, 3 months or so into the project, Hermann started nagging me (but only a bit) to get going doing pen&paper calculations. Until he saw my handwriting. "Suppose it's better you stick to computers, Thomas."
- Clearly saw the superiority of my approach when we looked into alternative gaugings.
  Would have been a crazy amount of work by hand, but trivial with my code.

SUGRA Symmetry Breaking

Hermann and Me

- Other projects during my time at AEI
  - First algorithm to admit a deep study of E_{10} and E_{11} root spaces in an SL(10)/SL(11) decomposition.
  - Bulk Witten Indices (with M. Staudacher)
  - N=4 SYM spin chain "Matrix QM" at 4 loop order via tailored symbolic algebra w. J. Plefka and T. Klose.
    - Fusing algebraic partial-term elimination ("a factor is zero") with graph elimination ("cannot be completed while staying planar").
    - Reduced ~10^{15} graphs to just under $7*10^9$. Doable with ~90 000 CPU-hours.
- Afterwards, first went to Bruxelles, but then left quantum gravity in 2005.
  Started working on numerical electrodynamics in Southampton.

SUGRA Symmetry Breaking

Hermann and Me

- Became a Lecturer in Engineering Physics at the University of Southampton in 2007 - mostly doing (finite element) computational field theory numerics for micromagnetism.
- Came back to working on SUGRA symmetry breaking in 2008, *using tools and techniques learned from the Engineers*. Shifted focus there due to a serious disagreement with colleagues. In 2009: First new equilibria for "de Wit-Nicolai Supergravity" since 1983 - including a N=1 U(1)xU(1) vacuum.
- In 2012, left Academia (and hence work on quantum gravity) for the 2nd time(!) - mostly due to a bad constellation of me disagreeing with ethically questionable behavior on three different fronts simultaneously.
- Joined Google in 2012; later joined Google Research.
- In 2018, Hermann contacted me again with a question about SUGRA symmetry breaking where he thought I would be the only one who could manage to do the calculation.

SUGRA Symmetry Breaking

Hermann and Me

- We both know the answer to his original 2018 question, but so far did not manage to write it up (and I feel somewhat guilty about that) - *but* got some a very nice by-products, as a consequence of him roping me back into quantum gravity:
- While we lack a completeness proof, we may now mostly know the full symmetry breaking structure of the SO(8)-gauged N=8 supergravity obtained by compactifying N=1 in D=10+1 to D=3+1 on the surface of an 8-ball ('7-spheres are tricky') - this is "the de Wit-Nicolai model".
- We also now have a good understanding of other cousins of this model, such as the "dyonic ISO(7)" gauging in D=4 obtained my compactifying mIIA SUGRA down from D=10.

SUGRA Symmetry Breaking

The Google Research Side

- At Google Research, our work has a heavy focus on ML and ML applications -
  but we also explore other things, such as non-ML data reduction/compression.
- Our perspective: much of academia is not yet fully aware of how the recent ML revolution brings new "hard-to-obtain" results within reach. *We at Google Research want to fix this*.
- This includes both ML per se but also creative use of tools and techniques developed in the context of ML research.
- During end-of-year "production freeze" in 2018: had a "Hackathon" week where we could propose projects for exploration and form cross-team groups around them. I proposed a "Supergravity with TensorFlow" project.

SUGRA Symmetry Breaking

The Google Research Side

- Wanted to know: "Is TensorFlow by now powerful enough to do the calculations that required tedious hand RM-AD in my 2008/2009 papers?"
- Answer back in 2018 was: "not quite so yet" - but knowing the calculation very well, I could work around problems and fill in the missing parts.
- During this Hackathon week, we found a new stable vacuum! N=8 SO(8)→N=1 SO(3)!
- At this point, it was clear that "we need to get this published". Took some extra effort beyond Hackathon week.
- Further work with academic collaborators: Deeper investigations into the symmetry breaking structure in various supergravities. (Non-Google collaborators: D. Berman, N. Bobev, F. F. Gautason, G. Inverso, K. Pilch.)

SUGRA Symmetry Breaking

The Problem

- N>1 SUGRA admits promoting part of the R-symmetry group to a local (gauge) symmetry.
- SUSY then forces us to include a potential for the scalar fields $\sim g^2$.
- In an isotropic "vacuum": Effectively a cosmological constant.
- QFT textbook lore: Unitarity mandates compactness of the gauge group.
  SUGRA: Loophole, due to vector kinetic term involving scalars ("non-renormalizability no worse than for SO(8)").
- Vacuum stability is a very subtle question!
  - EOM for Scalars: Potential (as a function of the scalar VeVs) needs to have a critical point.
  - Perturbative stability: For Potential<0, AdS geometry - localized finite-energy perturbations can be stable despite $m^2<0$ (→"BF Bound").
  - N>0 unbroken SUSY implies stability.
  - While e.g. SO(8) gauged N=8 SUGRA in D=4 is a consistent truncation,
    higher KK modes from N=1 D=11 SUGRA (for example) may induce instabilities.

SUGRA Symmetry Breaking

The Problem

- Vacuum stability - example
  - First broken-symmetry critical point for de Wit-Nicolai Supergravity: $SO(8) \to SO(3) \times SO(3)$.
  - "Drama" around stability: First considered unstable (but corresponding $SO(5) \to SO(3)$ solution of N=5 was known to be stable) - then, scalar mass spectrum showed BF-stability - but: general line of thought is that N=0 never should be stable. Indeed: Brane-Jet analysis (Pilch, Warner) and KK Spectroscopy (Malek, H.N., Samtleben) showed instability.
  - "Dyonic-ISO(7)": KK-stable N=0.
  - "Dyonic-ISO(7)" gauging has two critical points that sit *very* close together and do not saturate the BF-bound: P355983405 (N=1) and P355983403 (N=0). Stability of the N=0 solution?
    - Likely fiendishly hard to analyze, given $SO(7) \to Z_2 \times Z_2$ symmetry breaking.

SUGRA Symmetry Breaking

Structure of the Scalar Potentials of Maximal Supergravities

- Expressed in terms of Fermion Shifts A_1, A_2 (~"Yukawa"-type Gravitino-Higgs-Gravitino and Gravitino-Higgs-Fermion interaction terms): $P/g^2 \sim -(\#)A_1A_1+(\#)A_2A_2$.
- Maximal SUGRA: Scalar manifolds are coset manifolds, typically $E_{d(d)}/K(E_{d(d)})$.
- Modern understanding of gaugeability:
  - Determined by gauge group embedding tensor Θ; "spurionic" quantity.
  - SUSY imposes linear constraints on the E_d representation content of Θ.
  - Θ embeds gauge group generators into E_d; closure of gauge lie algebra imposes quadratic constraints.
  - A_1, A_2: Irreps of $K(E_d)$, extracted from "T-tensor", dressed-up (with "Vielbein") Θ-tensor.
  - Can always move any critical point on the scalar manifold to origin by adjusting ("E_d-boosting") Θ (dall'Agata, Inverso - "GTTO").

SUGRA Symmetry Breaking

Structure of the Scalar Potentials of Maximal Supergravities

- "Theta-tensor" formalism was first developed for maximal D=2+1 SUGRA (32 supercharges). (H.N., H. Samtleben)
  - There only: scalar/vector duality allows dualizing away unwanted vector fields, allowing much freedom in gauge group choice.
  - Of the maximal subgroups of global $E_{8(8)}$ symmetry, 13 are admissible gauge groups, including e.g.: $G_{2(-14)}$x$F_{4(-20)}$.
- "Theta-tensor" approach nicely systematizes analysis of various gaugings, such as in D=4 and D=5. (B. de Wit, H. Samtleben, M. Trigiante)
- Finding critical points - computational complication: We have a choice between two approaches, both hard:
  - Coordinate-parametrizing high-dimensional coset manifolds $E_d/K(E_d)$ - or:
  - Directly solving an algebraic equation system of quadratic constraints on Θ.

SUGRA Symmetry Breaking

Structure of the Scalar Potentials of Maximal Supergravities

- Finding "all" critical points - computational complication: We have a choice between two approaches, both hard:
  - Coordinate-parametrizing high-dimensional coset manifolds $E_d/K(E_d)$ - or:
  - Directly solving an algebraic equation system of quadratic constraints on Θ.
- In D=4: 70-dimensional scalar manifold M_{70}:=$E_{7(7)}/(SU(8)/Z_2)$. Theta-tensor in **912**-irrep of $E_{7(7)}$.
- Analytic coordinate-parametrization for M_{70} (such as: via "Euler angles") "beyond reach".
- In D=4, scalar potential then is quadratic in 3rd order polynomials of the entries of the "Vielbein" representing a point on M_{70}.
- Both routes admit group theoretic simplification: imposing "must retain symmetry S" constraints can reduce the number of parameters drastically - but at the expense of limiting our view.

SUGRA Symmetry Breaking

Structure of the Scalar Potentials of Maximal Supergravities

- 1982/1983: Analytic treatment of N=8 de Wit-Nicolai SUGRA by N. Warner:
  All critical points with $SO(8) \to G \supseteq SU(3)$: SO(8) N=8, $SO(7)^+$, $SO(7)^-$ (2x), G_2 N=1, SU(3)xU(1) N=2, SU(4).
  - SU(3)-invariant scalars: 6-parameter problem.
  - Nice simplifications available due to (anti)-self-dual 4-forms being equivalent to symmetric traceless matrices over spinors/co-spinors; can use SO(8) to diagonalize one.
  - Solution to 6-parameter problem must generalize to 70-dimensional scalar manifold.
- For "N=16" in D=2+1 with SO(8)xSO(8) gauging:
  - 128-dimensional scalar manifold.
  - Looking for $SO(8)_{diag} \to SU(3)$ scalars: 12-parameter problem.
  - In 2001, could not parametrize full 12d submanifold - but could check whether candidates generalize to 128-parameter setting.

SUGRA Symmetry Breaking

Structure of the Scalar Potentials of Maximal Supergravities

- In 2001/2002, used essentially the same methods as N. Warner for analyzing D=3.
- Found solutions corresponding to the known D=4 solutions for D=3.
- "Deep Analysis" considered out of reach.
- Two techniques I added to my toolbox while doing Engineering Research in Southampton:
  - Reverse Mode Automatic Differentiation
    - Back then, very relevant for engineering design optimization.
    - Think "jet engine geometry performance optimization with 300 parameters".
    - Nowadays: also relevant as key idea enabling the recent "Deep Learning" revolution in ML.
  - Identifying algebraic numbers from high precision numerics via PSLQ.
    - Found this by chance in a PhD thesis on chaotic dynamics.

SUGRA Symmetry Breaking

First steps towards deep analysis

- Even in 2002, had to resort to hybrid analytic/numerical techniques: Using numerics to discover an approximate solution, forming hypotheses about what parameters might be set to zero, analytically proving these hypotheses.
- RM-AD allows us to side-step analytic complications: "Collapsing" too-complicated-to-handle analytic expressions to floating point numbers makes it possible to numerically scan on full 70d coset manifold in D=4.
- Implementing RM-AD on top of high precision arithmetics allows making algebraic conjectures.
  (Note: Since exceptional groups and criticality constraints can be expressed in terms of intersecting varieties [with integer coefficients], all "physically interesting" quantities are algebraic numbers.)
- 2009: First new critical points for D=4 beyond N. Warner's list from the 1980s.
- Tedious coding, did not encourage exploration and tuning the search heuristic.

SUGRA Symmetry Breaking

SUGRA Symmetry Breaking as a Toy Problem

- Essentially algebraic in nature.
- Much of it can be explained to non-physicists
  (at the level of "what I need to know to understand how the calculation works").
- Highly nontrivial.
- Generally (as an optimization problem) nevertheless mostly non-malicious.
- Encourages development of generic techniques (algorithmics/numerics - including heuristics).
- Bit like "Rosenbrock's Banana function" (as a numerical optimizer performance test problem),
  but with more interesting structure.

- So: Always good to come back to, for many reasons.

SUGRA Symmetry Breaking

Further progress towards deep analysis

- Left a permanent academic position behind in 2012 - basically realized how unfixable some problems are.
- Context: Since 1997, was doing quite a bit of internet consultancy work on the side.
  Next most interesting thing on my list was non-academic: joined Google.
- 2012-2017 dominated by: work at Google (incl. teaching ML, writing patents, reviewing grant applications), also: Family. "Got very little sleep - and didn't read hep-th during that time."
- 2018: Started "abusing" ML tech to do SUGRA: RM-AD + accelerated linear algebra on GPU.
- (Some on-going internal nudging at Google to make TensorFlow also handle our physics use cases well.)
- 2022: Evolved a collection of tools and techniques to effectively analyze some M theory related questions with ML machinery.

SUGRA Symmetry Breaking

Current and Future Technology

- Imposing residual symmetry requirements is still an option.
- Problem generally is algebraic in nature:
    - In some low-dimensional cases, homotopy continuation methods (e.g.: Bertini2) and algebraic geometry based computer algebra (e.g.: Singular) are powerful enough to perform exhaustive analysis.
- Exploring "all" local minima of a D<=1000 scalar function is a generically interesting problem.
    - "Neighborhood analysis" techniques - originally developed for answering a question about maximal D=5 SUGRA.
    - Currently in the making: "multi-armed bandit neighborhood exploration".
    - Morse theory based "checksums"?

SUGRA Symmetry Breaking

Work in Progress

- TensorFlow based numerical all-gaugings scans for D=5 and (later) D=4.
- In D=5: Θ belongs to **351**-irrep of $E_{6(6)}$, need to solve quadratic gauge group closure constraints, then stationarity (independently or simultaneously), plus perhaps unbroken SUSY constraint.
- Overall quite doable, avoids some complications that show up in D=4: physically inequivalent ways to embed SO(8) (ω-deformation [dall'Agata, Inverso]) - so, when we find a critical point, we often can 'deform' it in a way that changes the physics (such as: mass spectra).
- While D=4 is conceptually trickier, it still looks feasible.

SUGRA Symmetry Breaking

How to think about it?

- Wherever QFT research takes us, these observations are clear:
  - QFT sometimes shows little respect for the limitations of "human brain hardware".
  - One class of interesting problems:
    Well-defined, equivalent to many-parameter optimization problems.
  - We should have well-and-widely-understood techniques to handle those.
  - Overall, getting SUGRA potentials under control is merely a stepping stone
    for handling more ambitious (and computationally challenging) problems.
  - We still barely started exploring the implications of supersymmetry!

SUGRA Symmetry Breaking

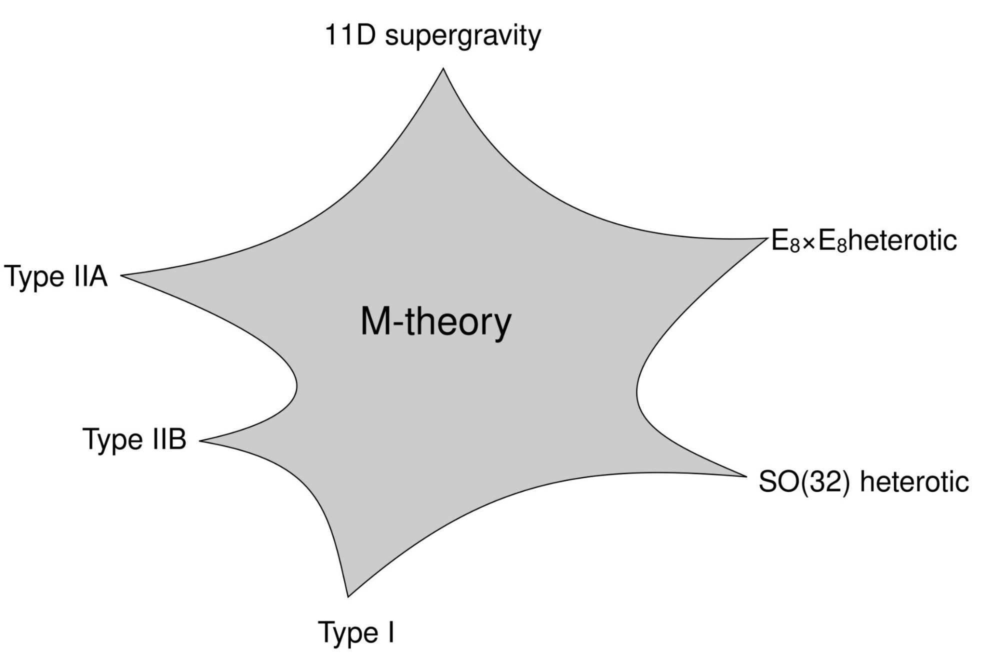
On Effort

- 1980s: ~6 parameters, ~weeks
- 2001: ~10 parameters, ~months
- 2008: ~200 parameters, ~a week
- 2020: ~1000 parameters, ~1 afternoon

(Right: D=2 SO(9) potential - Bossard, Ciceri, Inverso, Kleinschmidt) - 200 LoC TensorFlow code, ~3h.

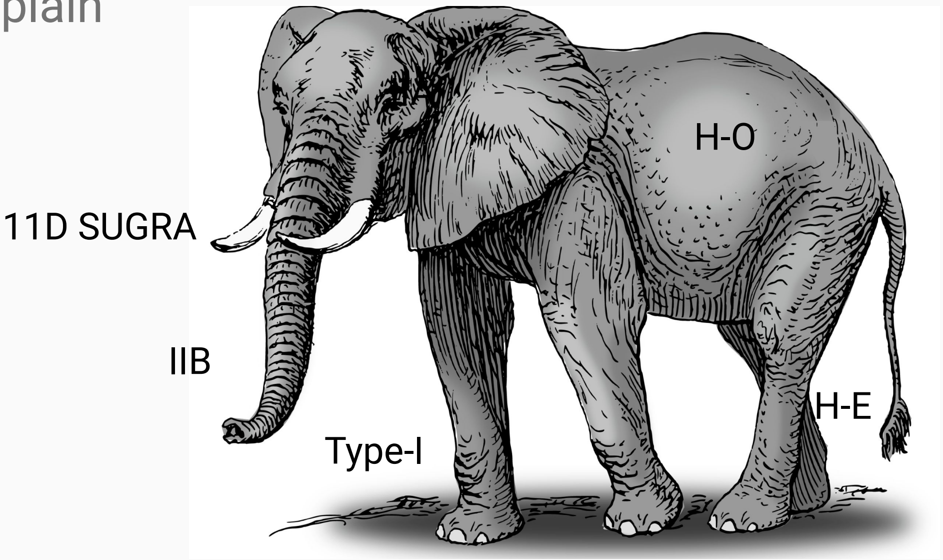
$$\begin{aligned} V_{\rm pot} = \frac{\mathsf{g}^2 e^{2\sigma}}{2\varrho^3}\delta_{IJ}\delta_{KL}\Bigg(& \left(2\mathsf{m}^{IK}\mathsf{m}^{JL} - \mathsf{m}^{IJ}\mathsf{m}^{KL}\right) + \frac{1}{2}\varrho^{-2/3}\left(a^{IPQ}a^{KRS}\mathsf{m}^{JL}\mathsf{m}_{PR}\mathsf{m}_{QS} - 2a^{IKP}a^{JLQ}\mathsf{m}_{PQ}\right) \\ & + 2\varrho^{-2}h^I{}_P h^K{}_Q \mathsf{m}^{Q[P}\mathsf{m}^{J]L} + \varrho^{-8/3}a^{IPR}h^J{}_P a^{KQS}h^L{}_Q\mathsf{m}_{RS} \\ & + \frac{\varrho^{-2}}{72}h^J{}_P a^{KQ_1Q_2}a^{LQ_3Q_4}a^{Q_5Q_6Q_7}\varepsilon_{Q_1\ldots Q_9}\mathsf{m}^{IQ_8}\mathsf{m}^{PQ_9} \\ & + \frac{3}{8}\varrho^{-4/3}a^{I[M_1M_2}a^{M_3M_4]J}a^{K[N_1N_2}a^{N_3N_4]L}\mathsf{m}_{M_1N_1}\mathsf{m}_{M_2N_2}\mathsf{m}_{M_3N_3}\mathsf{m}_{M_4N_4} \\ & + \frac{\varrho^{-2}}{2\cdot 144^2}a^{IN_1N_2}a^{JN_3N_4}a^{N_5N_6N_7}\varepsilon_{N_1\ldots N_9}a^{KP_1P_2}a^{LP_3P_4}a^{P_5P_6P_7}\varepsilon_{P_1\ldots P_9}\mathsf{m}^{N_8P_8}\mathsf{m}^{N_9P_9} \\ & + \frac{\varrho^{-8/3}}{576}a^{IRP}h^J{}_R a^{KN_1N_2}a^{LN_3N_4}a^{N_5N_6N_7}a^{N_8N_9Q}\varepsilon_{N_1\ldots N_9}\mathsf{m}_{PQ} \\ & + \frac{\varrho^{-8/3}}{1152^2}a^{IN_1N_2}a^{JN_3N_4}a^{N_5N_6N_7}a^{N_8N_9Q}\varepsilon_{N_1\ldots N_9}a^{KP_1P_2}a^{LP_3P_4}a^{P_5P_6P_7}a^{P_8P_9S}\varepsilon_{P_1\ldots P_9}\mathsf{m}_{QS}\Bigg)\,. \end{aligned} \tag{19}$$

M theory - The Current Picture



SUGRA Symmetry Breaking

We sometimes use this metaphor to explain the overall current situation

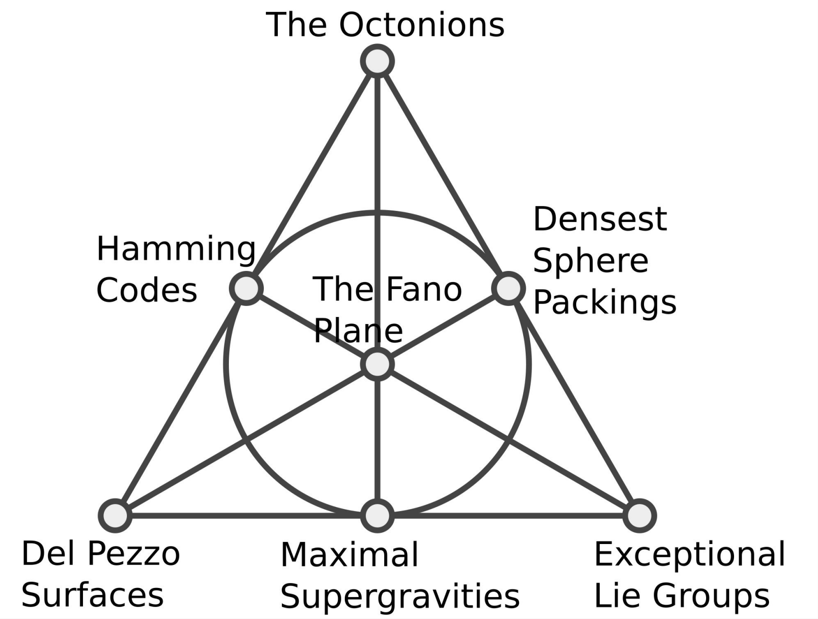


M theory Surprises

Would want to offer another such metaphor.

In the study of M Theory, one keeps running into the same "otherwise unusual" mathematical structures - in particular those shown in this (purely artistic - no deeper meaning) diagram.

(Mostly, the relations between these objects are accessible and understood - but some more so than others.)



SUGRA Symmetry Breaking

Two illustrations of what I mean with this diagram:

- Superpotential for D=4 SO(8) supergravity on SL(2)^7: Terms align with (7,4,3) Hamming code. Likewise, D=3: SO(8)xSO(8) supergravity on SL(2)^8: Terms align with (8,4,4) Hamming code.

$$\begin{aligned} \mathcal{W}_{\mathbb{Z}_2^3} = {} & \zeta_1\zeta_2\zeta_3\zeta_4\zeta_5\zeta_6\zeta_7 \\ & + \zeta_1\zeta_2\zeta_3\zeta_7 + \zeta_1\zeta_2\zeta_5\zeta_6 + \zeta_1\zeta_3\zeta_4\zeta_5 + \zeta_1\zeta_4\zeta_6\zeta_7 + \zeta_2\zeta_3\zeta_4\zeta_6 + \zeta_2\zeta_4\zeta_5\zeta_7 + \zeta_3\zeta_5\zeta_6\zeta_7 \\ & + \zeta_1\zeta_2\zeta_4 + \zeta_1\zeta_3\zeta_6 + \zeta_1\zeta_5\zeta_7 + \zeta_2\zeta_6\zeta_7 + \zeta_2\zeta_3\zeta_5 + \zeta_3\zeta_4\zeta_7 + \zeta_4\zeta_5\zeta_6 + 1\,, \end{aligned}$$

- M theory U-duality ~ Symmetries of del Pezzo surfaces
("A mysterious duality" - Iqbal, Neitzke, Vafa, hep-th/0111068)

SUGRA Symmetry Breaking

- We probably all have some puzzle pieces lying around where it is not quite clear yet *how they fit*.
- *A currently under-appreciated important puzzle piece might be the maximal D=3 models.*
- As Hermann likes to put it, "progress often came from a deeper understanding of an important symmetry".
- Relatively speaking, "E_8 is much smaller nowadays than it was 40 years ago" - thanks to major advances in computing.
- Many things have rapidly come well within reach of computational methods. We now can answer many questions that we could not discuss 10 years ago.

Lessons Learned So Far

- There is no "one established way to do physics": Explain the same theory to four different people who use very different approaches and get very different (complementary) insights.
- Giving people the space to tackle hard problems their own way is important.
- If a problem is tricky, "am I using the right language to think about it?" often is a useful guiding question.
- When Hermann says that a result is "computationally out of reach",
chances are it might be an interesting challenge.

To Conclude

Happy Birthday, Hermann

and thanks for many inspirational challenges