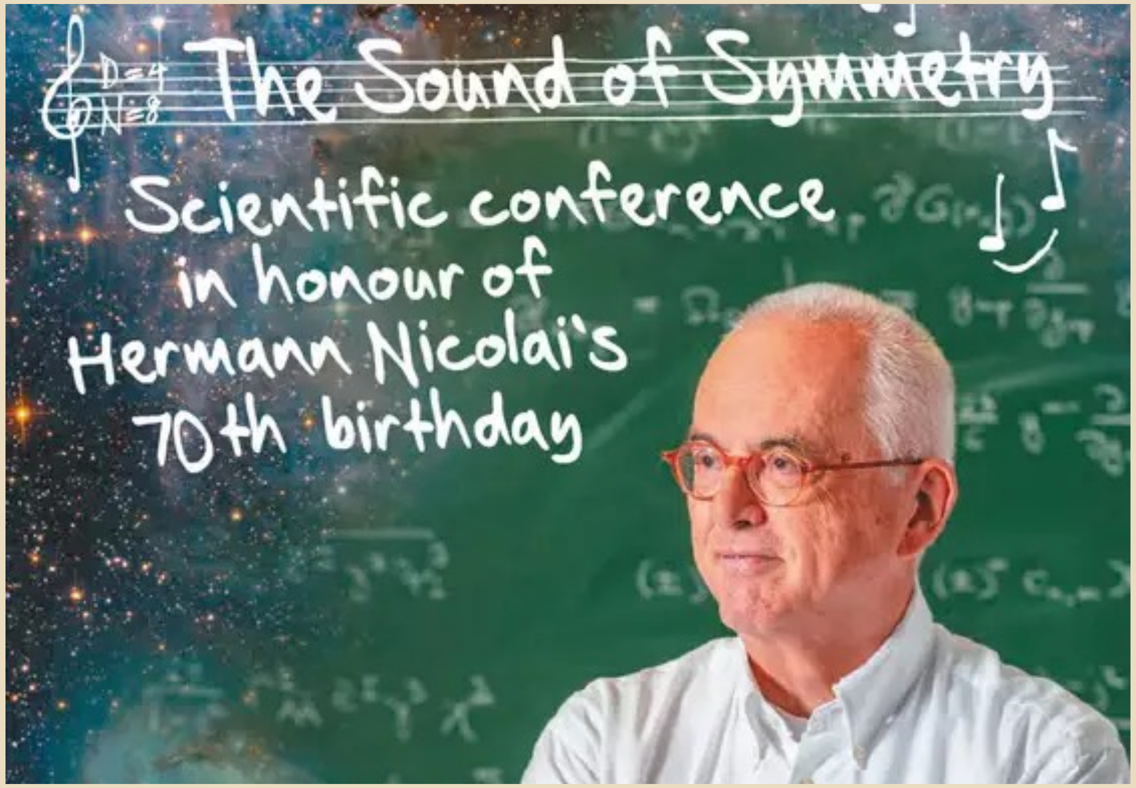
Kaluza-Klein spectrometry from exceptional field theory

Henning Samtleben, ENS de Lyon



Lab PHYS
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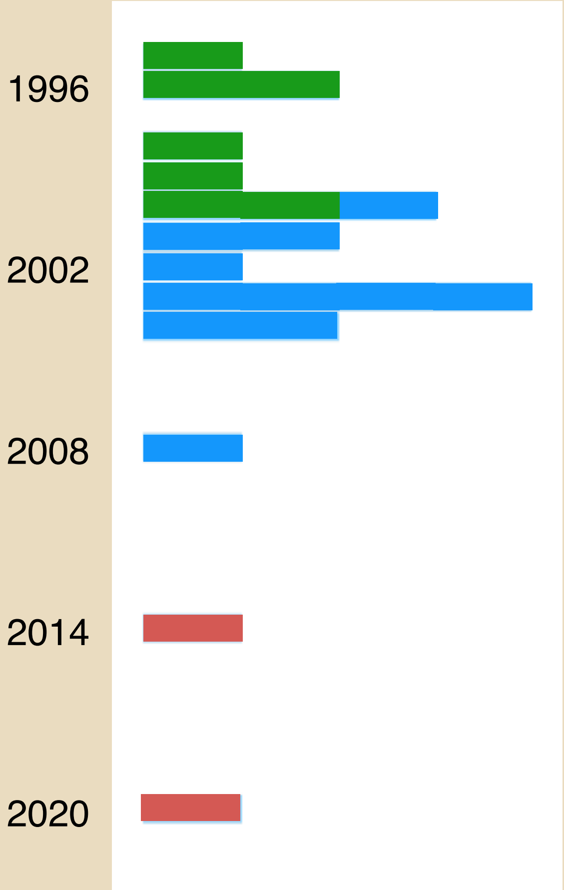
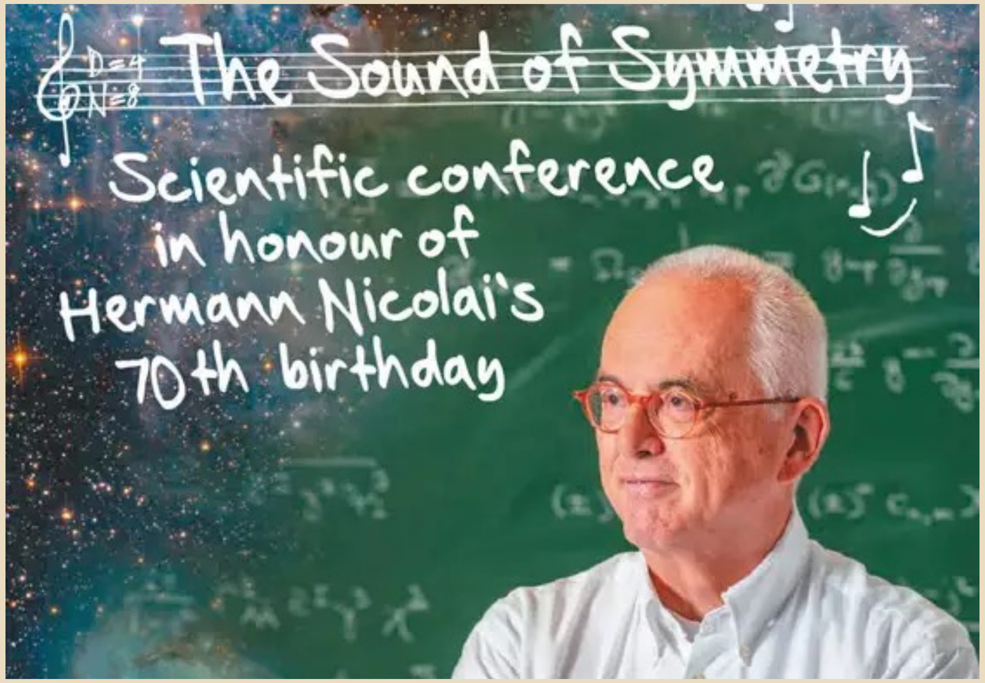
with Hermann through the dimensions — since 1995



2D: with Dmitrii Korotkin,
Kilian Koepsell

3D: with Thomas Fischbacher,
Bernard de Wit

4D: with Hadi and Mahdi Godazgar,
Olaf Hohm, Emanuel Malek



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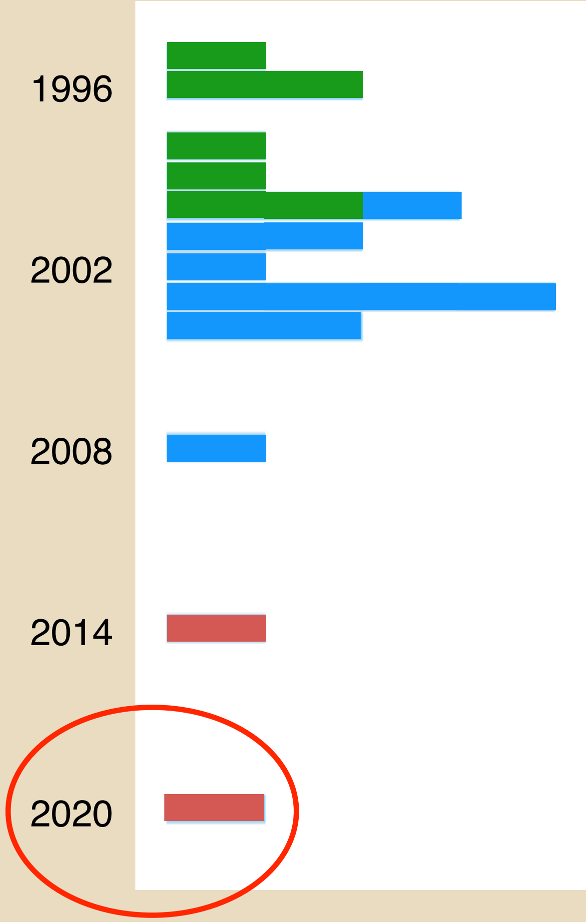

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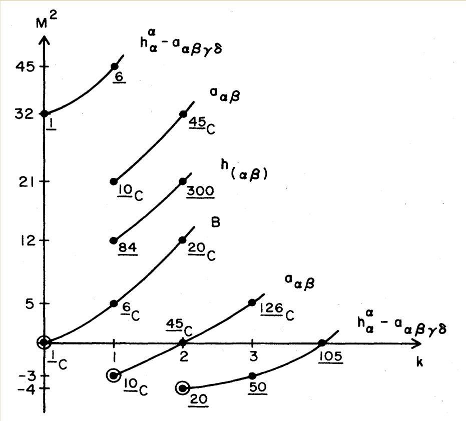
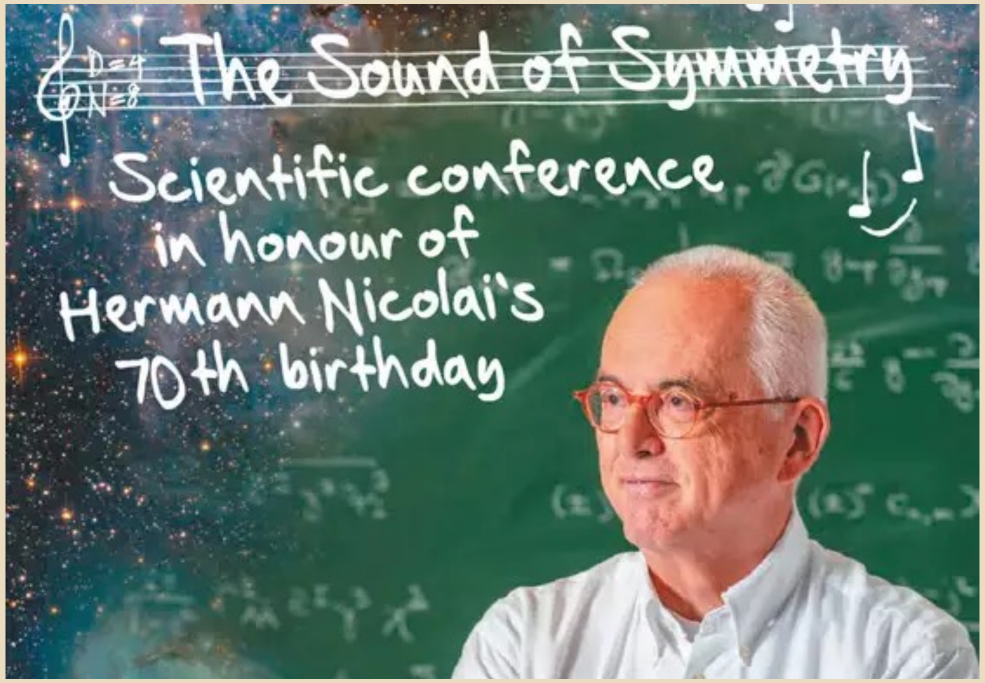
Kaluza-Klein spectrometry
— nonsusy AdS vacua —





plan: Kaluza-Klein spectrometry for supergravity

motivation

- compactification, Kaluza-Klein spectra

tools

- consistent truncations
- exceptional field theory
- Kaluza-Klein spectroscopy

examples

- $AdS_4 \times S^7$ and deformations
- non-supersymmetric AdS_4 vacua, perturbative stability
- S-fold vacua

based on work with Olaf Hohm, Emanuel Malek, Hermann Nicolai,
Adolfo Guarino, Alfredo Giambrone, Mario Trigiante

motivation compactification & Kaluza-Klein spectra

- background $\mathscr{M}_{10} = \mathscr{M}_4 \times \mathscr{M}_6$
- expanding fields in harmonics on the internal space

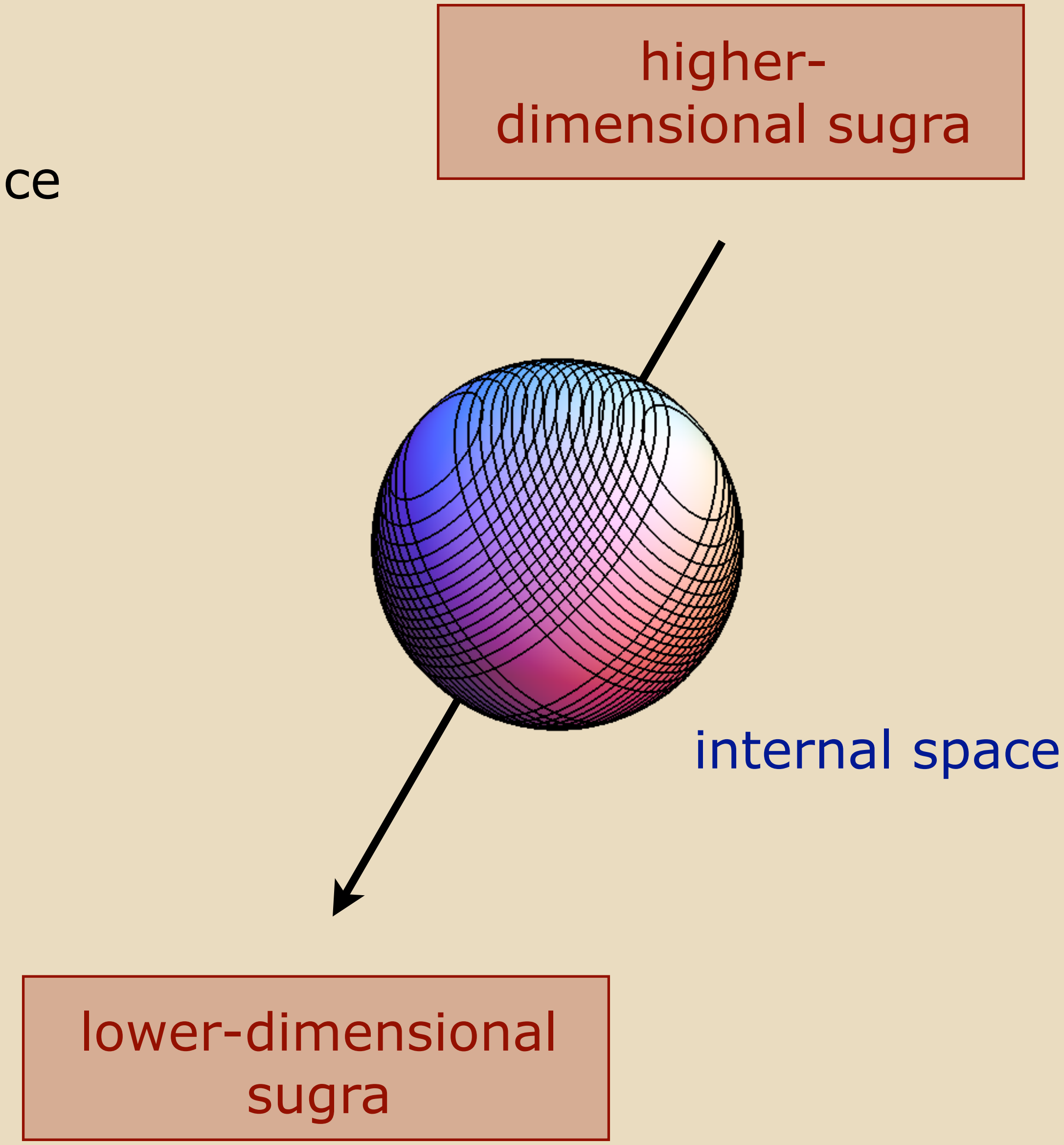
  e.g. scalar field

$$\phi(x,y) = \sum_{\Sigma} \phi_{\Sigma}(x)\, \mathcal{Y}^{\Sigma}(y)$$

fluctuations

harmonics

- such that the dynamics of the KK fluctuations is described by a lower-dimensional theory

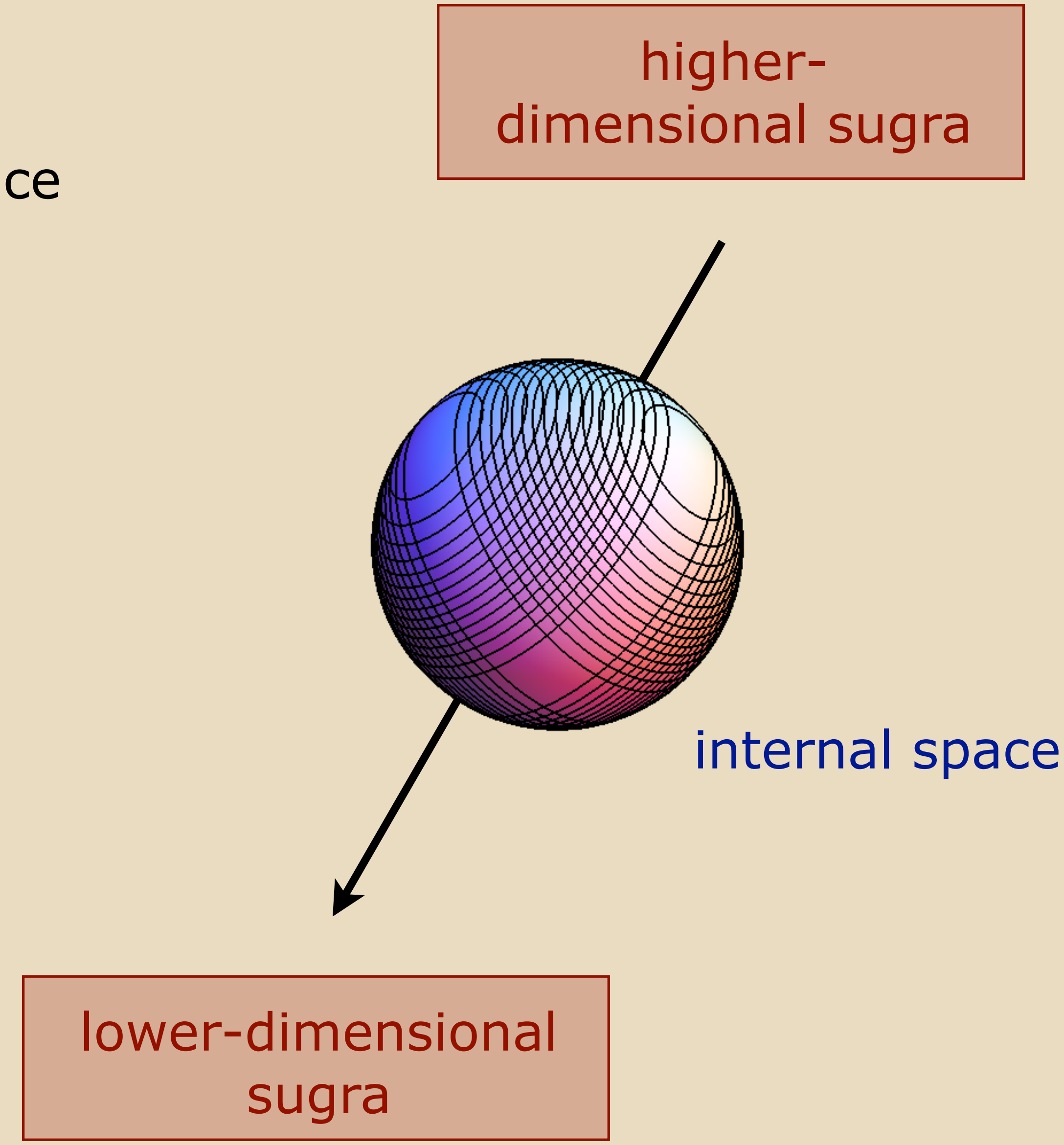


higher-dimensional sugra

internal space

lower-dimensional sugra

motivation compactification & Kaluza-Klein spectra



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fluctuations

harmonics

- such that the dynamics of the KK fluctuations is described by a lower-dimensional theory
- mass spectrum of the KK-fluctuations
- in general: complicated problem
  - diagonalize various Laplacians on the internal manifold
  - disentangle mass eigenstates from different higher-dimensional origin
  - flux compactifications: higher-dimensional p-forms
- important: phenomenology, stability, holography, …

motivation compactification & Kaluza-Klein spectra

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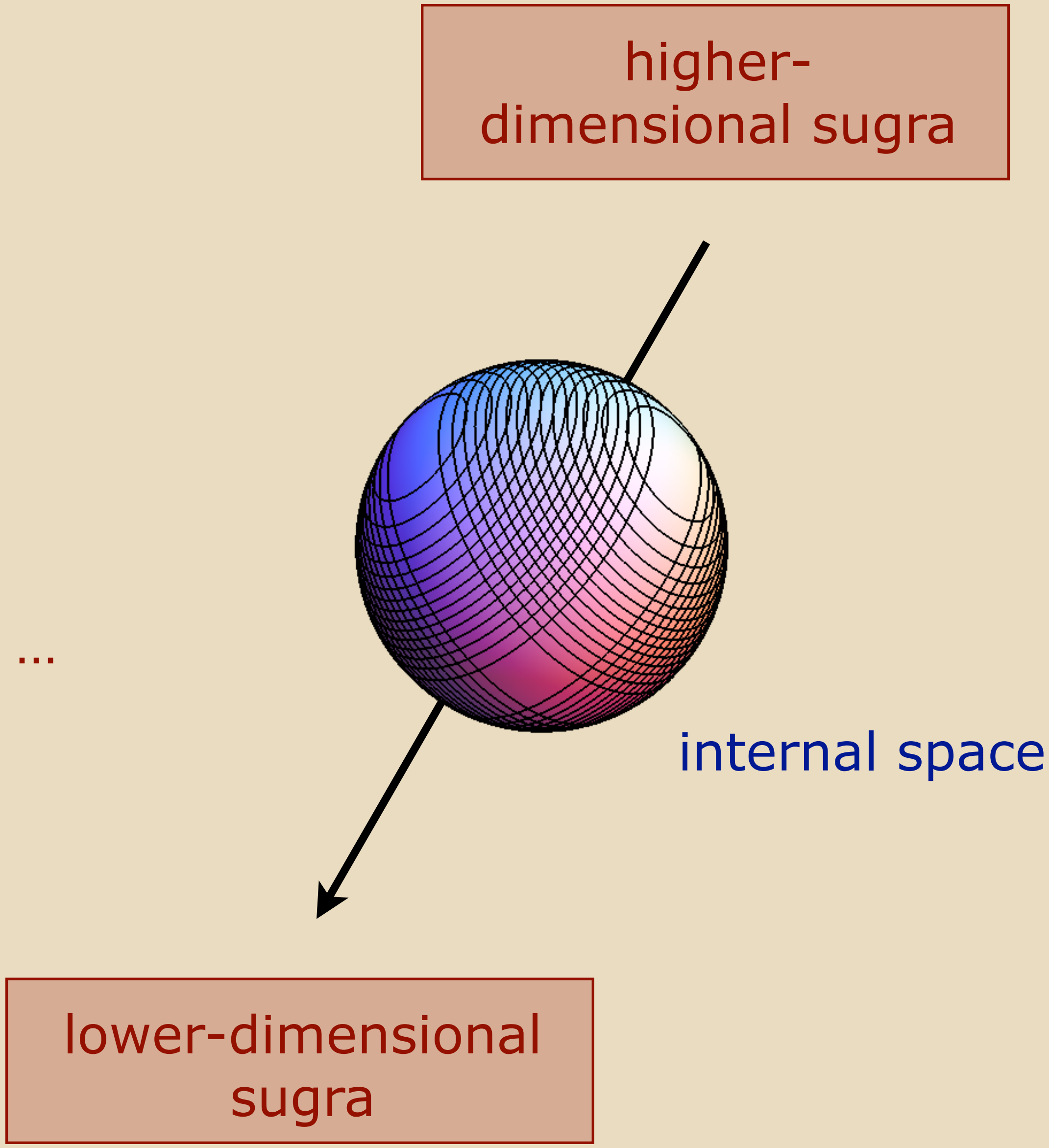

higher-dimensional sugra

internal space

lower-dimensional sugra

- some windows into the KK-spectrum:
  - universality in the spin-2 sector [Bachas, Estes]
    massless scalar wave equation in 10D
  - symmetry helps [Salam, Strathdee]
    symmetric spaces, large isometry groups
  - supersymmetry helps
    masses from group theory

motivation compactification & Kaluza-Klein spectra

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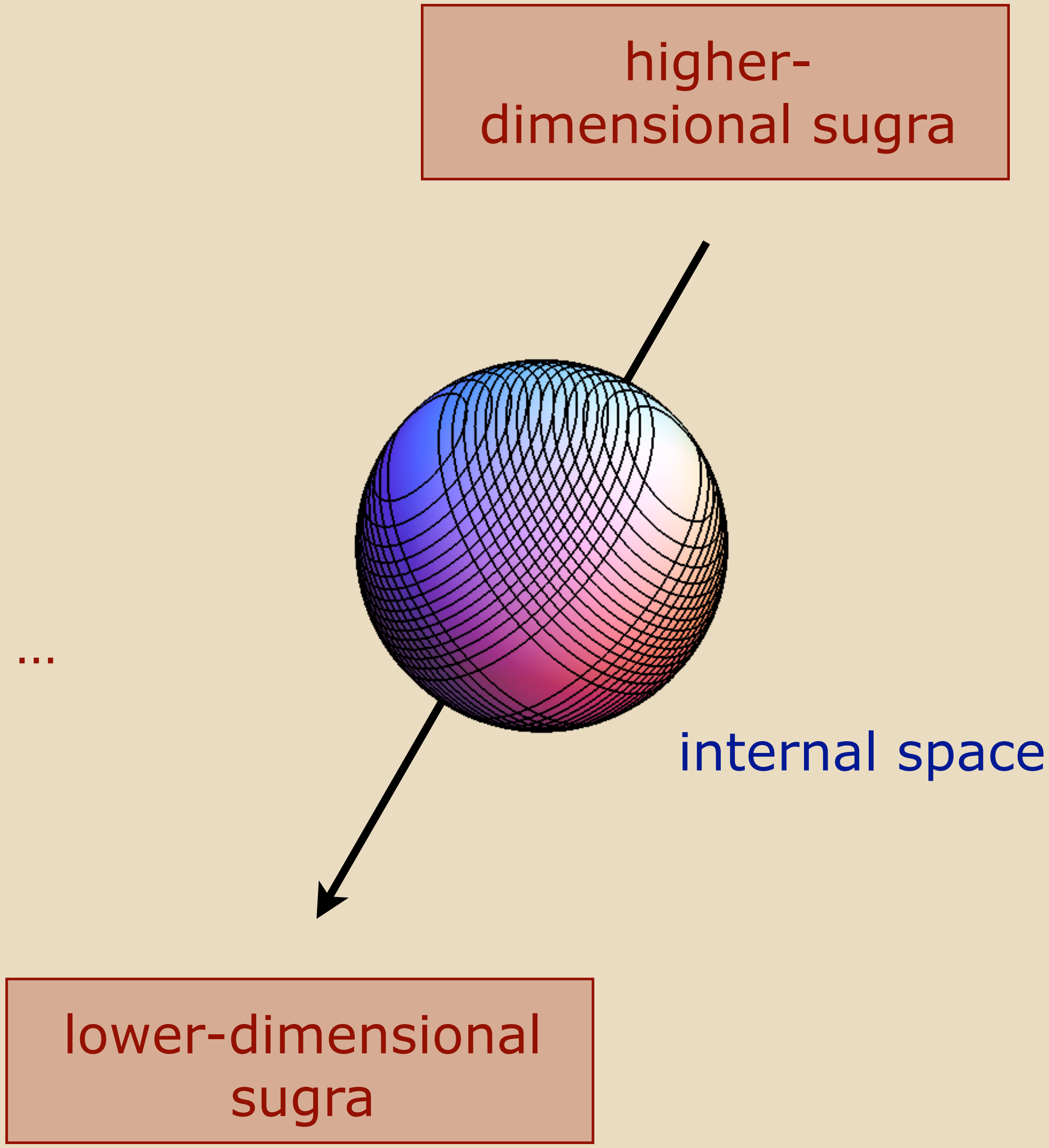

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- if the background has no (super-)symmetries, we need <u>new tools</u> !

tools consistent truncations

- (non-linear) truncation to a finite number of KK-modes
- described by a lower-dimensional supergravity
- such that any solution of the lower-dimensional theory lifts to a solution of the higher-dimensional theory

D=11 supergravity on AdS$_4$ x S^7 [de Wit, Nicolai 1987]

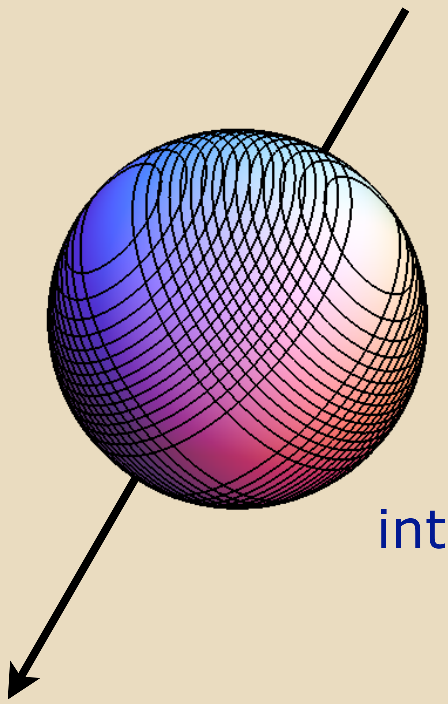
- non-linear reduction ansatz

$$\Delta^{-1} g^{mn}(x,y) = \frac{1}{8} K^{mIJ} K^{nKL}(y) \Big[(u^{ij}{}_{IJ} + v^{ijIJ})(u_{ij}{}^{KL} + v_{ijKL}) \Big](x)\,.$$

$$A_{mnp}(x,y) = -\frac{\sqrt{2}}{96}\; i\Delta\, g_{pq}(x,y)\, K_{mn}{}^{IJ} K^{qKL}(y) \Big[(u^{ij}{}_{IJ} - v^{ijIJ})(u_{ij}{}^{KL} + v_{ijKL}) \Big](x)$$

$$A_{nm_1\ldots m_5} - \frac{\sqrt{2}}{4} A_{n[m_1 m_2} A_{m_3 m_4 m_5]}$$

$$= -\frac{\sqrt{2}}{16\cdot 6!}\, \Delta\, g_{np} \left(K_{m_1\ldots m_5}{}^{IJ} - 6\cdot 6!\, \mathring{\zeta}_{m_1\ldots m_5 q} K^{qIJ} \right) K^{pKL} \Big[(u_{ij}{}^{IJ} + v_{ijIJ})(u^{ij}{}_{KL} + v^{ijKL}) \Big]$$



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internal space

lower-dimensional sugra

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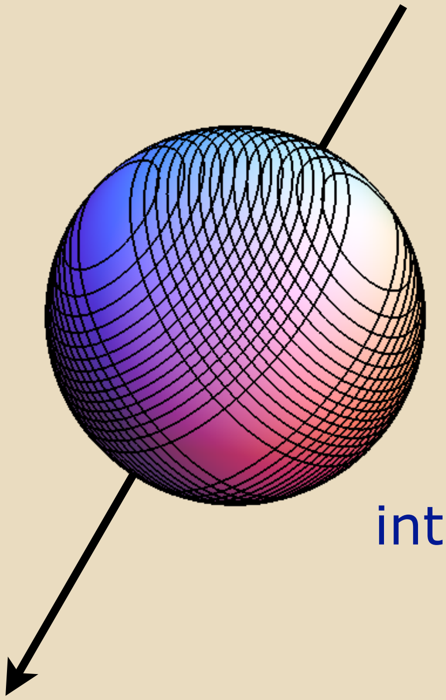

internal space

lower-dimensional sugra

D=11 supergravity on AdS₄ x S⁷ [de Wit, Nicolai 1987]

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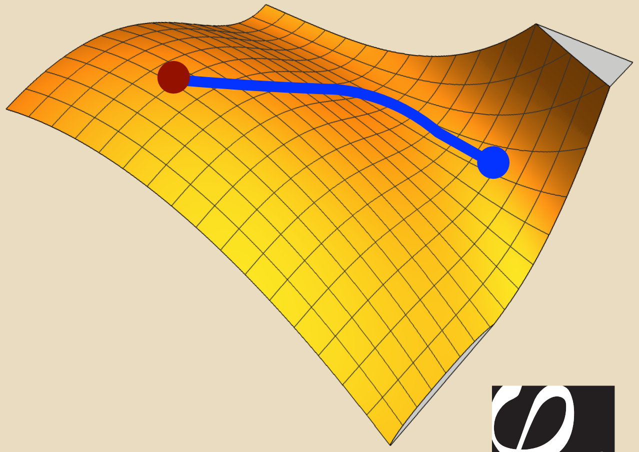
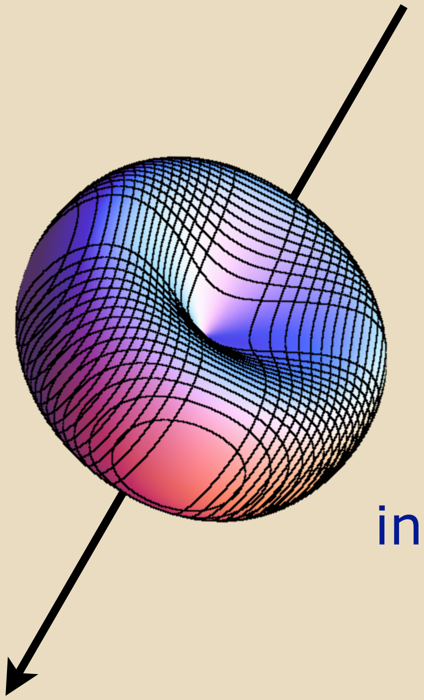


- every stationary point of the D=4 scalar potential lifts to a D=11 background of the form $\mathscr{M}_{11} = \mathrm{AdS}_4 \times \mathscr{M}_7$
- around these backgrounds, we can compute the masses of the 70 scalars from the $\mathscr{N} = 8$ supergravity multiplet

tools consistent truncations

D=11 supergravity on AdS4 x S7 [de Wit, Nicolai 1987]

higher-dimensional sugra



lower-dimensional sugra

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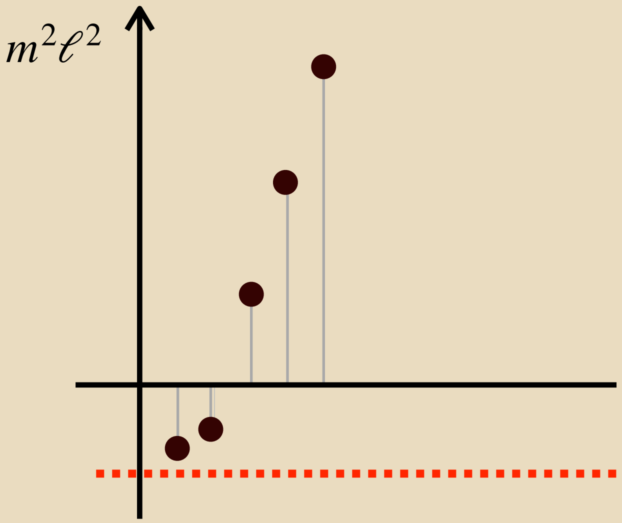
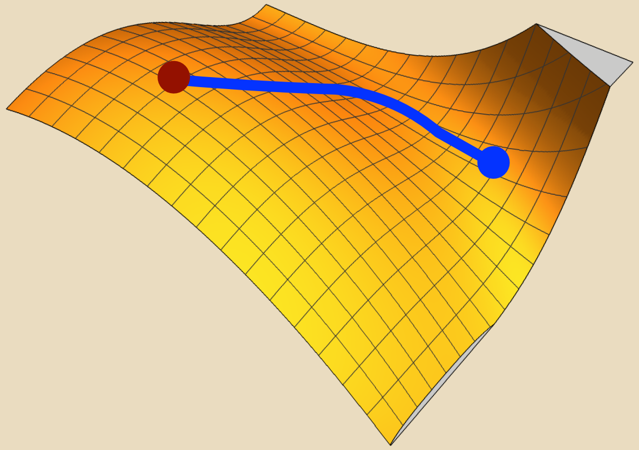
e.g.





- has allowed to detect instabilities in almost all non-supersymmetric AdS4 vacua [Comsa, Firsching, Fischbacher]
- in order to address the full KK spectrum, we need to combine with new tools (not much sound of symmetry yet)

tools exceptional field theory (ExFT)

duality covariant formulation of D=11 supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

[Hohm, HS]

- all fields live on external space-time : $\{x^\mu\}$ $\mu = 0, \dots, 3$
  internal (exceptional) space : $\{Y^M\}$ $M = 1, \dots, 56$

> embedding $\partial_m \longrightarrow \partial_M$ subject to the section constraint $(t_\alpha)^{MN}\, \partial_M \otimes \partial_N = 0$
covariant restriction from 56 down to 7 (6) coordinates

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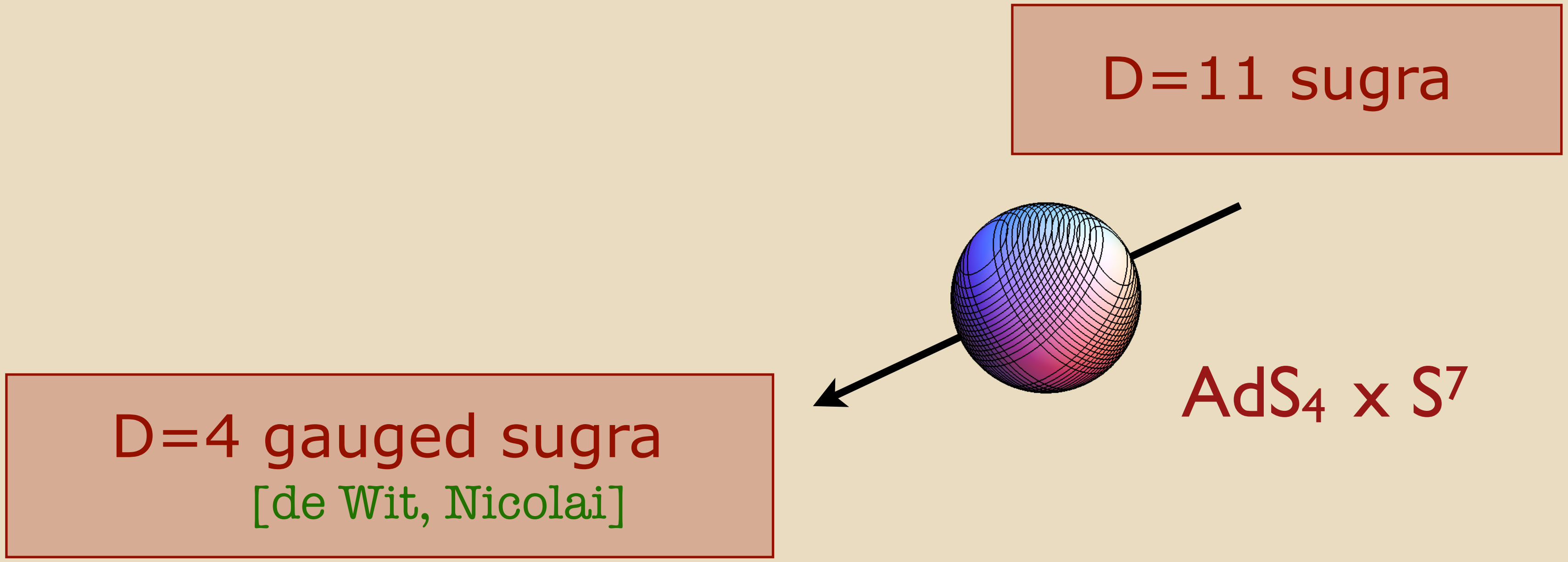
    covariant restriction from 56 down to 7 (6) coordinates
- non-abelian gauge structure
  - (infinite-dimensional) non-abelian gauge structure:
    the internal diffeomorphisms (Kaluza-Klein) + tensor gauge symmetry
    $\longrightarrow$ generalized diffeomorphisms [Coimbra, Strickland-Constable, Waldram]

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu} A_{\nu]}{}^M - 2\,[A_\mu, A_\nu]_{\text{E}}^M - 12\,(t^\alpha)^{MN}\, \partial_N B_{\mu\nu\,\alpha} - \frac{1}{2}\, \Omega^{MN}\, B_{\mu\nu\,N}$$

tools consistent truncations from ExFT



D=11 sugra

$AdS_4 \times S^7$

D=4 gauged sugra

[de Wit, Nicolai]

tools consistent truncations from ExFT

ExFT

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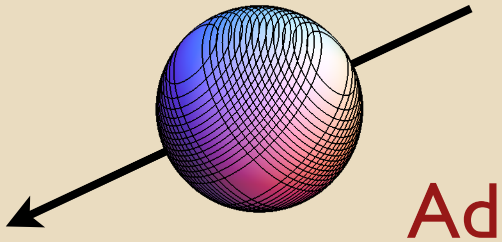
dictionary

generalized Scherk-Schwarz reduction of ExFT

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) = \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

D=11 sugra



AdS4 x S7

D=4 gauged sugra
[de Wit, Nicolai]

- E7(7) valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$
- system of consistency equations
  $$\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{912}} \stackrel{!}{=} \rho\, X_{MN}{}^K$$
  embedding tensor of the D=4 gauged supergravity
- powerful tool to construct consistent truncations

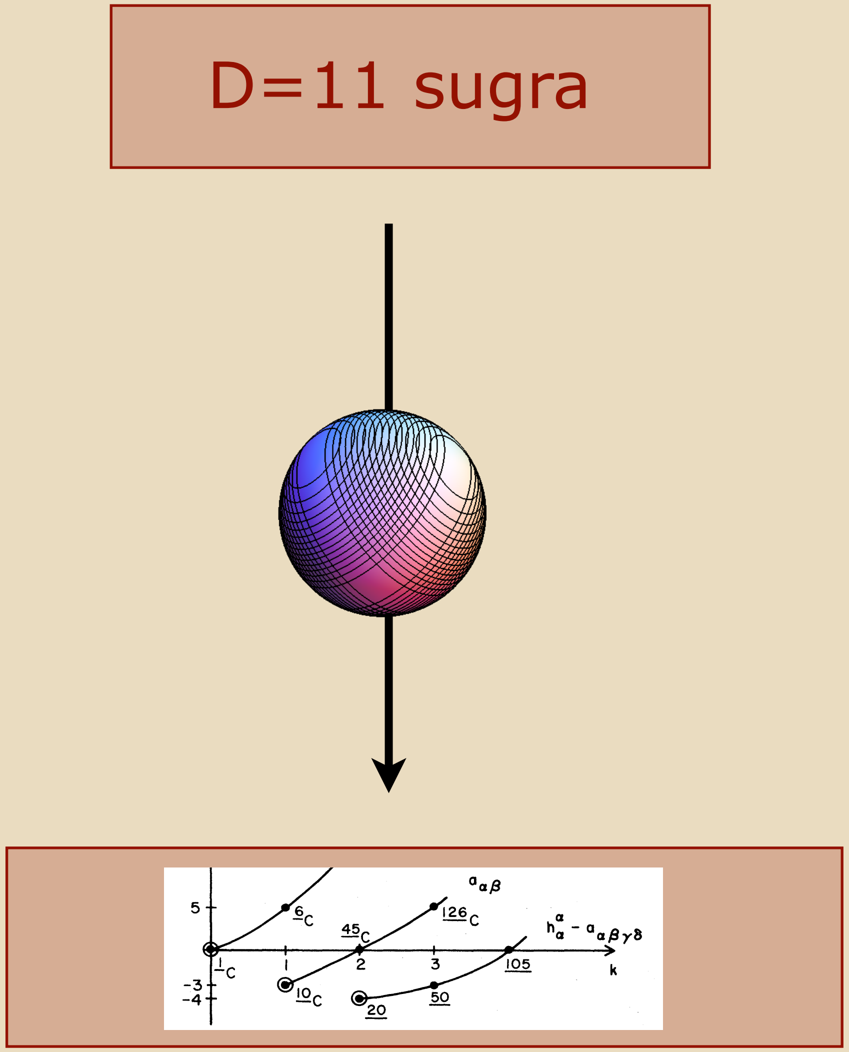
tools Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

- consistent truncation to lowest KK-multiplet

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x)$$

- extend to the higher Kaluza-Klein modes (linearized)



D=11 sugra

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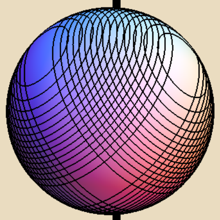
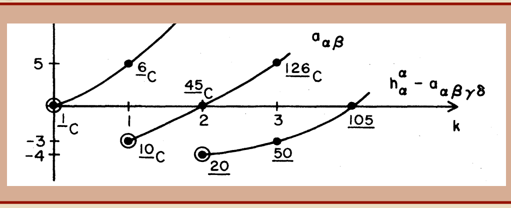
D=11 sugra





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$$\begin{aligned}
\mathcal{A}_\mu{}^M(x,Y) \;&=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma \\
\mathcal{M}_{MN}(x,Y) \;&=\; U_M{}^K(Y)\,U_N{}^L(Y)\,\Big(\delta_{KL} + \sum_\Sigma j_{KL,\Sigma}(x)\,\mathcal{Y}^\Sigma\Big)
\end{aligned}$$

  - with fluctuations $A_\mu{}^{K,\Sigma}$, $j_{KL,\Sigma}$,
    and the tower of <u>scalar</u> harmonics $\mathcal{Y}^\Sigma$

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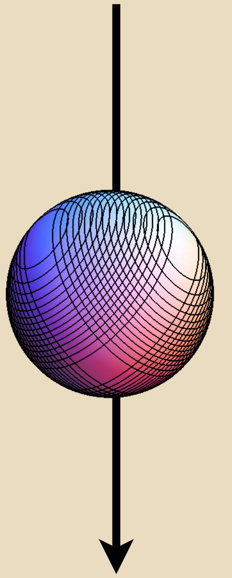
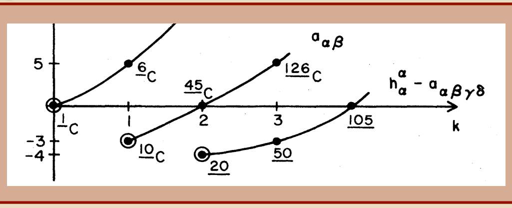
D=11 sugra





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> with fluctuations $A_\mu{}^{K,\Sigma}$, $j_{KL,\Sigma}$,
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- (lowest KK multiplet) $\otimes$ (scalar harmonics) $\xrightarrow{\text{ExFT field equations}}$ KK spectrum

tools Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

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- plug into the ExFT action and linearize in fluctuations
- e.g. mass matrix for vector fluctuations $A_\mu{}^{M,\Sigma}$

$$\begin{aligned} M_{M\Sigma,N\Omega} \;\propto\; & \frac{1}{3}\,X^{\rm s}_{ML}{}^K\,X^{\rm s}_{NK}{}^L\,\delta^{\Sigma\Omega}+2\,\big(X^{\rm s}_{MK}{}^N-X^{\rm s}_{NM}{}^K\big)\,\mathcal{T}_{K,\Omega\Sigma} \\ & -6\,\big(\mathbb{P}^K{}_M{}^L{}_N+\mathbb{P}^M{}_K{}^L{}_N\big)\,\mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma}+\frac{8}{3}\,\mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma} \end{aligned}$$

in terms of

- symmetrized D=4 embedding tensor $X^{\rm s}_{MN}{}^K \equiv X_{MN}{}^K+X_{MK}{}^N$
- adjoint projector $\mathbb{P}^M{}_N{}^K{}_L \;=\; (t^\alpha)_N{}^M\,(t_\alpha)_L{}^K$
- representation of scalar harmonics $\mathcal{K}_M{}^m\partial_m\mathcal{Y}^\Sigma = \mathcal{T}_{M,\Sigma\Omega}\,\mathcal{Y}^\Omega$

tools Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

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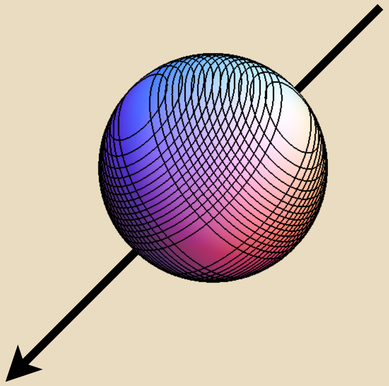
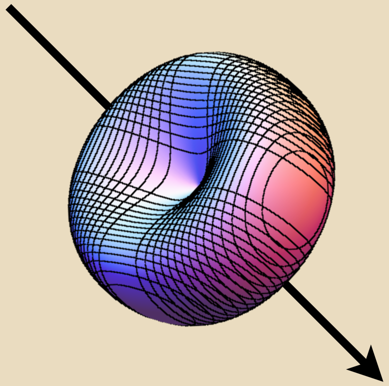
- representation of scalar harmonics $\mathcal{K}_M{}^m\partial_m\mathcal{Y}^\Sigma = \mathcal{T}_{M,\Sigma\Omega}\,\mathcal{Y}^\Omega$

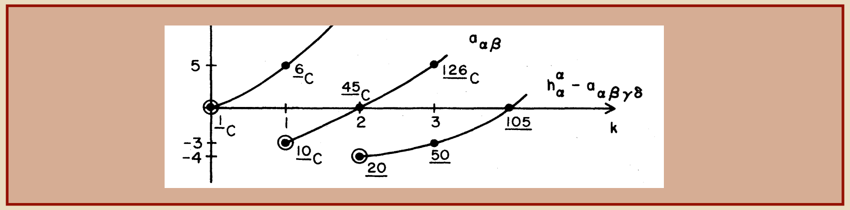
- similar for the scalar mass matrix
- entire KK mass spectrum!

examples: AdS₄ x S⁷ and deformations

D=11 sugra



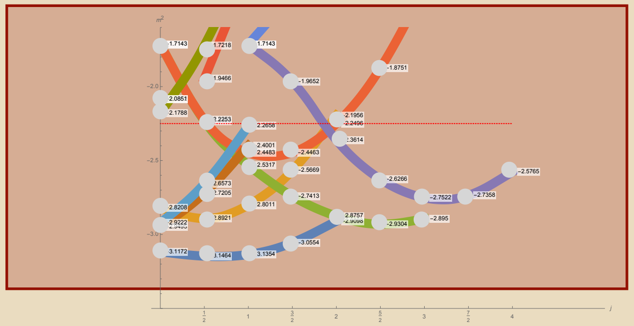




- simple and compact (re-)derivation of the supergravity spectrum on S^7

  [1980's: Biran, Casher, Englert, Nicolai, Rooman, Spindel, Sezgin]

- direct identification of BPS multiplet components within D=11 supergravity



- powerful tools for <u>non-supersymmetric</u> vacua (where masses are not controlled by symmetry)

  [E. Malek, H. Nicolai, HS]

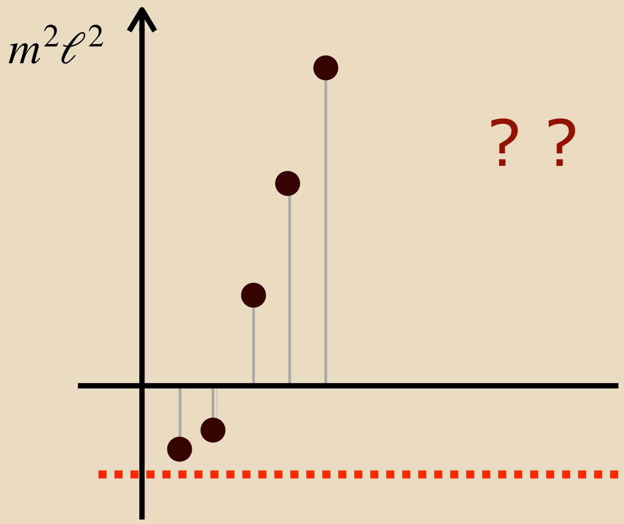
example: non-supersymmetric AdS$_4$ vacua SO(3) x SO(3)

- in D=4 SO(8) supergravity, the supergravity potential has been carefully scanned for AdS$_4$ vacua

  [Comsa, Firsching, Fischbacher: "SO(8) Supergravity and the Magic of Machine Learning"]

  - all non-supersymmetric vacua are unstable already within D=4 supergravity,
  - i.e. have instabilities within the lowest Kaluza-Klein multiplet

- except for a distinguished SO(3) x SO(3) invariant extremal point [Warner]
  - stable within D=4 supergravity [Fischbacher, Pilch, Warner]
  - uplift to D=11 supergravity [Godazgar, Godazgar, Krüger, Nicolai, Pilch]



example: non-supersymmetric AdS_4 vacua SO(3) x SO(3)

- in D=4 SO(8) supergravity, the supergravity potential has been carefully scanned for AdS_4 vacua

  [Comsa, Firsching, Fischbacher: "SO(8) Supergravity and the Magic of Machine Learning"]

  - all non-supersymmetric vacua are unstable already within D=4 supergravity,
  - i.e. have instabilities within the lowest Kaluza-Klein multiplet

- except for a distinguished SO(3) x SO(3) invariant extremal point [Warner]
  - stable within D=4 supergravity [Fischbacher, Pilch, Warner]
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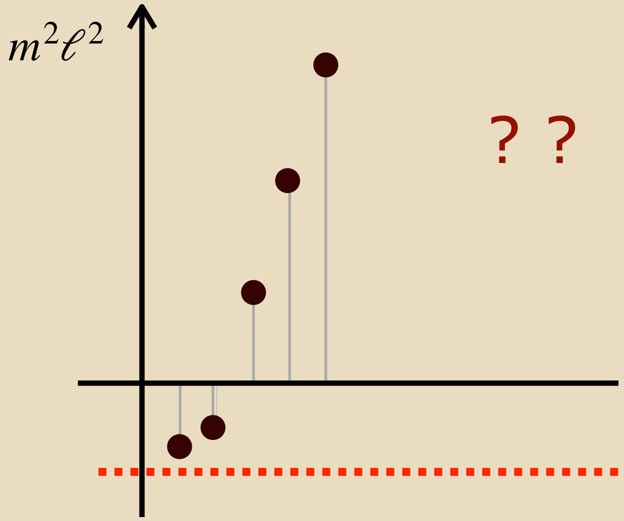
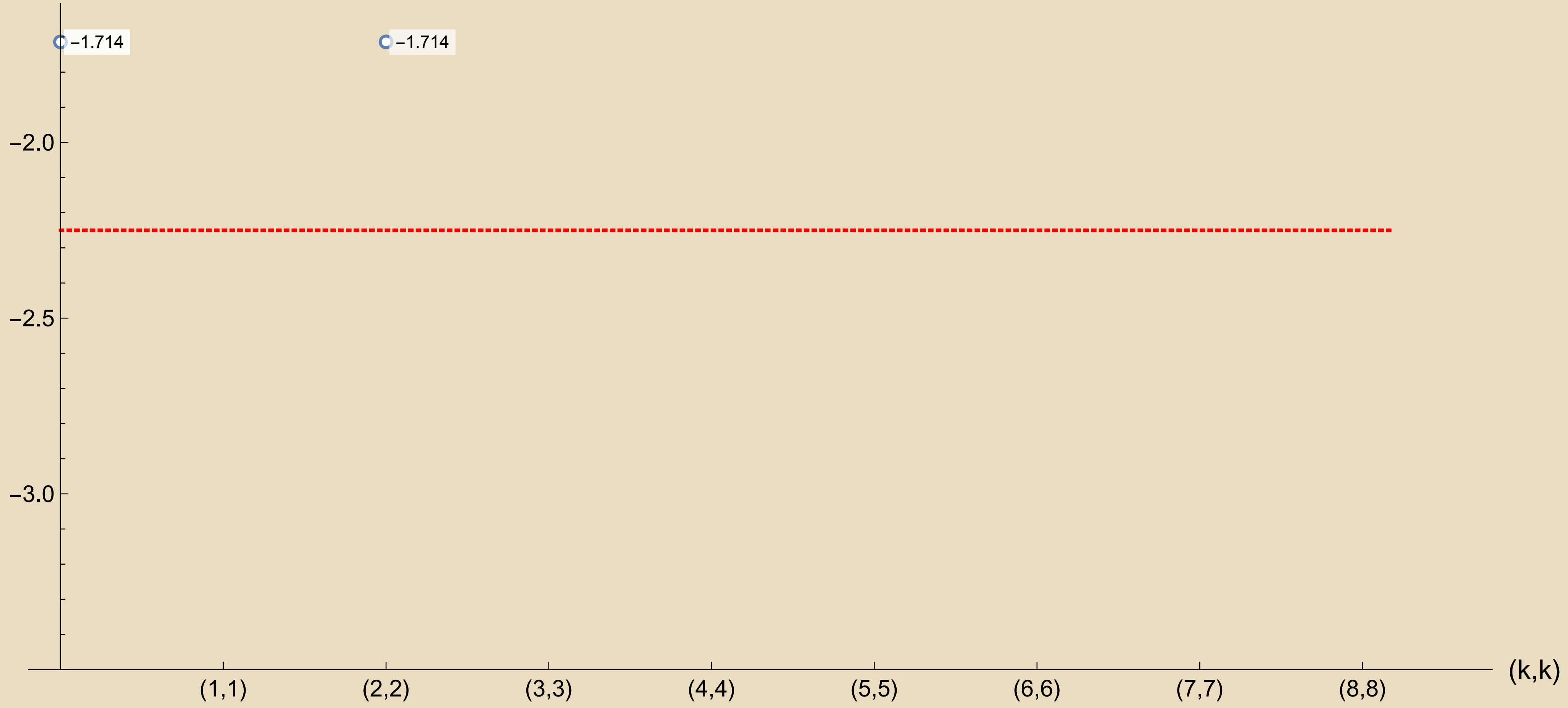


⟶ apply the ExFT formulas!

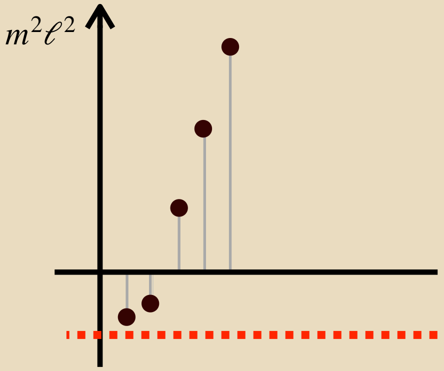
example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- scalar spectrum: the N=8 supergravity multiplet (70 scalars)

$$\mathcal{M}_{MN} = U_M{}^{\underline{K}} U_N{}^{\underline{L}} \left(\delta_{\underline{KL}} + j_{\underline{KL}}(x)\right)$$



$\mathbf{70} \longrightarrow$



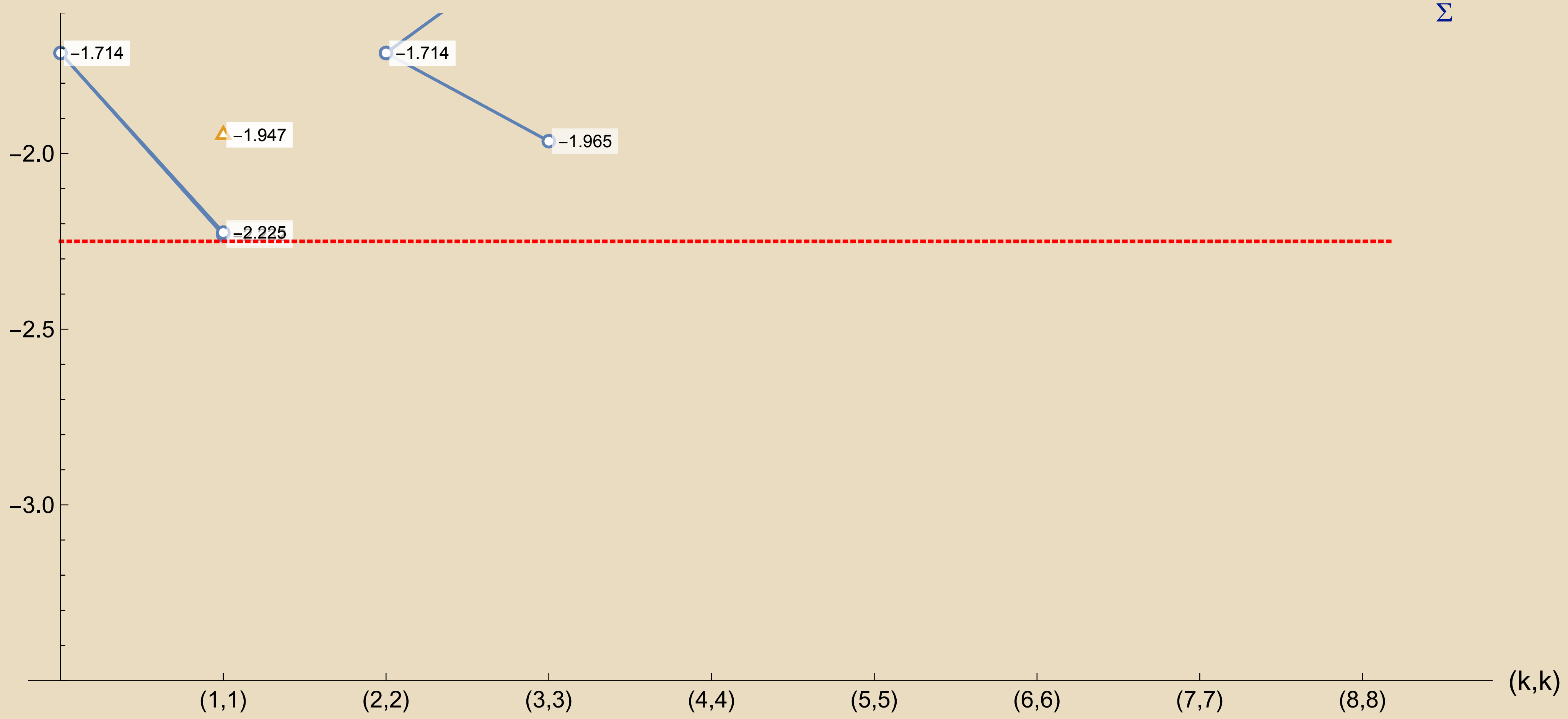
negative masses but above the BF bound

[Fischbacher, Pilch, Warner]

example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- Kaluza-Klein level 1: 560 scalars (including spin-1 and spin-2 Goldstone modes)

$$\mathcal{M}_{MN} = U_M{}^{\underline{K}} U_N{}^{\underline{L}} \Big(\delta_{\underline{KL}} + \sum_{\Sigma} \mathcal{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x) \Big)$$

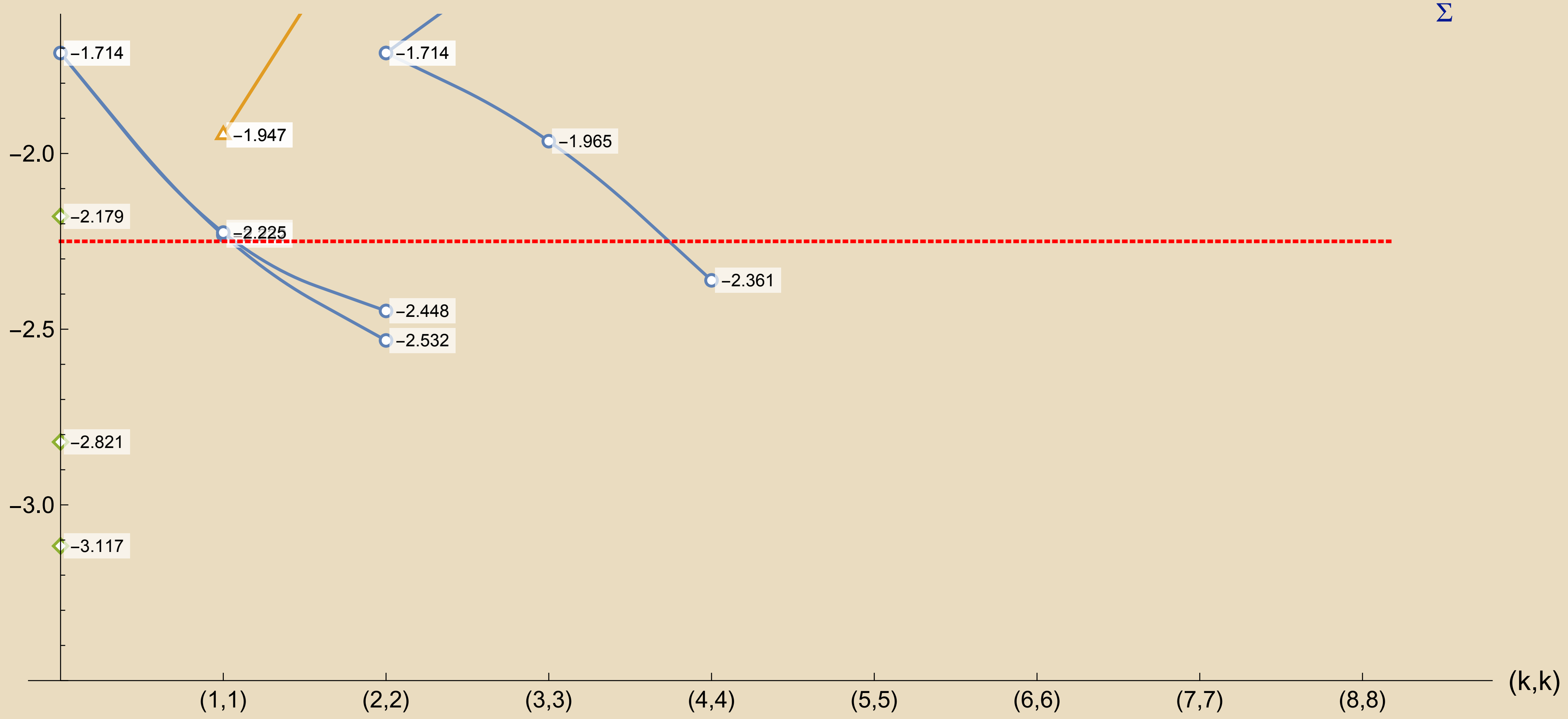


still all the negative masses remain above the BF bound

example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- Kaluza-Klein level 2: 2450 scalars (including spin-1 and spin-2 Goldstone modes)

$$\mathcal{M}_{MN} \;=\; U_M{}^{\underline{K}}\, U_N{}^{\underline{L}} \Big(\delta_{\underline{KL}} + \sum_{\Sigma} \mathcal{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x) \Big)$$

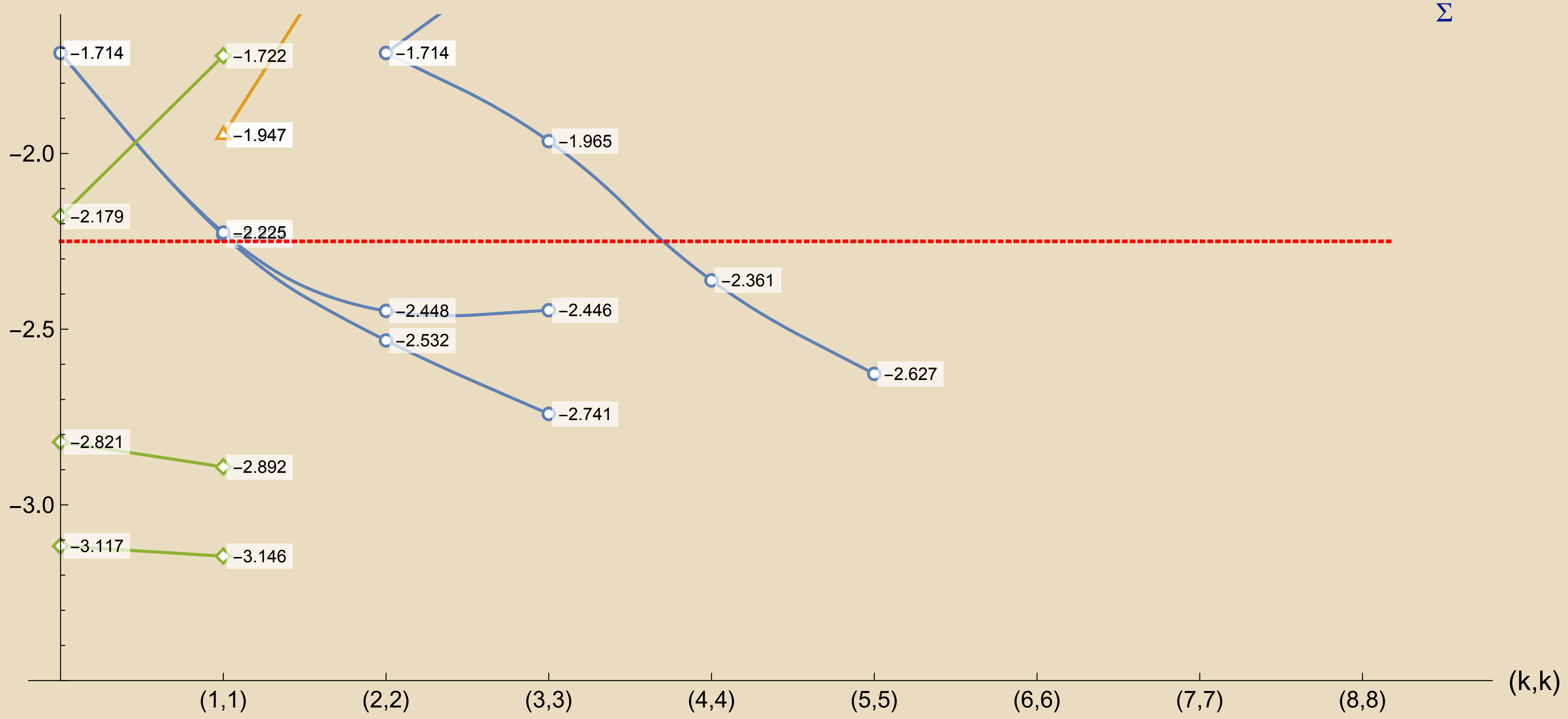


unstable modes arise!

example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- Kaluza-Klein level 3: 7840 scalars (including spin-1 and spin-2 Goldstone modes)

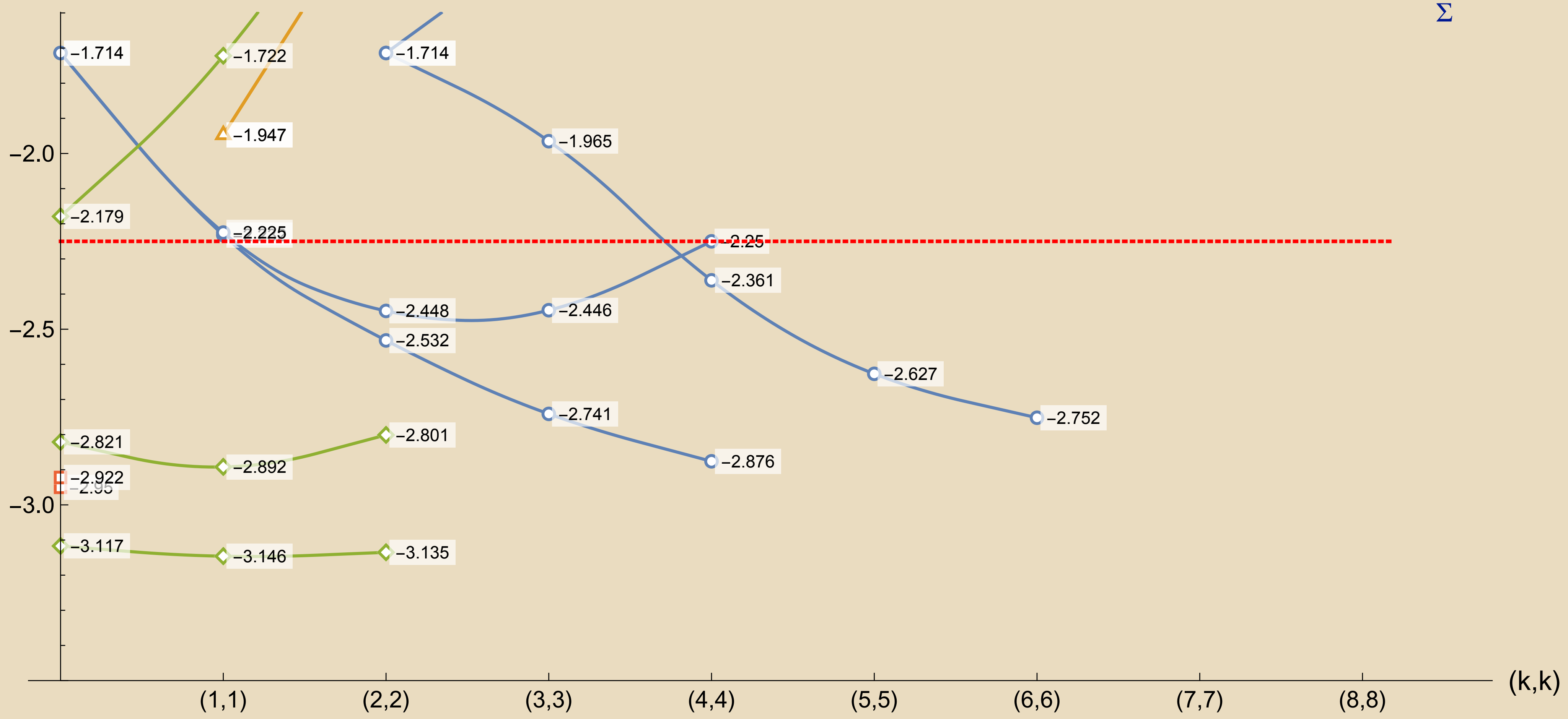
$$\mathcal{M}_{MN} = U_M{}^{\underline{K}} U_N{}^{\underline{L}} \Big(\delta_{\underline{KL}} + \sum_{\Sigma} \mathcal{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x) \Big)$$



example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- Kaluza-Klein level 4: 20580 scalars (including spin-1 and spin-2 Goldstone modes)

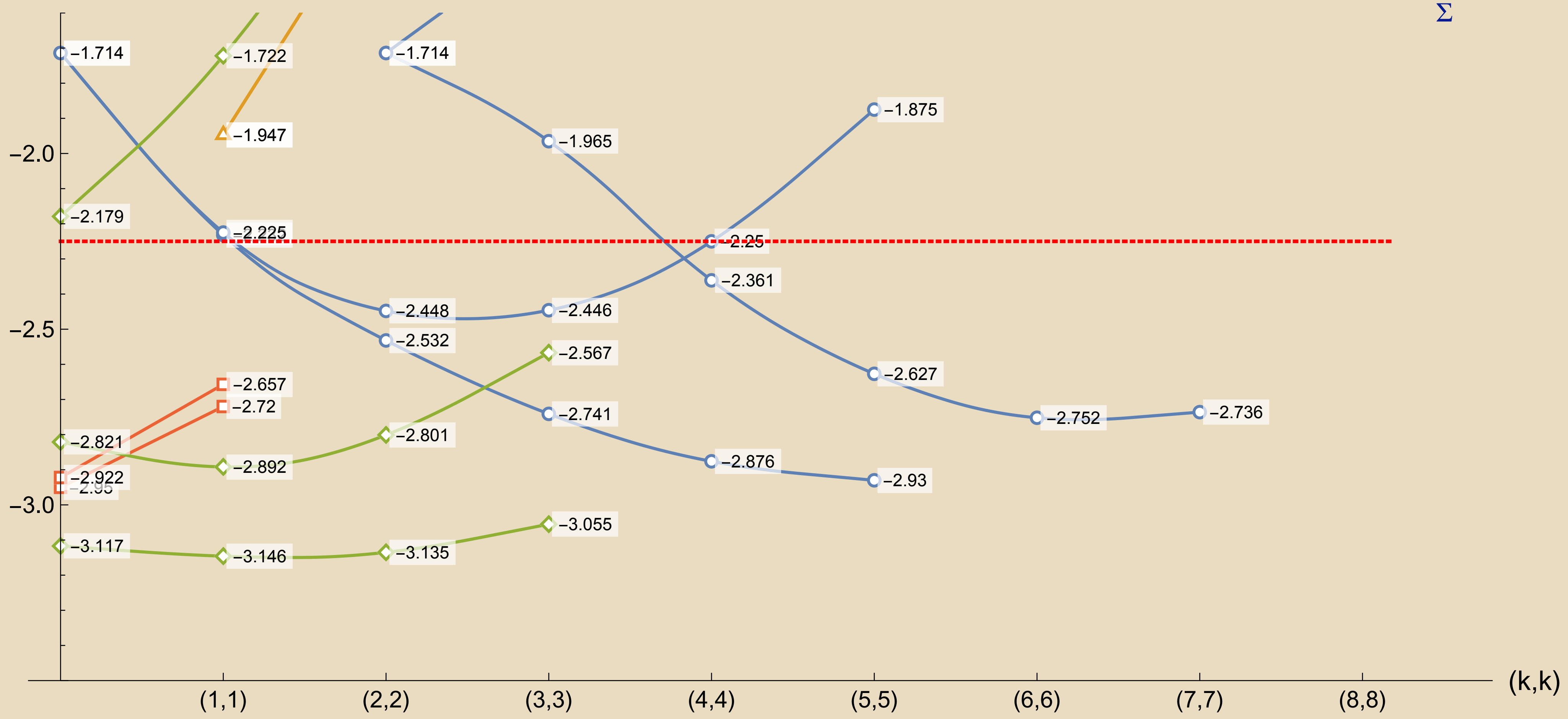
$$\mathscr{M}_{MN} = U_{M}{}^{\underline{K}}\, U_{N}{}^{\underline{L}} \left(\delta_{\underline{KL}} + \sum_{\Sigma} \mathscr{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x)\right)$$



example: non-supersymmetric AdS₄ vacua SO(3) x SO(3)

- Kaluza-Klein level 5: 47040 scalars (including spin-1 and spin-2 Goldstone modes)

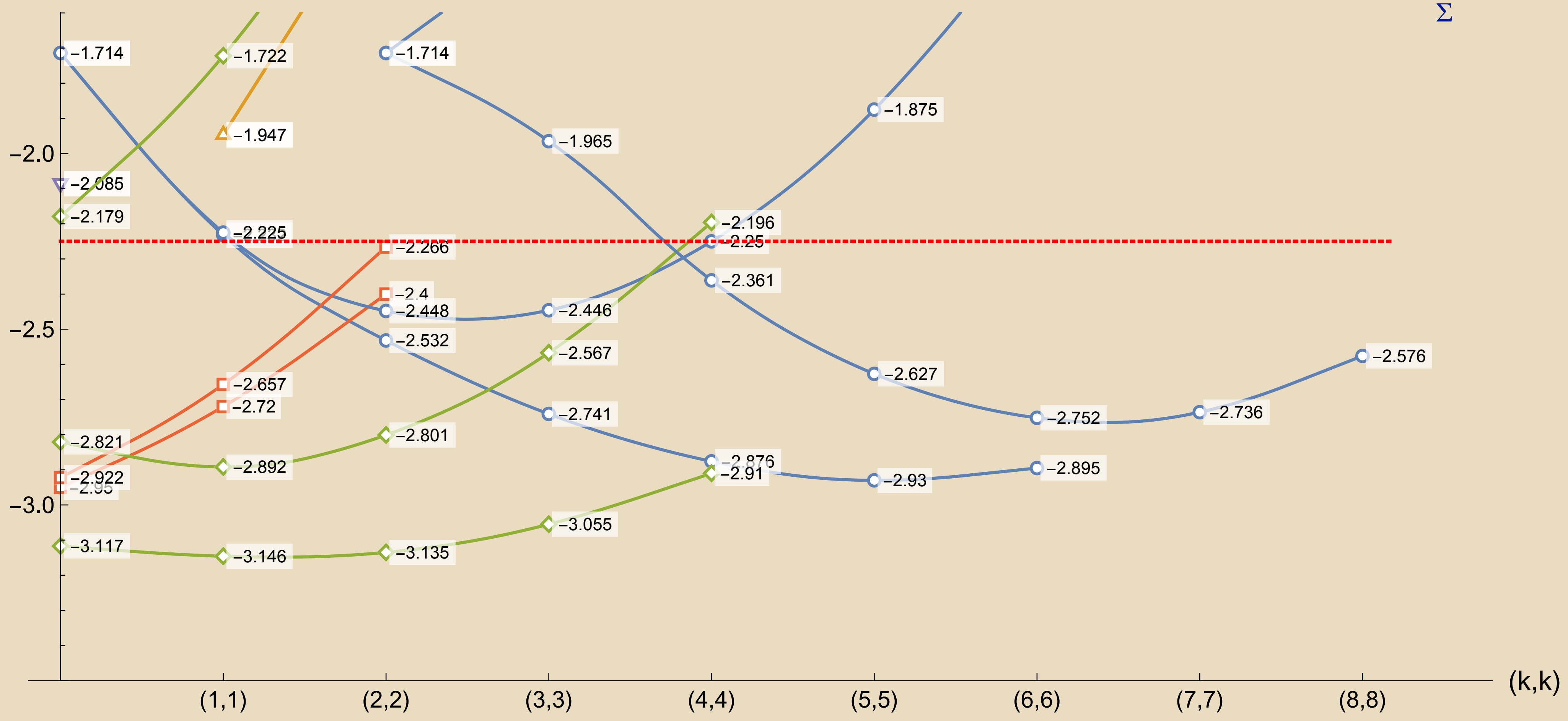
$$\mathscr{M}_{MN} \;=\; U_M{}^{\underline{K}}\, U_N{}^{\underline{L}} \left(\delta_{\underline{KL}} + \sum_{\Sigma} \mathcal{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x)\right)$$



example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- Kaluza-Klein level 6: 97020 scalars (including spin-1 and spin-2 Goldstone modes)

$$\mathcal{M}_{MN} = U_M{}^{\underline{K}} U_N{}^{\underline{L}} \Big(\delta_{\underline{KL}} + \sum_{\Sigma} \mathcal{Y}(y)^{\Sigma}\, j_{\underline{KL},\Sigma}(x) \Big)$$

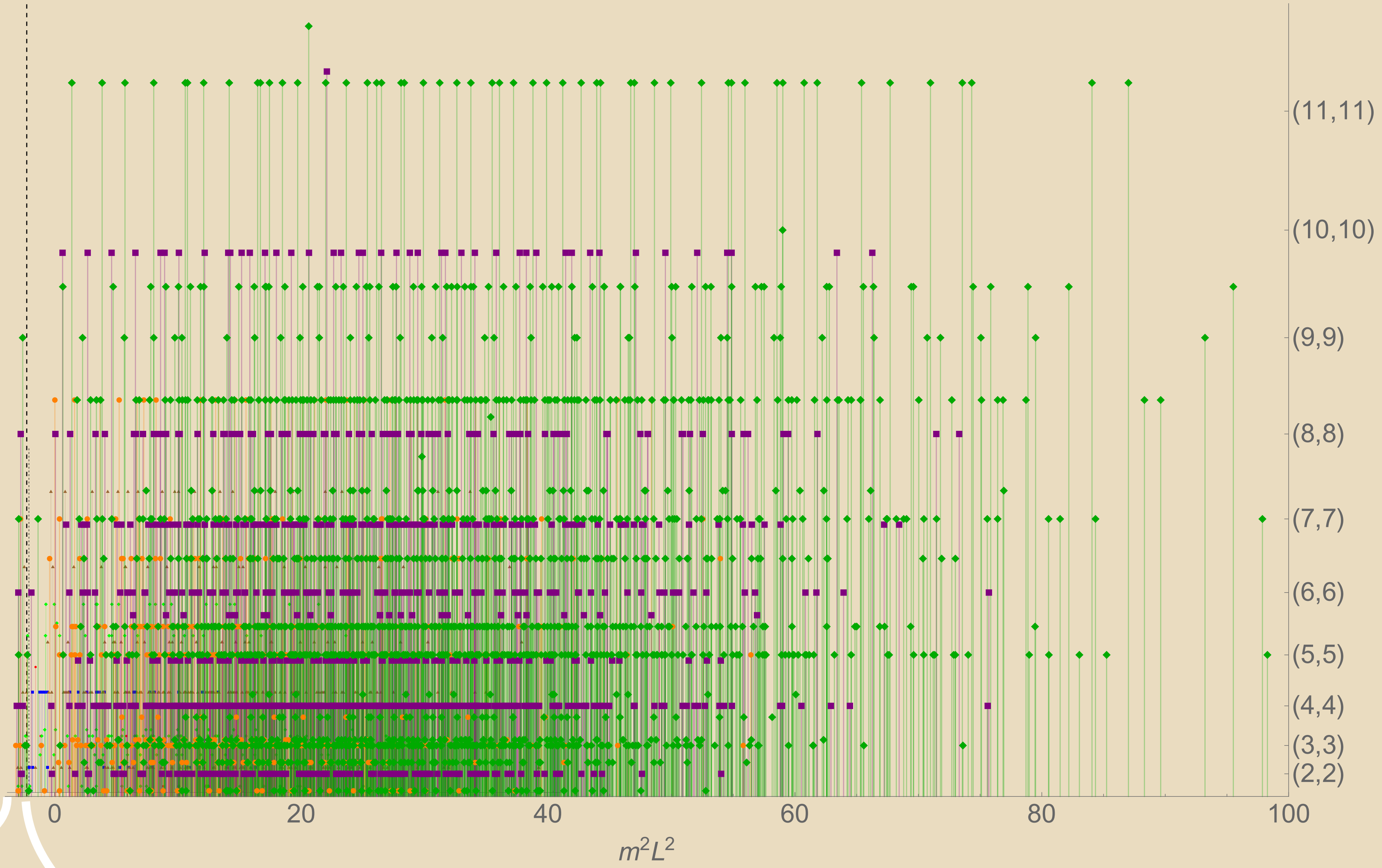


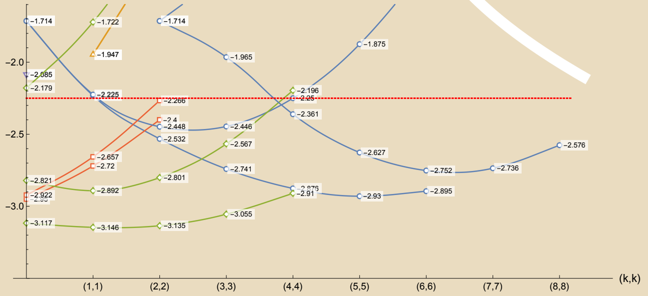
Henning Samtleben

ENS Lyon

example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- full scalar Kaluza-Klein spectrum up to level 6



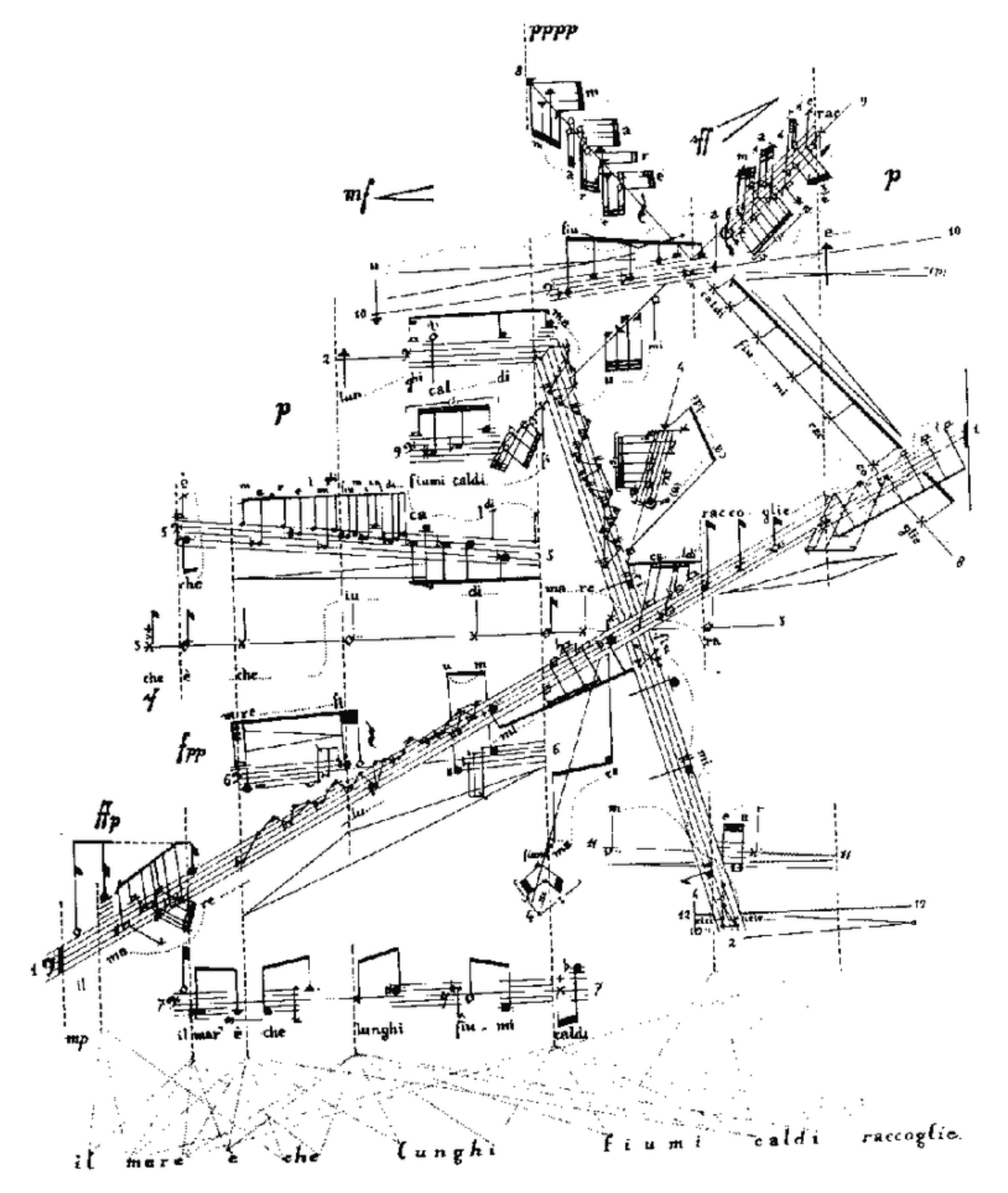


relation to brane-jet instabilities ? [Bena, Pilch, Warner]

example: non-supersymmetric AdS$_4$ vacua SO(3) x SO(3)

- full sound of symmetry

  but unstable!



example: stable non-supersymmetric AdS$_4$ vacua

[A. Guarino, E. Malek, HS]

example: stable non-supersymmetric AdS_4 vacua

- massive IIA admits a consistent truncation on S^6 [Guarino, Jafferis, Varela]

  ⟶ to (dyonic) ISO(7) gauged supergravity [Dall'Agata, Inverso]

  with $\mathcal{N} = 3$ AdS_4 vacuum

- the D=4 scalar potential carries a wealth of AdS vacua:

  ⟶ non-supersymmetric vacua, stable within D=4 supergravity

- most symmetric: $\mathcal{N} = 0$ G_2 vacuum, deformed S^6

  ⟶ no brane-jet instabilities [Guarino, Tarrio, Varela]

example: stable non-supersymmetric AdS$_4$ vacua

- massive IIA admits a consistent truncation on S^6 [Guarino, Jafferis, Varela]
  $\longrightarrow$ to (dyonic) ISO(7) gauged supergravity [Dall'Agata, Inverso]
  with $\mathcal{N}=3$ AdS$_4$ vacuum

- the D=4 scalar potential carries a wealth of AdS vacua:
  $\longrightarrow$ non-supersymmetric vacua, stable within D=4 supergravity

- most symmetric: $\mathcal{N}=0$ G$_2$ vacuum, deformed S^6
  $\longrightarrow$ no brane-jet instabilities [Guarino, Tarrio, Varela]

- ExFT analysis yields the full KK spectrum!
  - analytic mass formula for all scalars:

$$\begin{aligned}
\langle \mathscr{J}\,\mathscr{M}\,\mathscr{J}\rangle \;\propto\; & \frac{1}{5}\left(X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BE}}{}^{\underline{F}}+X_{\underline{EA}}{}^{\underline{F}}X_{\underline{EB}}{}^{\underline{F}}+X_{\underline{EF}}{}^{\underline{A}}X_{\underline{EF}}{}^{\underline{B}}+5\,X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BF}}{}^{\underline{E}}\right)\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Sigma} \\
& +\frac{2}{5}\left(X_{\underline{AC}}{}^{\underline{E}}X_{\underline{BD}}{}^{\underline{E}}-X_{\underline{AE}}{}^{\underline{C}}X_{\underline{BE}}{}^{\underline{D}}-X_{\underline{EA}}{}^{\underline{C}}X_{\underline{EB}}{}^{\underline{D}}\right)\mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{CD},\Sigma} \\
& -\frac{4}{5}\left(X_{\underline{AC}}{}^{\underline{D}}\mathscr{T}_{\underline{B},\Omega\Sigma}+6\,X_{\underline{AC}}{}^{\underline{B}}\mathscr{T}_{\underline{D},\Omega\Sigma}\right)\mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{CD},\Omega} \\
& -\frac{4}{5}\left(X_{\underline{CA}}{}^{\underline{B}}\mathscr{T}_{\underline{C},\Omega\Sigma}+6\,X_{\underline{BC}}{}^{\underline{A}}\mathscr{T}_{\underline{C},\Omega\Sigma}\right)\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Omega} \\
& +12\,\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Omega}\mathscr{T}_{\underline{A},\Omega\Lambda}\mathscr{T}_{\underline{B},\Lambda\Sigma}-\mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{AB},\Omega}\mathscr{T}_{\underline{C},\Omega\Lambda}\mathscr{T}_{\underline{C},\Lambda\Sigma}
\end{aligned}$$

$$\boxed{m^2\,\ell^2=(n+2)\,(n+3)-\frac{3}{2}\,\mathscr{C}_{[n_1,n_2]}}$$

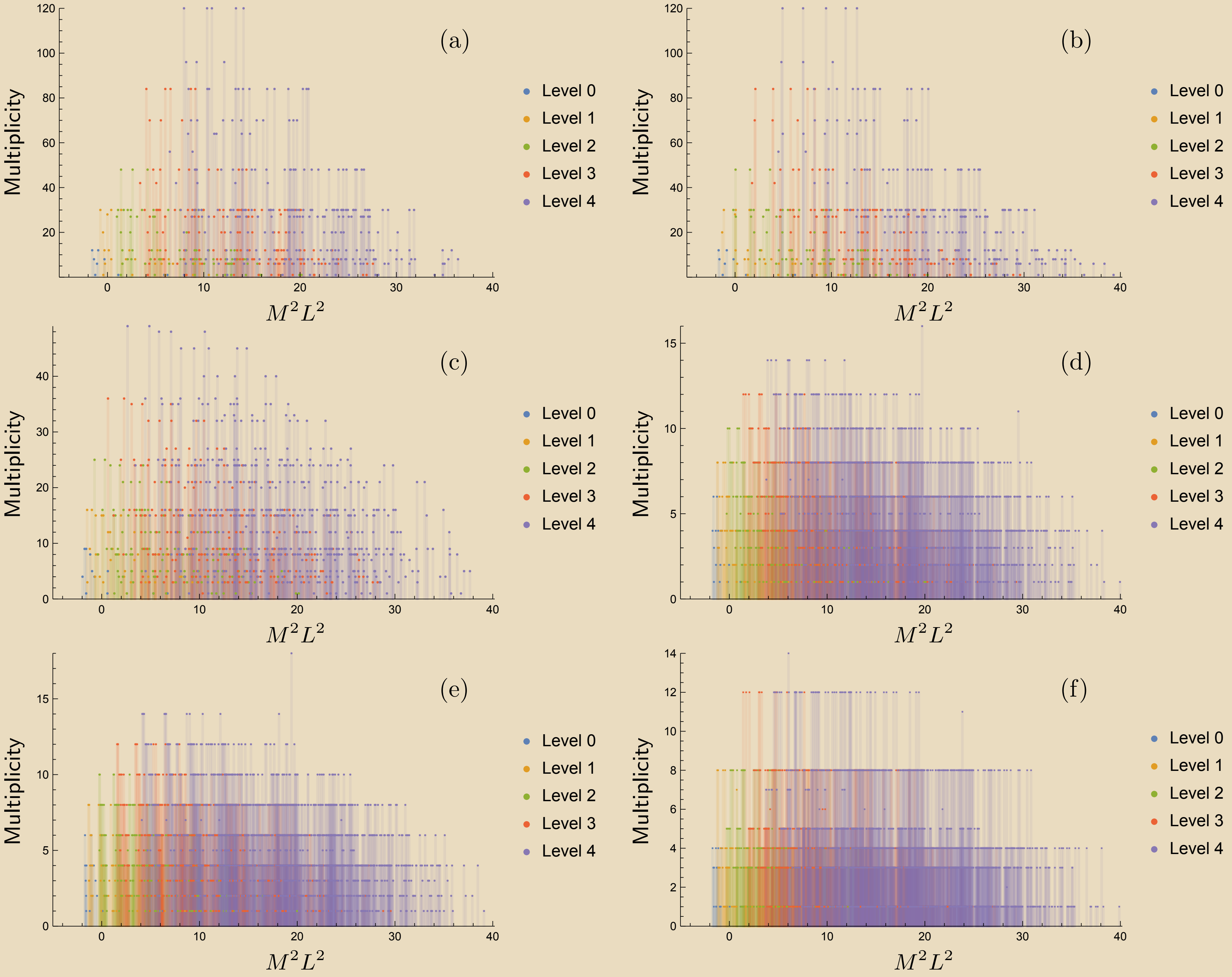
KK level n, G$_2$ Casimir $\mathscr{C}_{[n_1,n_2]}$

- proves stability of the KK spectrum: $m^2\,\ell^2 \geq m_{\rm BF}^2\,\ell^2$

- (perturbatively) stable non-supersymmetric AdS$_4$ vacuum

example: stable non-supersymmetric AdS$_4$ vacua

- likewise: KK-spectra for more non-supersymmetric vacua (numerical) with remaining SU(3), SO(4), U(2), SO(3): all (perturbatively) stable!



example: IIB S-fold backgrounds

[A. Giambrone, E. Malek, HS, M. Trigiante]

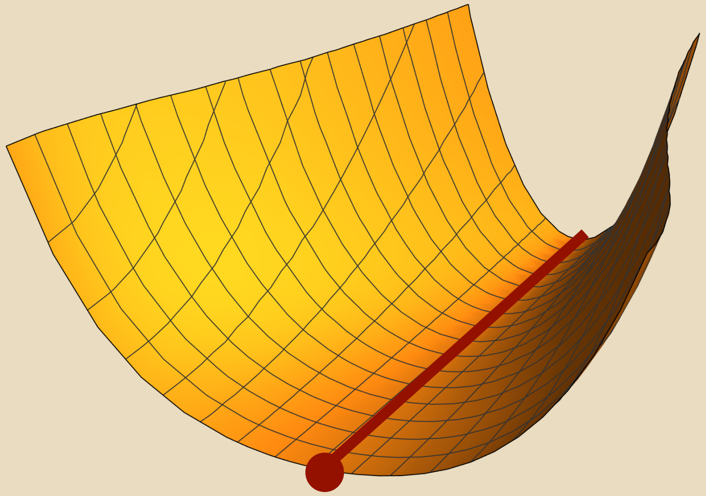
example: IIB S-fold backgrounds

- IIB supergravity admits a consistent truncation on $S^5 \times S^1$ [Inverso, HS, Trigiante]

  $\longrightarrow$ to (dyonic) $(\mathrm{SO}(6) \times \mathrm{SO}(1,1)) \ltimes T^{16}$ gauged supergravity [Dall'Agata, Inverso]

  with $\mathcal{N} = 4$ AdS_4 vacuum

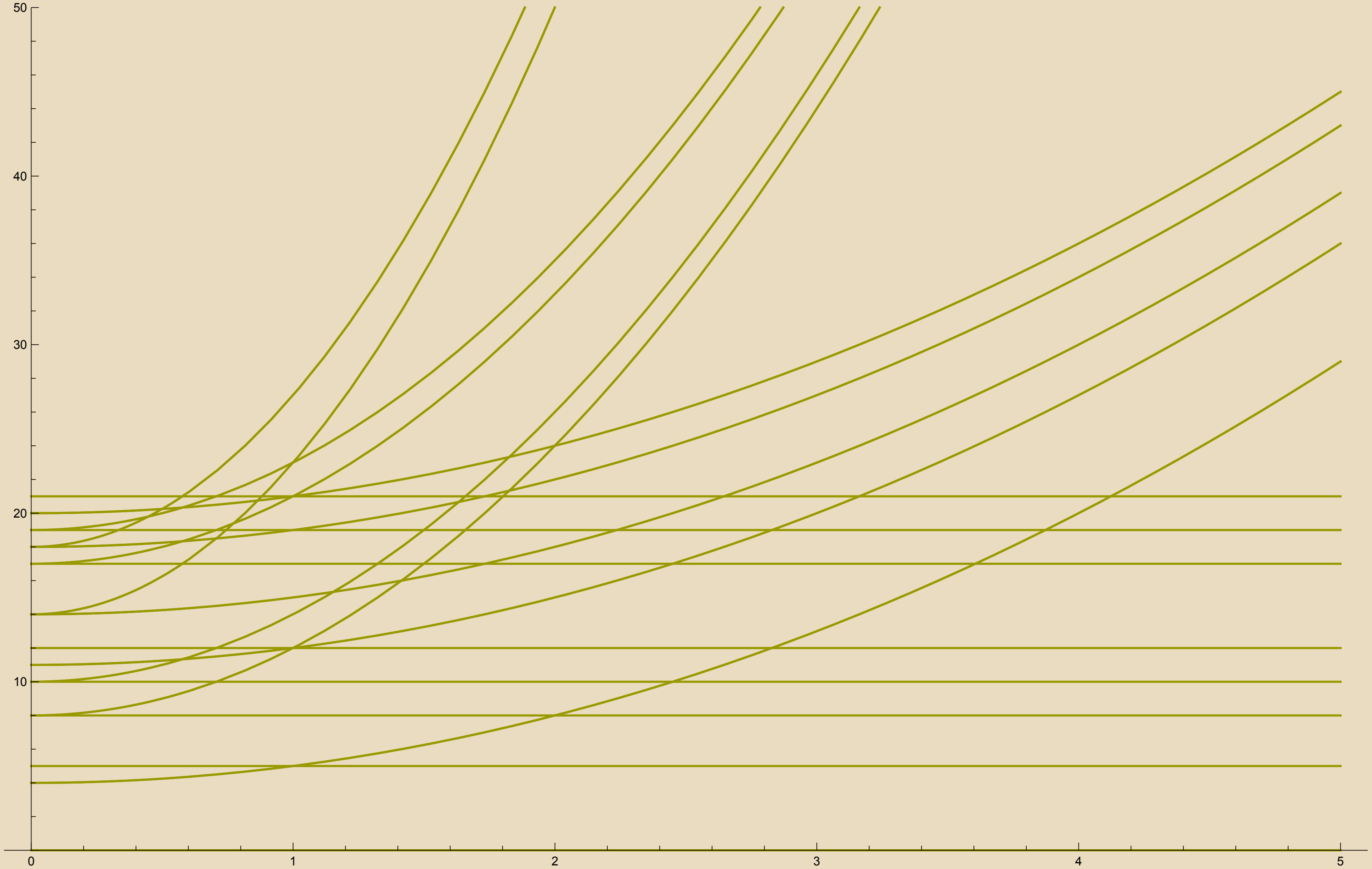
- the D=4 scalar potential carries a wealth of AdS vacua:
  - in particular, a 1-parameter family $\{\chi\}$ of $\mathcal{N} = 2$ vacua [Guarino, Sterckx, Trigiante]
  - with $\mathrm{U}(1)^2$ symmetry
  - and symmetry enhancement $\mathrm{U}(1)^2 \longrightarrow \mathrm{U}(2)$ at $\chi = 0$
  - with uplift to a IIB S-fold background



$\longrightarrow$ compute the modulus-dependent KK-spectrum

example: IIB S-fold backgrounds

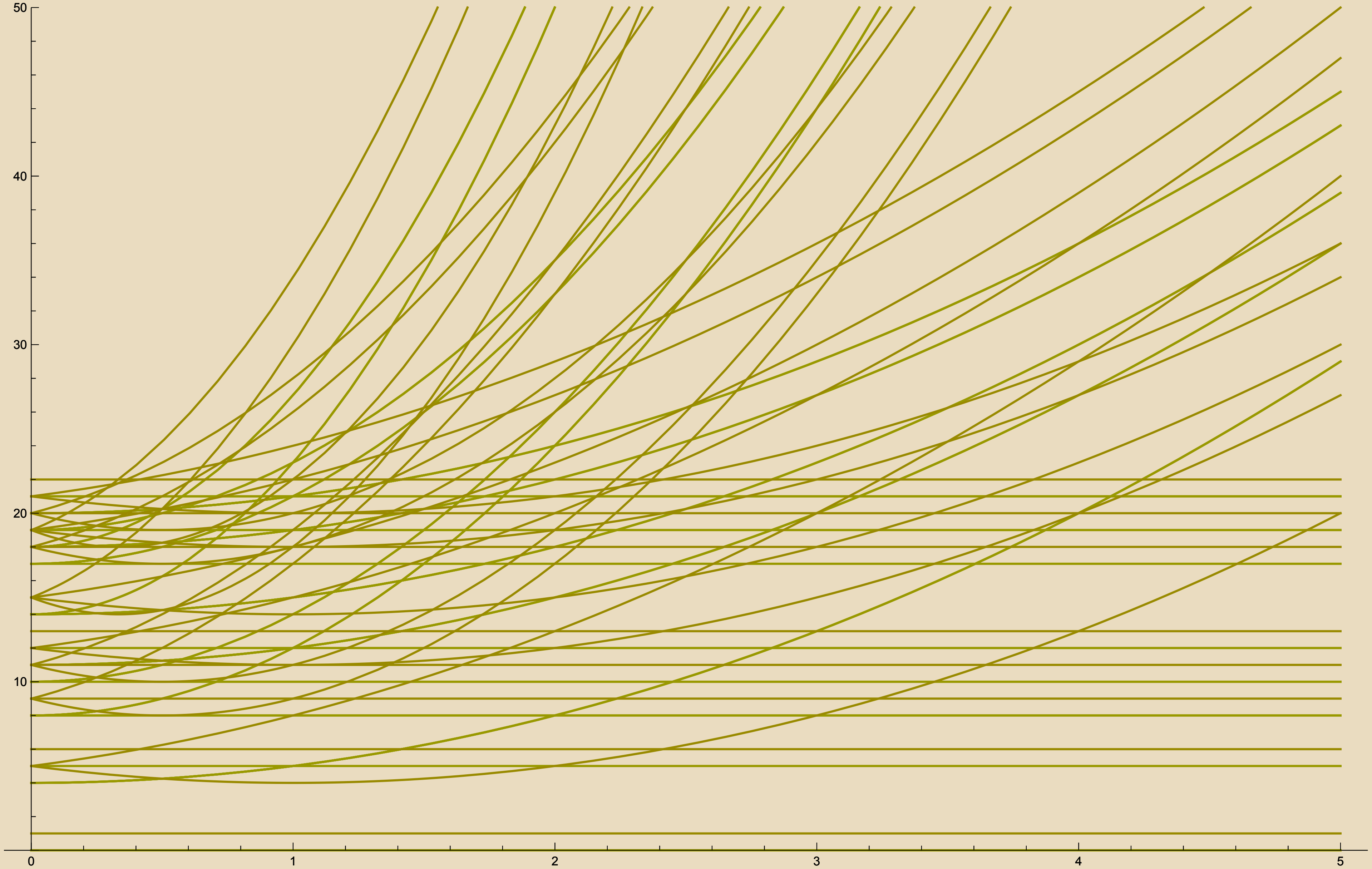
- the spin-2 spectrum as a function of the modulus χ in units of inverse S¹ radius $\frac{2\pi}{T}$



> Kaluza-Klein level $n = 0$

example: IIB S-fold backgrounds

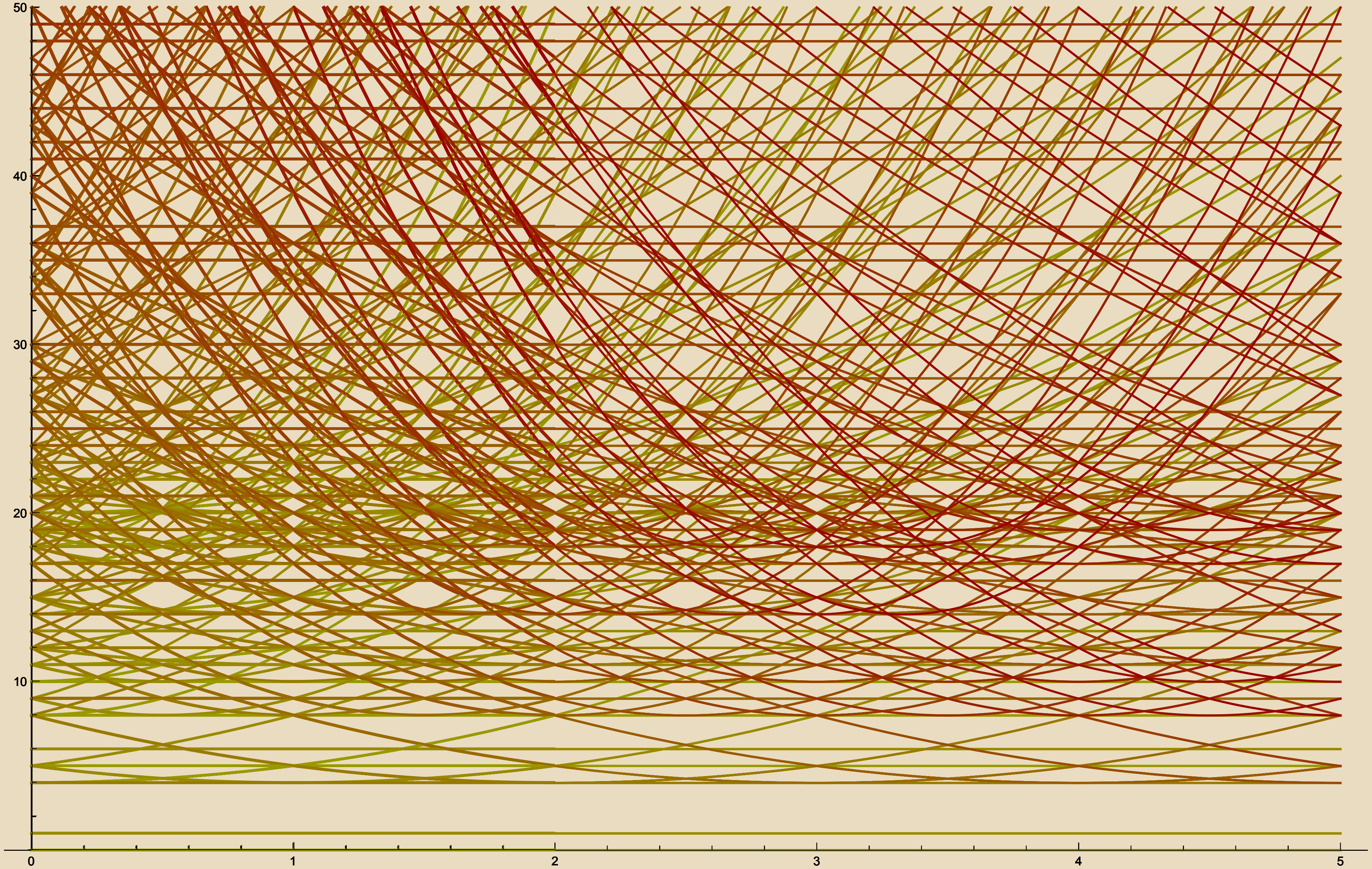
- the spin-2 spectrum as a function of the modulus χ in units of inverse S¹ radius $\frac{2\pi}{T}$



> Kaluza-Klein level $n \leq 1$

example: IIB S-fold backgrounds

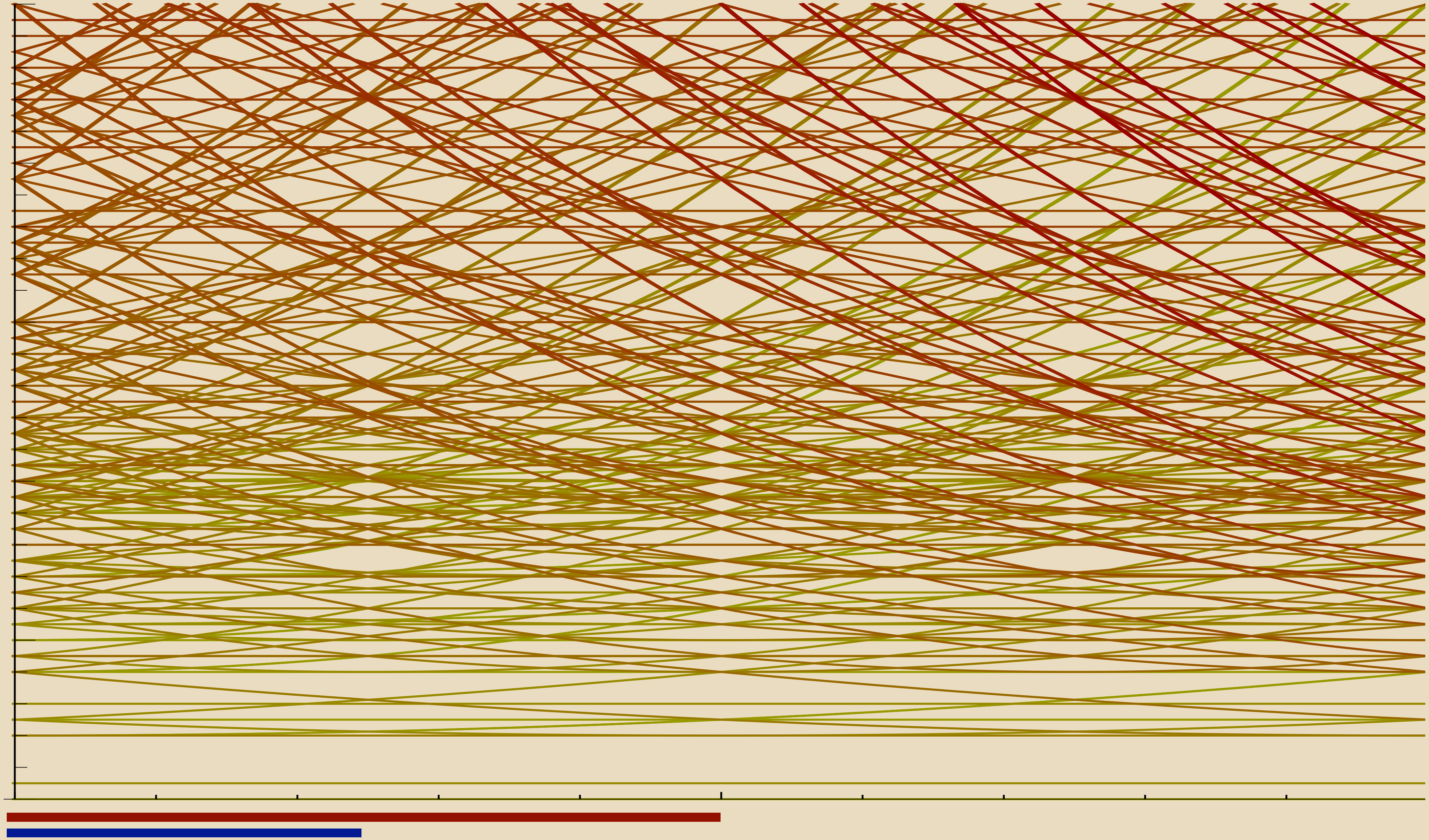
- the spin-2 spectrum as a function of the modulus χ in units of inverse S^1 radius $\frac{2\pi}{T}$



> Kaluza-Klein level $n \leq 10$

example: IIB S-fold backgrounds

- the spin-2 spectrum as a function of the modulus χ in units of inverse S¹ radius $\frac{2\pi}{T}$



- periodic in $\chi \to \chi + \frac{2\pi}{T}$
- symmetry enhancement at $\chi = \frac{\pi}{T}$: $\mathrm{U}(1)^2 \longrightarrow \mathrm{U}(2)$

conclusions

- new tools for the analysis of Kaluza-Klein spectra from ExFT
- Kaluza-Klein spectra entirely encoded in
  - embedding tensor $X_{MN}{}^{K}$ of the lower-dimensional supergravity
  - representation $(\mathscr{T}_M)_\Sigma{}^\Lambda$ of the scalar harmonics

$$
\begin{aligned}
M_{M\Sigma,N\Omega} \;\propto\; & \frac{1}{3}\, X^{\rm s}_{ML}{}^{K}\, X^{\rm s}_{NK}{}^{L}\, \delta^{\Sigma\Omega} + 2\left(X^{\rm s}_{MK}{}^{N} - X^{\rm s}_{NM}{}^{K}\right) \mathcal{T}_{K,\Omega\Sigma} \\
& -6\,\left(\mathbb{P}^{K}{}_{M}{}^{L}{}_{N} + \mathbb{P}^{M}{}_{K}{}^{L}{}_{N}\right) \mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma} + \frac{8}{3}\, \mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma} \qquad \text{etc.}
\end{aligned}
$$

- access to vacua with few or no (super-)symmetries
- applications to holography, stability analysis, moduli-dependence
- new windows into $\mathcal{N}=8$ supergravity

conclusions

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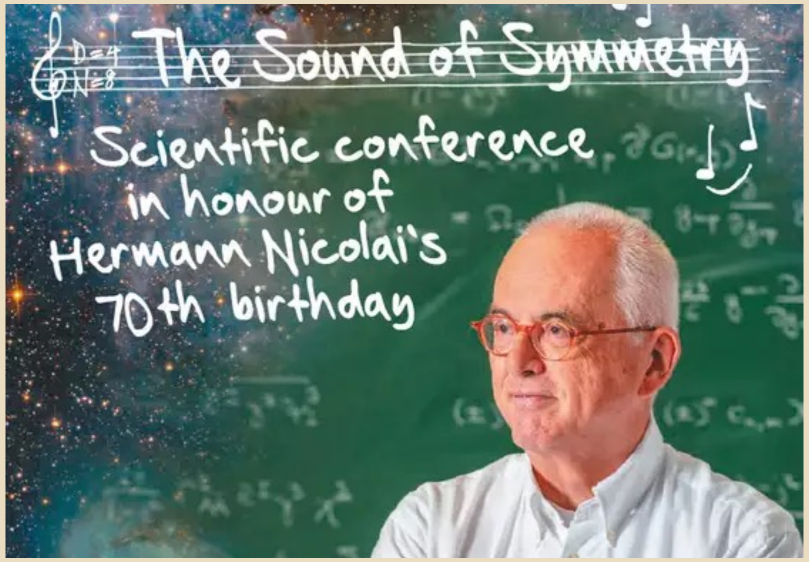

- much more to explore!

Happy Birthday Hermann!