Baryogenesis and Magnetogenesis: Two sides of the same coin?

Parity violation and broken conformal invariance

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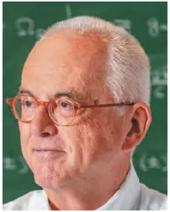
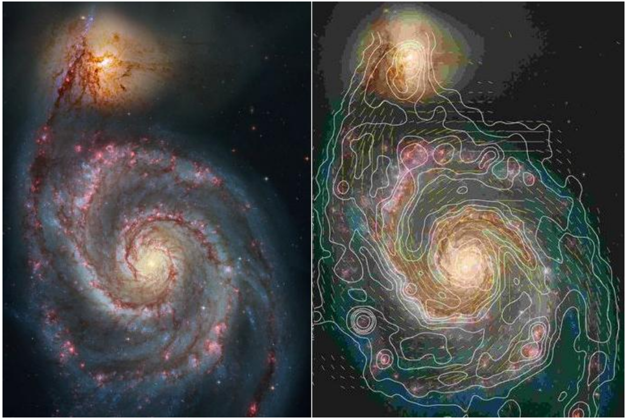
With: Ashu Kushwaha 2008.10825, 2103.05339



HermannFest @ AEI 13 - 16 September 2022

Two unresolved problems

Cosmological magnetic fields

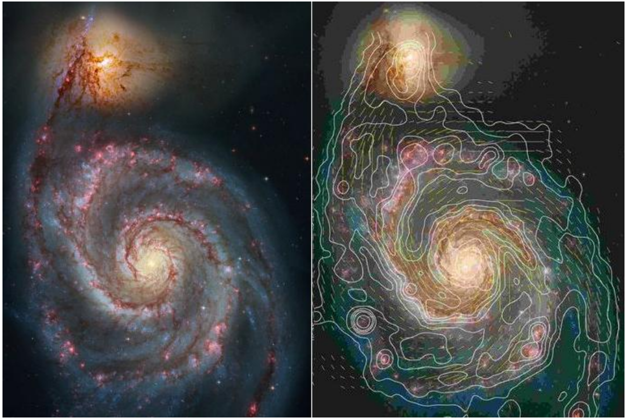


M51 Galaxy showing large scale Magnetic fields Credit: Max Planck Institute, Bonn

$$1\ G = 10^{-4}\ T$$

- large scale magnetic fields $10^{-6} - 10^{-8} G$ from kpc to Mpc

Cosmological magnetic fields



M51 Galaxy showing large scale Magnetic fields Credit: Max Planck Institute, Bonn

- large scale magnetic fields $10^{-6} - 10^{-8} G$ from kpc to Mpc
- Magnetic field energy for the above strengths is

$$\rho_B \sim 10^{-12} - 10^{-15} \, ergcm^{-3}$$

is comparable to CMB photon energy density in the current epoch: $\rho_\gamma \sim 10^{-13} \, ergcm^{-3}$

Cosmological magnetic fields

One LHC magnet	10^5 G
The Earth	10^{-1} G
Stars	$1 - 10^{15}$ G
Molecular cloud	10^{-3} G
Interstellar medium	10^{-6} G
Cluster of galaxies	$10^{-7} - 10^{-6}$ G
Voids	$\geqslant 10^{-16}$ G

Stronger field

Larger scale

Voids

Voids are empty regions! If the voids contain magnetic fields, the origin of these fields can only be from Early-Universe!

Typical constraints

Astrophysical/ Cosmological objects

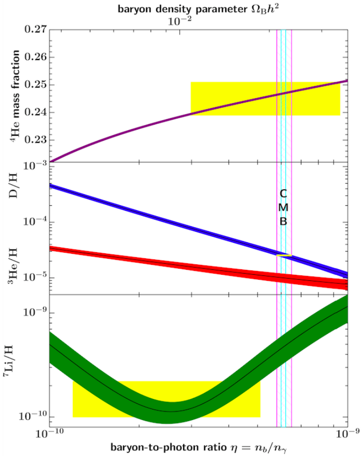
Object	Typical strength	Typical coherence scale
Galaxy	10^{-6} G	$1 - 10$ kpc
Clusters	$10^{-7} - 10^{-6}$ G	$10 - 1000$ kpc

Non-detected magnetic fields

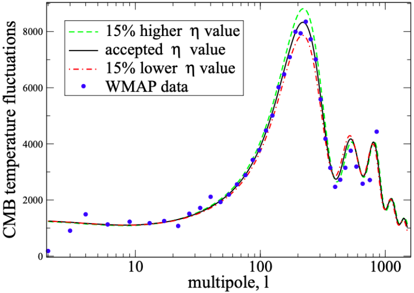
Constraints	Typical strength	Typical coherence scale
IGM	$< 10^{-8} - 10^{-11}$ G	$1 - 50$ Mpc
CMB	$< 10^{-8} - 10^{-9}$ G	1 Mpc
BBN	$< 10^{-6}$ G	> 10 Mpc

What is the origin of large scale magnetic fields?

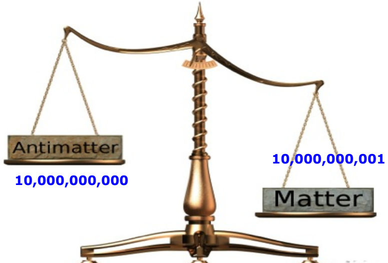
Matter-Antimatter asymmetry (Baryon Asymmetry Parameter)



Particle Data Group (2019)



Cline (2006)



$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$

Sakharov's conditions 1967

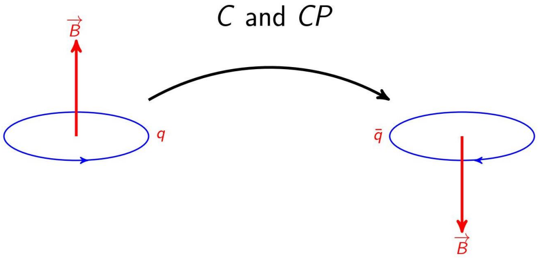
Sakharov proposed three necessary conditions for creating the baryon asymmetry:

- Baryon number violation
- Charge (C) and charge parity (CP) violation
- Departure from thermal equilibrium

Origin of the baryon asymmetry ($\eta_B \sim 10^{-10}$) is still an unresolved problem in particle physics.

In 1996, Davidson pointed out an interesting relation between the primordial magnetic fields and Sakharov's conditions. Davidson's conditions are:

- Since B is odd under C and CP, the presence of magnetic field will lead to CP violation
- Magnetic field chooses a direction $\Longrightarrow$ breaks isotropy SO(3).
- There should be some out-of-thermal-equilibrium dynamics.



Presence of generic magnetic fields satisfy only two of Sakharov's conditions
There is a key missing ingredient.

Helical magnetic fields

Helical and non-helical fields

- For massless particle, helicity is the projection of the direction of spin (clockwise or anti-clockwise) along the direction of propagation. $+1(-1)$ denote right (left) handed helicity modes.
- Electromagnetic field has two transverse degrees of freedom which can be associated with left circular and right circular polarization.
- Same propagation (**speed or dispersion relation**) of both polarization modes lead to non-helical, and differently propagating modes lead to helical fields.
- If both the polarization modes propagate differently $\rightarrow$ Helicity imbalance

How to create helicity imbalance?

Helical magnetic fields

- Lorentz force, $\vec{F} = m\frac{d\vec{v}}{dt} = \vec{E} + \vec{v} \times \vec{B}$ implies that under parity transformation (changing the sign of coordinate system): $\vec{E} \longrightarrow -\vec{E}$, $\vec{B} \longrightarrow \vec{B}$.
- Because standard EM action, $F_{\mu\nu}F^{\mu\nu} \propto B^2 - E^2$, is quadratic in $\vec{E}$ and $\vec{B}$, it is invariant under parity symmetry.
- $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\,\vec{E} \cdot \vec{B}$ is parity non-invariant, where (the dual) $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$.

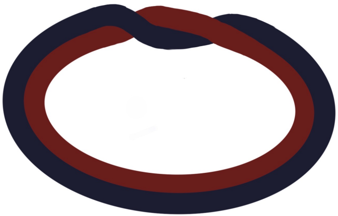
$F_{\mu\nu}\tilde{F}^{\mu\nu}$ can create Helicity imbalance

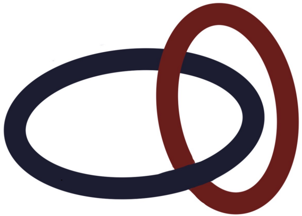
- Generating cosmological magnetic fields require broken conformal invariance without breaking gauge invariance!

Need to break CI

Magnetic helicity in early Universe

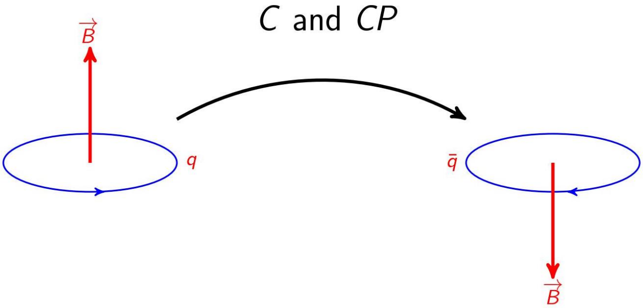
- Magnetic helicity ($\mathcal{H}_M$) is defined as: $\int d^3x \overrightarrow{A} \cdot \overrightarrow{B}$ and $\overrightarrow{B} \cdot \overrightarrow{\nabla} \times \overrightarrow{B}$.
- Magnetic helicity is a conserved quantity that describes field topology.
- It measures the twist (self-helicity) and linkage (mutual helicity) of magnetic field lines.
- Helical structures are observed in the magnetic fields of solar active regions!
- Primordial $\mathcal{H}_M$ provides direct indication of parity violation CP violation Vachaspati '01.





Missing link between Sakharov and Davidson's conditions

Davidson conditions: Broken symmetries in the presence of magnetic field



Davidson's conditions :

- There should be some out-of-thermal-equilibrium dynamics
- Breaks C, CP and $SO(3)$

Presence of non-helical magnetic fields satisfy only two Sakharov conditions

Missing ingredient: Helical magnetic fields

Kushwaha & SS '21

- Chiral anomaly is [Barrie & Kobakhidze, '14]

$$\nabla_\mu J^\mu_A = -\frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma} + \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \tag{1}$$

  where J^μ_A is the chiral current.

- **First term on RHS vanishes (up to I order) in flat FRW universe**. Chern-Simons gravity can lead to gravitational birefringence and chiral current. Alexander et al. (2006)

- In the presence of helical magnetic fields, net chiral current is non-zero.

- Baryon number density $n_B = n_b - n_{\bar{b}} = a(\eta)\langle 0|J_A^0|0\rangle = \frac{e^2}{4\pi^2} a(\eta) n_{CS}$, where Chern Simon number density is

$$n_{CS} = \frac{1}{a^4} \int_{\mu}^{\Lambda} \frac{dk}{k} \frac{k^4}{2\pi^2} \left(|A_+|^2 - |A_-|^2 \right) \tag{2}$$

- For **non-helical fields** $|A_+| = |A_-| \implies n_{CS} = 0$.
- For **helical fields**, $n_{CS} \neq 0$ implies an imbalance between baryons and anti-baryons $\implies$ Baryon number violation

Presence of helical magnetic fields can lead to non-zero n_{CS}.

- Davidson's conditions with Helical fields can provide a route for Baryon number violation.

Helical magnetic fields from Riemann coupling

Model

- **Necessary condition :** Conformal invariance breaking + parity violation

$$S = \overbrace{-\frac{M_{\mathrm{P}}^2}{2}\int d^4x\sqrt{-g}\,R}^{\text{Einstein-Hilbert term}} + \overbrace{\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]}^{\text{Inflaton}}$$
$$-\frac{1}{4}\int d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu} - \underbrace{\frac{1}{M^2}\int d^4x\,\sqrt{-g}\,R_{\mu\nu}{}^{\alpha\beta}F_{\alpha\beta}\tilde{F}^{\mu\nu}}_{\text{Conformal breaking}} \qquad (3)$$

where M is the energy scale, which sets the scale for the breaking of conformal invariance.

- Non-minimal coupling to the Riemann tensor generates sufficient primordial helical magnetic fields at **all observable scales.**

▸ Need to break CI

Evolution equations

- Consider Flat FRW universe : $ds^2 = a^2(\eta)\,(d\eta^2 - \delta_{ij}dx^i dx^j)$.

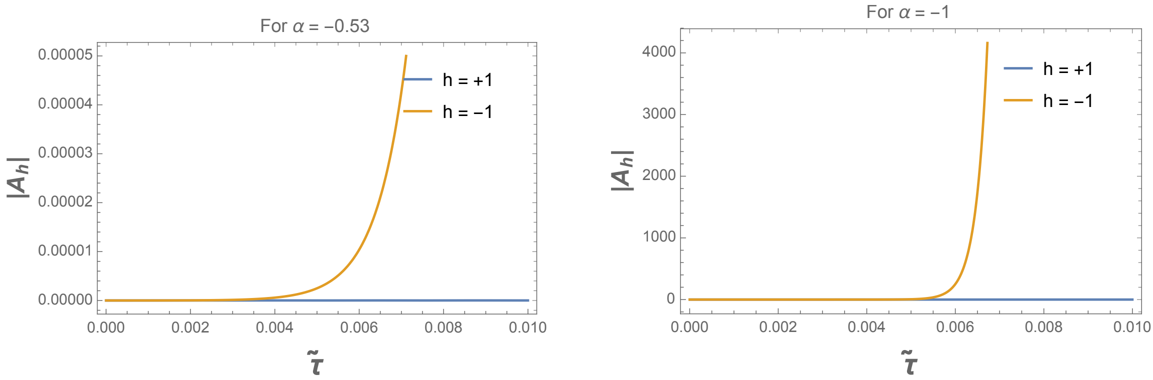
- In the Coulomb gauge ($A^0 = 0, \partial_i A^i = 0$), the EOM in helicity basis is:

$$A_h'' + \left[k^2 - \frac{4kh}{M^2}\,\Gamma(\eta)\right] A_h = 0 \qquad \Gamma(\eta) = \frac{a'''}{a^3} - 3\frac{a''a'}{a^4} = \frac{1}{a^2}\left(\mathcal{H}'' - 2\mathcal{H}^3\right)$$

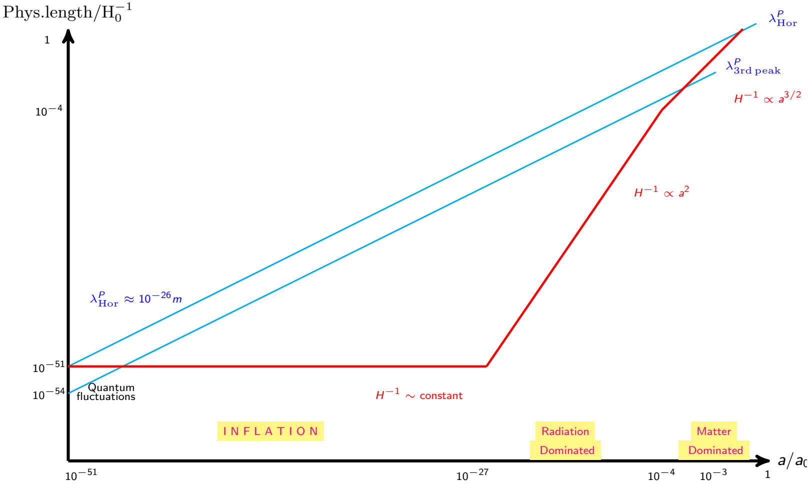
- Γ vanishes for exact de-sitter case! Non-zero for power-law/slow-roll inflation.

Evolution on two helicity modes

Taking $\mathcal{H} \sim {\eta_0}^{-1} \sim 10^{14}\mathrm{GeV}$, and $M \sim 10^{17}\mathrm{GeV}$, α related to power-law inflation, gives



Only one helicity mode survives, leading to non-zero helicity!



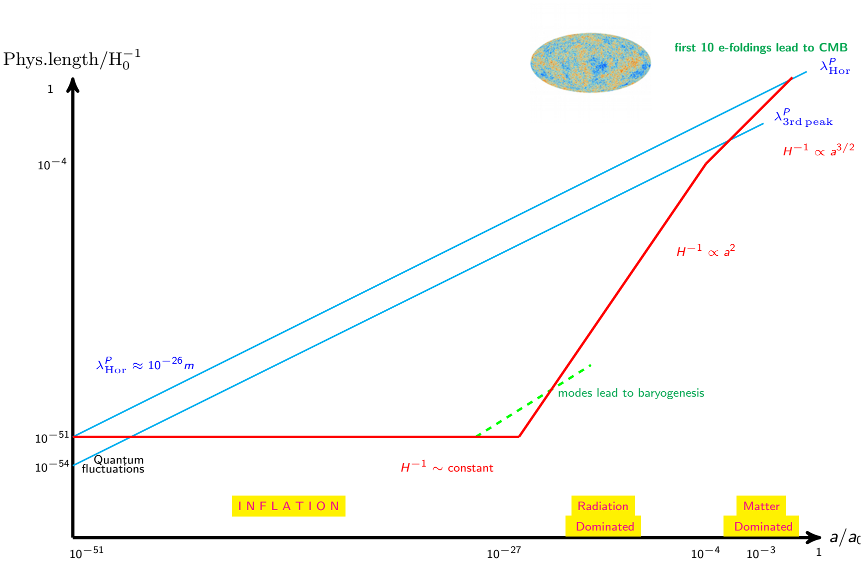
Helical magnetic fields that re-entered the horizon at two different epochs:

$$B|_{50\ \rm MPc} \sim 10^{-18}\ G\ (z \sim 20)\ ;\ \ B|_{1\ \rm MPc} \sim 10^{-15}\ G\ \text{(present day)}$$

Baryogenesis from helical magnetic fields

Baryogenesis from helical magnetic fields

Kushwaha & SS '21



Baryon asymmetry parameter

- The modes that re-enter very early during the radiation-dominated epoch are responsible for the generation of baryon asymmetry.
- Therefore we consider the modes which left the horizon around 5 to 10 e-foldings, Chern-Simon number density is

$$n_{CS} = \frac{1}{2\pi^2\, a^4(\eta)} \int_\mu^\Lambda dk \left(|C|^2\, k^{3+\frac{1}{2\alpha}} + \left| C_2 \frac{\mathcal{F}^{-1}}{\pi} \Gamma\left(\frac{1}{2\alpha}\right) \right|^2 k^{3-\frac{1}{2\alpha}} \tau^{-\frac{2}{\alpha}} \right). \tag{4}$$

- Since entropy density per comoving volume is conserved, the quantity n_B/s is better suited for theoretical calculations.
- Assuming that there was no significant entropy production after reheating phase, entropy density in the radiation-dominated epoch is:

$$s \simeq \frac{2\pi^2}{45} g\, T_{\rm RH}^3 \tag{5}$$

- Baryon asymmetry parameter

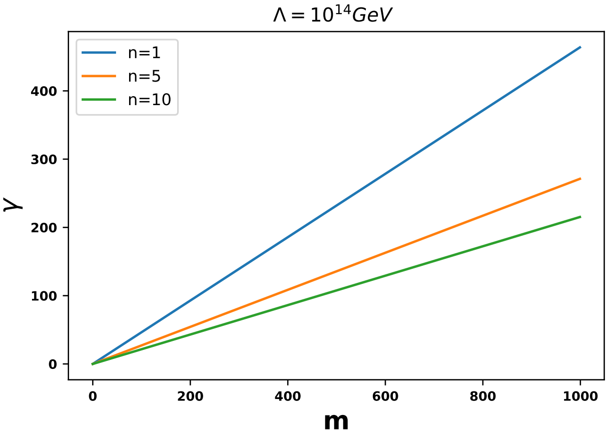
$$\eta_B = \frac{n_B}{s} \approx 10^{-2}\left(\frac{M}{M_P}\right)^3\left(\frac{\Lambda}{T_{\mathrm{RH}}}\right)^3 \tag{6}$$

- Using the parametrization

$$\eta_B = n \times 10^{-10},\ M = m \times 10^{14}\,GeV,\ \Lambda = \delta \times 10^{12}\,GeV,\ T_{RH} = \gamma \times 10^{12}\,GeV \tag{7}$$

equation for baryon asymmetry parameter becomes:

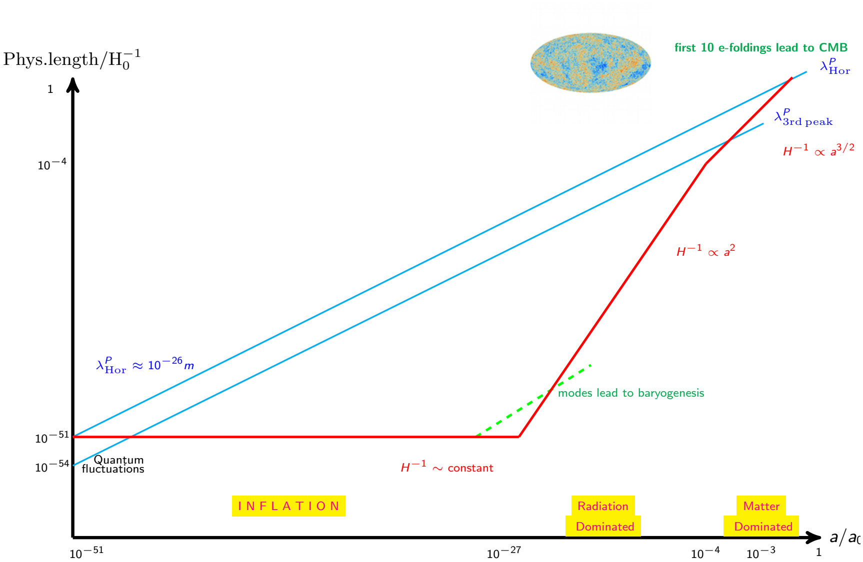
$$\boxed{\frac{m^3 \times \delta^3}{\gamma^3} \approx n\,10^7} \tag{8}$$



- For a range of values of γ, δ, and m, BAU can have values between 10^{-10} to 10^{-9}.
- The analysis shows that $M \sim 10^{17}\,GeV$ is consistent with baryogenesis.

Baryogenesis from helical magnetic fields

Kushwaha & SS '21



Conclusion and Future work

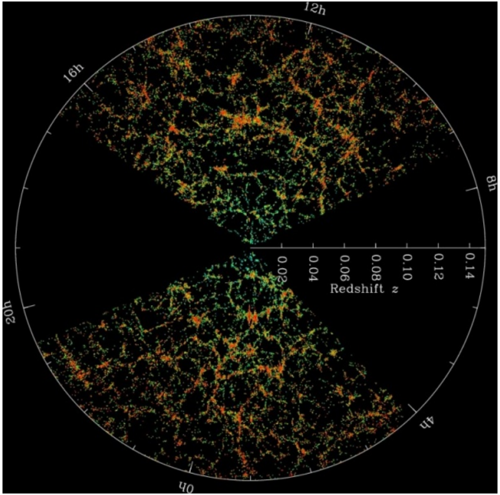
- Our model does not require the coupling of the electromagnetic field with the scalar field. Hence, there are no extra degrees of freedom and will not lead to a strong-coupling problem.
- Since the curvature is large in the early Universe, the coupling term will introduce non-trivial corrections to the electromagnetic action.
- We have explicitly shown that Davidson's conditions are necessary but not sufficient. The key missing ingredient is the requirement of helical magnetic fields.
- The BAU parameter predicted by our model is independent of any specific inflation model and reheating dynamics; however, it depends on the scale at which inflation ends and reheating temperature.
- Currently, we are studying the effects on the asymmetry generated **leptons**.

Happy birthday Hermann

Backup slides

Voids and Magnetic fields

Return

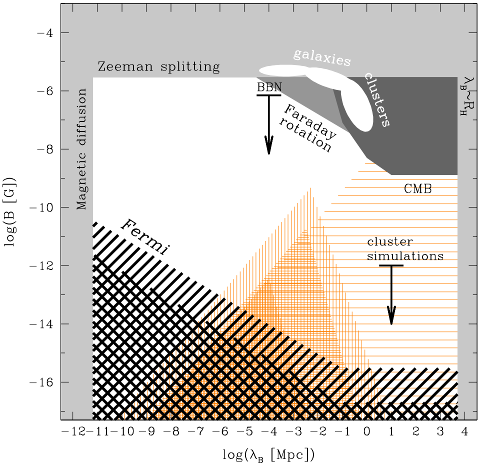


SDSS (2014)

- Zoomed view of the Galaxies and Voids $(z \sim 1 \Longrightarrow \sim 1Gpc)$
- Dark patches are empty spaces (voids) contain H, He and rare dust particles!
- Since gravity attracts, clusters and superclusters grow over time. Thus voids become more and more empty over the time!
- H, He and ions are remnant of Early-Universe.

Voids and Magnetic fields

Return



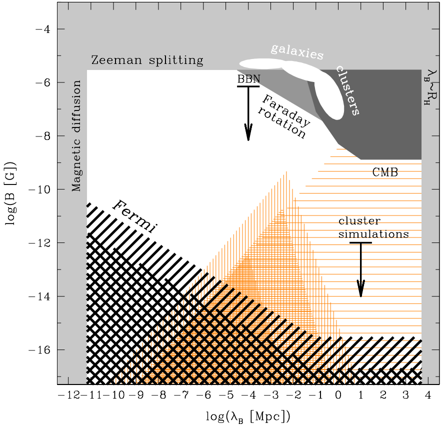
Neronov & Vovk (2014)

Lower bound in voids $\sim 10^{-6}$ nG

- Observations of Blazars show an interesting feature TeV (γ-rays) are received but GeV not received.
- TeV photons pass through voids before reaching us
- Create e^-, e^+ pairs when interacting with CMB photons.
- Pairs will travel in the same direction and should produce an observable electromagnetic cascade.

Voids and Magnetic fields

Return



Neronov & Vovk (2014)

Lower bound in voids $\sim 10^{-6}$ nG

- What is the physical mechanism for the origin of these fields?
- Voids are empty regions!
- Magnetic fields generated in late Universe are generated in proto-galaxies and spilled to IGM. Can not generate magnetic fields in voids.
- If the voids contain magnetic fields, the origin of these fields can only be from Early-Universe!

How to generate magnetic fields ?

Problem with magnetic field generation during inflation

Return

- EM action for an arbitrary 4-D metric

$$S_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}\,g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and A^μ is electromagnetic four vector.

- Under conformal transformation $\tilde{g}_{\mu\nu} = \omega^2(x)g_{\mu\nu}$

$$\tilde{S}_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} = S_{\mathrm{EM}}$$

- EM action is conformally invariant, and hence equations of motion for magnetic fields

- Flat FRW line element:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right] \tag{9}$$

with $dt = a(\eta)d\eta$ is

$$ds^2 = \underbrace{a^2(\eta)[d\eta^2 - dx^2 - dy^2 - dz^2]}_{\text{conformally flat}} \tag{10}$$

- FRW models are conformally flat: $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$
- EM action in conformal FRW metric is

$$\mathcal{S} = -\frac{1}{4}\int d^4x\eta^{\alpha\mu}\eta^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}$$

Which is same as in Minkowski space-time.

- Hence in conformally flat FRW background, $B \sim \frac{1}{a^2}$
- for inflation $a(t) = e^{Ht}$, so after the end of inflation (for 60 e-foldings), $B \sim e^{-120}$

We need to break the conformal invariance of EM action

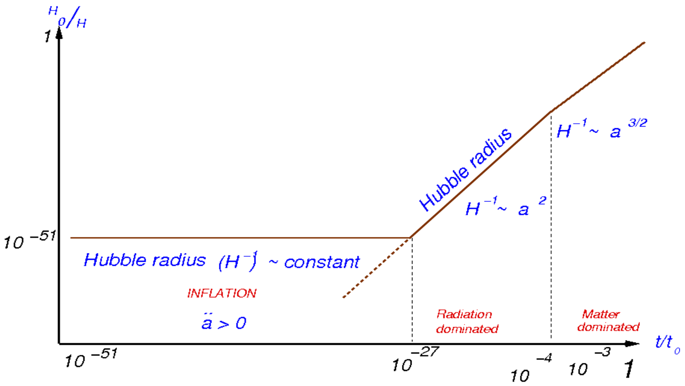
Perturbation theory perspective

- Consider a small fluctuations on top of FRW background:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu} \qquad \delta g_{\mu\nu} = \delta g^{(S)}_{\mu\nu} + \delta g^{(V)}_{\mu\nu} + \delta g^{(T)}_{\mu\nu} \qquad \left|\frac{\delta g_{\mu\nu}}{g^{(0)}_{\mu\nu}}\right| \ll 1$$

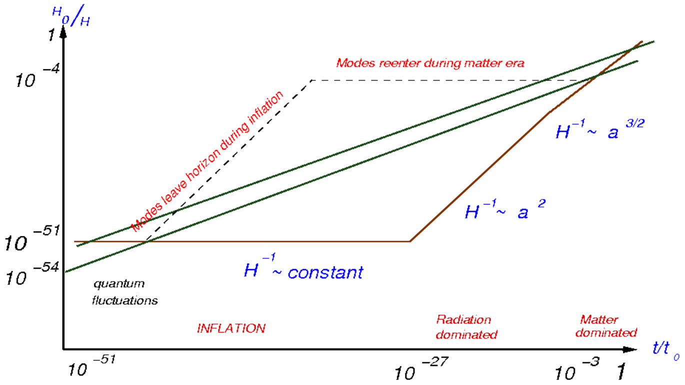
- Scalar, Vector and Tensor perturbations decouple; evolve independently
- Einstein's equations become $$G^{(0)}_{\mu\nu} + \delta G_{\mu\nu} = \frac{1}{M^2_{\mathrm{Pl}}}\left[T^{(0)}_{\mu\nu} + \delta T_{\mu\nu}\right]$$
- Scalar (Density) and Tensor (gravitational waves) perturbations grow
- Vector perturbations decay; can grow in closed universe

Inflation can not generate B-fields



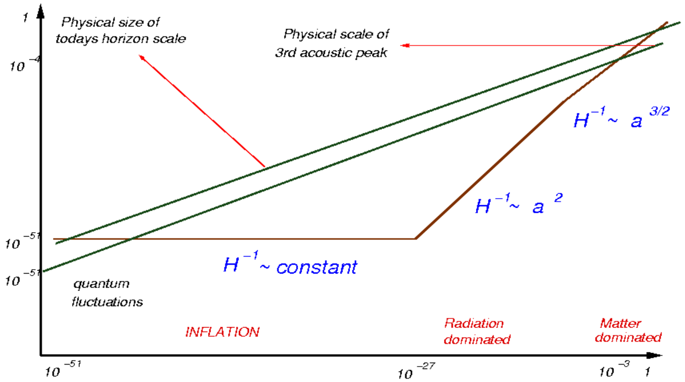
- Simple case $a(t) \propto e^{Ht}$ (de Sitter)
- $H^{-1} \simeq$ constant during inflation; increases in standard cosmology
- Energy scale of inflation $\sim 10^{14}\mathrm{GeV}$

Inflation can not generate B-fields



- Quantum fluctuations are exponentially stretched exponentially
- Causal connection to the past

Inflation can not generate B-fields



Perturbations generated

1. Scalar perturbations — density perturbations [Starobinsky '81, Guth and Pi '82]
2. Gravitational fluctuations — Free gravitational waves [Grischuck '75]
3. No Vector perturbations

Conformal transformation

$$\tilde{g}_{\mu\nu} = \omega^2(x) g_{\mu\nu} \quad \Longrightarrow \quad \tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + C^{\lambda}_{\mu\nu} \tag{11}$$

where $C^{\lambda}_{\mu\nu} = \omega^{-1} \left(\delta^{\lambda}_{\mu} \nabla_{\nu} \omega + \delta^{\lambda}_{\nu} \nabla_{\mu} \omega - g_{\mu\nu} g^{\rho\lambda} \nabla_{\rho} \omega \right)$

$$F_{\mu\nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} = \partial_{\mu} A_{\nu} - \Gamma^{\lambda}_{\mu\nu} A_{\lambda} - \partial_{\nu} A_{\mu} + \Gamma^{\lambda}_{\nu\mu} A_{\lambda} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \tag{12}$$

$$\tilde{R}^{\lambda}_{\sigma\mu\nu} = R^{\lambda}_{\sigma\mu\nu} + \nabla_{\mu} C^{\lambda}_{\nu\sigma} - \nabla_{\nu} C^{\lambda}_{\mu\sigma} + C^{\lambda}_{\mu\rho} C^{\rho}_{\nu\sigma} - C^{\lambda}_{\nu\rho} C^{\rho}_{\mu\sigma} \tag{13}$$

$$\begin{aligned} \tilde{R}_{\mu\nu} = R_{\mu\nu} &- [2\delta^{\alpha}_{\mu} \delta^{\beta}_{\nu} + g_{\mu\nu} g^{\alpha\beta}] \omega^{-1} (\nabla_{\alpha} \nabla_{\beta} \omega) \\ &+ [4\delta^{\alpha}_{\mu} \delta^{\beta}_{\nu} - g_{\mu\nu} g^{\alpha\beta}] \omega^{-2} (\nabla_{\alpha} \omega)(\nabla_{\beta} \omega) \end{aligned} \tag{14}$$

$$\tilde{R} = \omega^{-2} R - -6 g^{\alpha\beta} \omega^{-3} (\nabla_{\alpha} \nabla_{\beta} \omega) \tag{15}$$

$$\tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \phi = \nabla_{\mu} \nabla_{\nu} \phi - (\ \delta^{\alpha}_{\mu}\ \delta^{\beta}_{\nu}\ +\ \delta^{\beta}_{\mu}\ \delta^{\alpha}_{\nu}\ )\ \omega^{-1}\ (\nabla_{\alpha} \omega)(\nabla_{\beta} \omega) \tag{16}$$

Back

Energy densities

Gauge field decomposition:

$$A^i(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \varepsilon^i_\lambda \left[A_\lambda(k,\eta) b_\lambda(\vec{k}) e^{ik\cdot x} + A^*_\lambda(k,\eta) b^\dagger_\lambda(\vec{k}) e^{-ik\cdot x} \right] \tag{17}$$

The EM energy densities with respect to the comoving observer are:

$$\rho_B(\eta,k) \equiv -\frac{1}{2}\langle 0|B_\mu B^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^5}{a^4} \left(|A_+(\eta,k)|^2 + |A_-(\eta,k)\ |^2 \right) \tag{18}$$

$$\rho_E(\eta,k) \equiv -\frac{1}{2}\langle 0|E_\mu E^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^3}{a^4} \left(|A'_+(\eta,k)|^2 + |A'_-(\eta,k)|^2 \right) \tag{19}$$

$$\rho_h(\eta,k) \equiv -\langle 0|A_\mu B^\nu|0\rangle = \int \frac{dk}{k} \frac{1}{2\pi^2} \frac{k^4}{a^3} \left(|A_+(\eta,k)|^2 - |A_-(\eta,k)|^2 \right). \tag{20}$$

where spectral energy density is given by $\frac{d\rho_\Upsilon}{d\mathrm{lnk}}$ for $\Upsilon \in (B,E,h)$

Using the fact that we can approximate the super-horizon modes by power law, we have

$$A_+(\tau, k) = C\, k^{\frac{1}{4\alpha}} - C_2 \frac{\mathcal{F}^{-1}}{\pi} \Gamma\left(\frac{1}{2\alpha}\right) k^{-\frac{1}{4\alpha}} \tau^{-\frac{1}{\alpha}} \tag{21}$$

where

$$\mathcal{F}(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}}, \tag{22}$$

$$C(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}} \left[\frac{C_1}{\Gamma\left(1+\frac{1}{2\alpha}\right)} - \frac{C_2}{\pi}\Gamma\left(-\frac{1}{2\alpha}\right)\cos\left(\frac{\pi}{2\alpha}\right)\right], \tag{23}$$

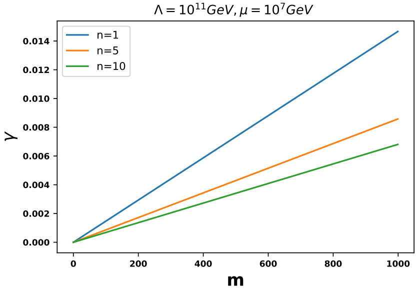
and the approximate values are $|\mathcal{F}| \sim 10^{-\frac{5}{\alpha}}\ \mathrm{GeV}^{-1/4\alpha}, |\mathrm{C}| \sim 10^{-\frac{5}{\alpha}-\frac{11}{2}}\,\mathrm{GeV}^{-\frac{1}{4\alpha}-\frac{1}{2}}$.

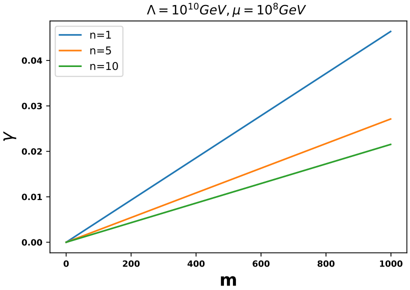
Power spectrum

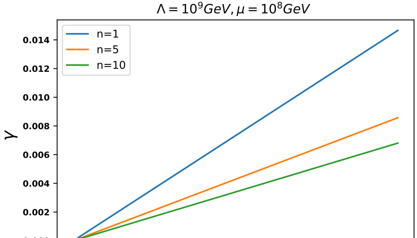
$$\left.\frac{d\rho_B}{d\ln k}\right|_{k_*\sim\mathcal{H}} \propto |C|^2\, k_*^{3+4\alpha+\frac{1}{2\alpha}} + \left|C_2\frac{\mathcal{F}^{-1}}{\pi}\Gamma\left(\frac{1}{2\alpha}\right)\right|^2 \frac{(2\alpha-1)^2}{4\eta_0^2}\, k_*^{1+4\alpha-\frac{1}{2\alpha}} \tag{24}$$

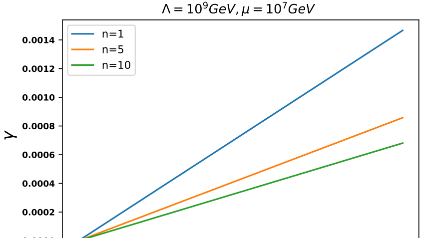
- It has two branches
  - The first branch (setting $C_2 = 0$) has **scale-invariant spectrum** for $\alpha = -\frac{1}{2}, -\frac{1}{4}$.
  - Second branch (setting $C = 0$) has **scale invariant spectrum** for $\alpha = -\frac{1}{2}, \frac{1}{4}$.
- Physically allowed values of $\alpha \leq -1/2$. Hence, $\alpha = \pm 1/4$ **is ruled out.**
- For slow-roll inflation ($\alpha = -\frac{1}{2} - \epsilon$), the two branches scale differently: $k_*^{-2\epsilon}$ (first branch) and $k_*^{-6\epsilon}$ (second branch).
- Since ϵ is positive, this implies that our model produces **more power on the large scales.** $\longrightarrow$ **Red spectrum for slow roll inflation**

Plots for lower energy scales of Λ and μ









(Helical fields) Models in the literature

Scalar field coupled models: $f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$ where $f(\phi)$ is time-dependent coupling function.

- **Problems with these models :**
  - **Strong coupling** - Coupling between charged particles and the EM field is so strong that theory can not be treated perturbatively.
  - **Back-reaction** - Overproduction of gauge fields affect the background inflationary dynamics
  - Because magnetic fields are produced near the end of inflation, strength of the **fields generated depends on the reheating scale.**

To resolve strong coupling and back-reaction problem $f(\phi)$ is assumed to increase during inflation and decrease back to its initial value post inflation.

Durrer et al.(2011), Sharma et al.(2018)

Helical magnetic field generation

- For power law inflation: $a(\eta) = \left(-\frac{\eta}{\eta_0}\right)^{\beta+1}$, de-sitter $\beta = -2$, we have

$$A_h'' + \left[k^2 - \frac{8kh}{M^2}\frac{\beta(\beta+1)(\beta+2)}{\eta_0^3}\left(\frac{-\eta_0}{\eta}\right)^{(2\beta+5)}\right]A_h = 0 \tag{25}$$

- Sub-horizon mode $|-k\eta| >> 1$ solution is: $A_h = \frac{1}{\sqrt{k}}e^{-ik\eta}$
- For super-horizon mode $|-k\eta| << 1$, with dimensionless variable, $\tau = \left(-\frac{\eta_0}{\eta}\right)^{\alpha}$ and $\alpha = \beta + \frac{3}{2}$

$$A_+(\tau, k) = \tau^{-\frac{1}{2\alpha}} J_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_1 + \tau^{-\frac{1}{2\alpha}} Y_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_2 \tag{26a}$$

$$A_-(\tau, k) = \tau^{-\frac{1}{2\alpha}} J_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_3 + \tau^{-\frac{1}{2\alpha}} Y_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right) C_4 \tag{26b}$$

Electromagnetic energy density

To identify whether these modes lead to back-reaction on the metric, we define R, which is the ratio of the total energy density of the fluctuations and background energy density during inflation: Talebian et al.(2020)

$$R = \frac{(\rho_B + \rho_E)|_{k_* \sim \mathcal{H}}}{6 M_P^2 H^2} \tag{27}$$

α	ρ (in GeV^4)	R
$-\frac{1}{2} - \epsilon$	$\sim 10^{64}$	$\sim 10^{-4}$
$-\frac{3}{4}$	$\sim 10^{62}$	$\sim 10^{-6}$
-1	$\sim 10^{61}$	$\sim 10^{-7}$
-3	$\sim 10^{59}$	$\sim 10^{-9}$

No back-reaction on the background metric.

Estimating the strength of helical magnetic fields

- Assuming instantaneous reheating, and the Universe becomes radiation dominated after inflation. Due to flux conservation, the magnetic energy density will decay as $1/a^4$: Subramanian (2016)

- Using the fact that the relevant modes exited Hubble radius around 30 e-foldings of inflation, with energy density $\rho_B \approx 10^{64}\mathrm{GeV}^4$, the primordial helical fields at GPc scales is:

$$B_0 \approx 10^{-20}\mathrm{G} \tag{28}$$

- Helical magnetic fields that re-entered the horizon at two different epochs:

$$B|_{50\ \mathrm{MPc}} \sim 10^{-18}\ G\ (z \sim 20)\ ;\ \ B|_{1\ \mathrm{MPc}} \sim 10^{-14}\ G\ (z \sim 1000)$$