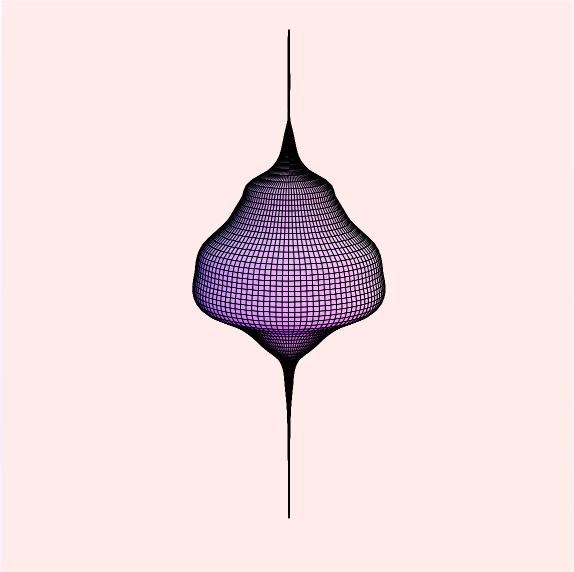
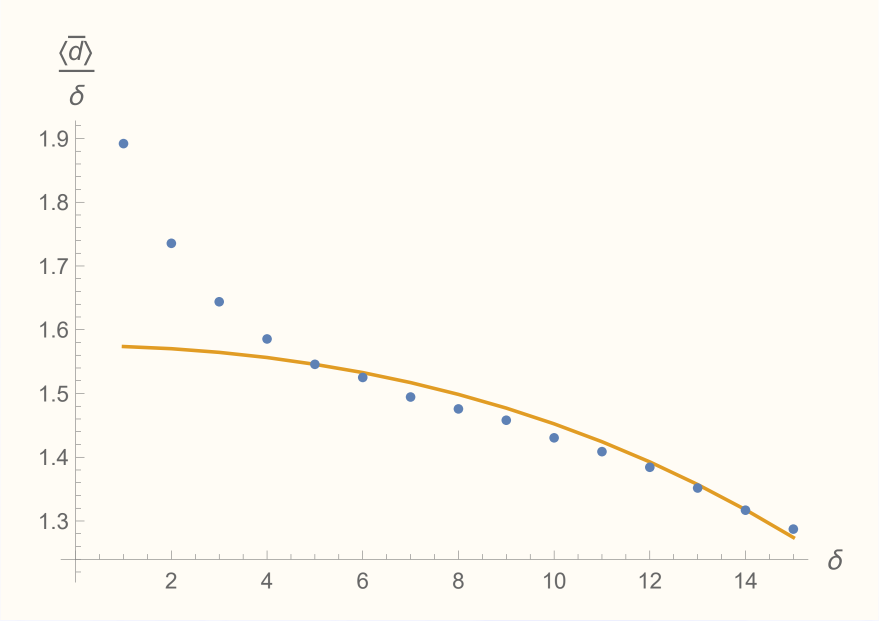


J. Ambjørn, J. Jurkiewicz, R.L., PRL 93 (**2004**) 131301



N. Klitgaard, R.L., EPJ C 80 (**2020**) 990

Quantum Gravity Demystified

Renate Loll
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14 Sep 2022

My perspective on quantum gravity

Aim: construct a fundamental theory of quantum gravity as a non-perturbative, diffeomorphism-invariant quantum field theory of dynamical geometry and study its properties in a Planckian regime.

This presents major technical, physical and conceptual challenges: dealing with QFT infinities and the absence of a fixed background spacetime, devising appropriate numerical and renormalization methods, (re-)deriving the classical limit and phenomenology.

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This is possible. Major advances towards this goal have been made in the research program of ***Causal Dynamical Triangulations (CDT).*** It sets a concrete frame of reference - beyond matters of taste and style - for what we may reasonably expect to be able to achieve in quantum gravity.

Why should you care?

quantum gravity: nontrivial, unexpected results despite non-exotic ingredients; functioning computational framework (*= our "lab"*) to evaluate quantum observables beyond perturbation theory; *"CDT is to gravity what lattice QCD is to nonabelian gauge theory"*

symmetry: diffeomorphism symmetry is very different from local gauge symmetry; we finally understand how to implement it consistently in a nonperturbative quantum theory of gravity

"demystification": quantum (field) theory and general relativity are perfectly compatible; CDT provides bottom-up realization of QG: causal structure is essential, unitarity is realized

cosmology: most likely phenomenological predictions will involve early-universe quantum physics, but derived from the full theory *without* an a priori symmetry reduction (unlike quantum cosmology)

What's the problem with quantum gravity?

- General Relativity = theory ***of*** spacetime, not ***on*** (a fixed) spacetime
- quantum theory based on perturbative split $g_{\mu\nu}(x)= \eta_{\mu\nu}^{Mink} + h_{\mu\nu}(x)$ on a fixed Minkowskian background is nonrenormalizable

M. Goroff, A. Sagnotti, NPB 266 (1986) 709

- standard relativistic quantum field theory (QFT) not applicable, no blueprint beyond perturbation theory (except nonperturbative lattice QCD, ***but*** this has a fixed background, different gauge symmetry)
- no experiments or observations to guide theory-building
- (nonperturbative) QG ≲ 2000: large variety of approaches;
  "**We don't know *what* to compute, and we don't know *how*.**"
- QG ≳ 2000 (post extended-objects era): renaissance of "good old QFT"/the path integral, *we have learned how and what to compute*

R.L. et al.: "Quantum Gravity in 30 Questions", arXiv: 2206.06762

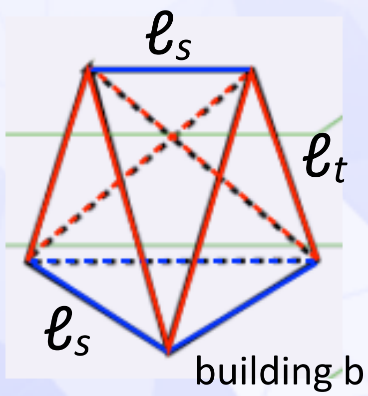
Causal Dynamical Triangulations: the basics

$$Z=\int_{g\in\frac{Lor(M)}{Diff(M)}}\mathcal{D}g\ \mathrm{e}^{iS_{\mathrm{grav}}[g]}$$

- gravitational path integral over metric d.o.f., *nonperturbative* (NP), background-independent, Lorentzian signature, 4D, not “grand-unified”
- building on Euclidean “dynamical triangulations”, define a new NP Lorentzian 2D path integral: *exactly soluble* ⇒ *signature matters!*

J. Ambjørn, R.L., preprint AEI-064, NPB 536 (1998) 407

- CDT combines emphasis on geometry with path integral covariance (no split $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, no 3 + 1 decomposition)
- uses a regularized version of the space of geometries, $\mathcal{G}(M)= Lor(M)/Diff(M)$: piecewise flat, simplicial manifolds T
- minimal GR ingredients + standard Q(F)T methods, adapted to dynamical geometry + numerical methods = **new territory near ℓ_{Pl}**



building block of 4D CDT

- 2D random geometry is a hot topic in maths! **S. Sheffield, arXiv:2203.02470**

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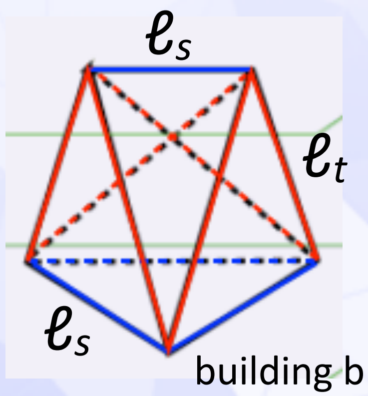

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Putting quantum gravity on a lattice, correctly

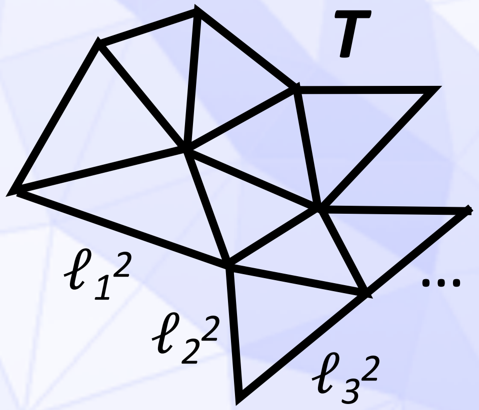
General strategy: lattice acts as a regulator, with UV cutoff a; search for a continuum limit by approaching a second-order phase transition in the limit $a \to 0$ while renormalizing bare couplings appropriately; attain *universality* (independence of regularization); this is **not** "discrete QG"

- "reaches where other methods don't", subject to numerical limitations; if it exists, continuum theory is essentially *unique*

- "naïve" lattice QG (≥ 1979): put various first-order formulations of GR (tetrad $e_\mu{}^A$ + spin connection $\omega_\mu{}^{AB}$) on a fixed hypercubic lattice; problem: diffeomorphism symmetry badly broken; no interesting results

- "not-so-naïve" lattice QG (≥ 1981): based on "GR without coordinates" $(M, g_{\mu\nu}(x)) \to (T, \{\ell_i{}^2, i=1,\dots,n\})$, $S_{grav}[g_{\mu\nu}] \to S^{Regge}(T, \{\ell_i{}^2\})$

T. Regge, Nuovo Cim. A19 (1961) 558



- diffeo-invariance manifest, work directly on $\mathcal{G}(M)$; CDT ($\ell^2 = \pm a^2$) implementation is labelling-invariant

The path integral (PI) according to CDT

$$Z = \int_{\mathcal{G}(M)} \mathcal{D}g\ \mathrm{e}^{iS_{\mathrm{grav}}[g]} \to Z^{\mathrm{CDT}} = \lim_{a\to 0} \sum_{\substack{inequiv.\\ causal\\ triang.\,T}} \frac{1}{C(T)}\, \mathrm{e}^{iS^{\mathrm{Regge}}[T]}$$

bare action

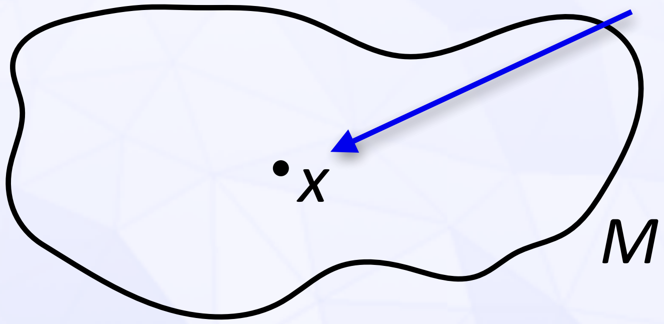
discrete symmetries of T

- usually, can't evaluate complex PI, do Euclidean *∫Dg exp(-S^{eu})* instead
  - ☑ CDT has a well-defined analytic continuation ("Wick-rotation")
- usually, hard to renormalize compatible with diffeomorphism symmetry
  - ☑ CDT has no residual symmetries, has a geometric cutoff a
- usually, PI highly divergent, no unique renormalization;
  - ☑ numerical evidence of exponential bound on # of configurations
- usually, cannot do any computations, PI not Gaussian
  - ☑ CDT amenable to Monte Carlo simulation; get quantitative results
- usual problem: why should PI lead to a unitary theory?
  - ☑ CDT reflection-positive w.r.t. discrete "proper time"

CDT quantum gravity: results

- we have a computational framework — what can we do with it?
- physics of ***quantum spacetime*** is captured by invariant ***quantum observables*** $\hat{\mathcal{O}}$:

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z}\int \mathcal{D}g\ \mathcal{O}[g]\,\mathrm{e}^{-S_{\rm grav}[g]}$$



"the point x" is an unphysical concept

- observables in Yang-Mills theory are local scalars, like $F^{\mu\nu}F_{\mu\nu}$, but observables in pure gravity are nonlocal integrals of scalars, like $\int_M d^4x\sqrt{g}\,R(x)$
- "expectation management": your favourite (semi-)classical question will not a have Planckian implementation (this is a feature)
- quantum gravity signature: CDT predicts a reduction 4 → 2 of the ***spectral dimension*** @ℓ_{Pl}, J. Ambjørn, J. Jurkiewicz, R.L., PRL 95 (2005) 171301 reproduced across approaches — universal in QG? S. Carlip, CQG 34 (2017) 193001

Key result: emergence of classicality from CDT

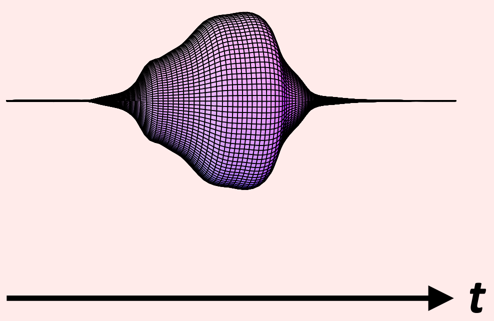
The measured average shape $\langle V_3(t) \rangle$ (spatial volume as a function of proper time) of the dynamically generated quantum spacetime in CDT matches that of a classical de Sitter space.

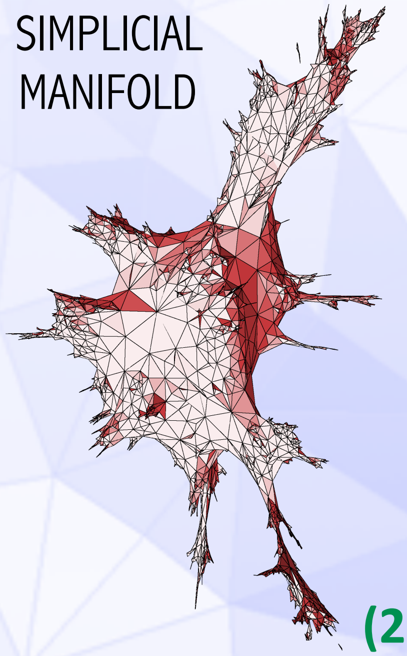
J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., PRL 100 (2008) 091304, PRD 78 (2008) 063544

Since the global shape of the universe is just a single mode of the metric, we cannot conclude that it ***is*** a (Euclidean) de Sitter space S^4, with line element

$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2_{(3)} \,.$$

MC snapshot of the shape **❮***V(t)***❯** *of the universe*

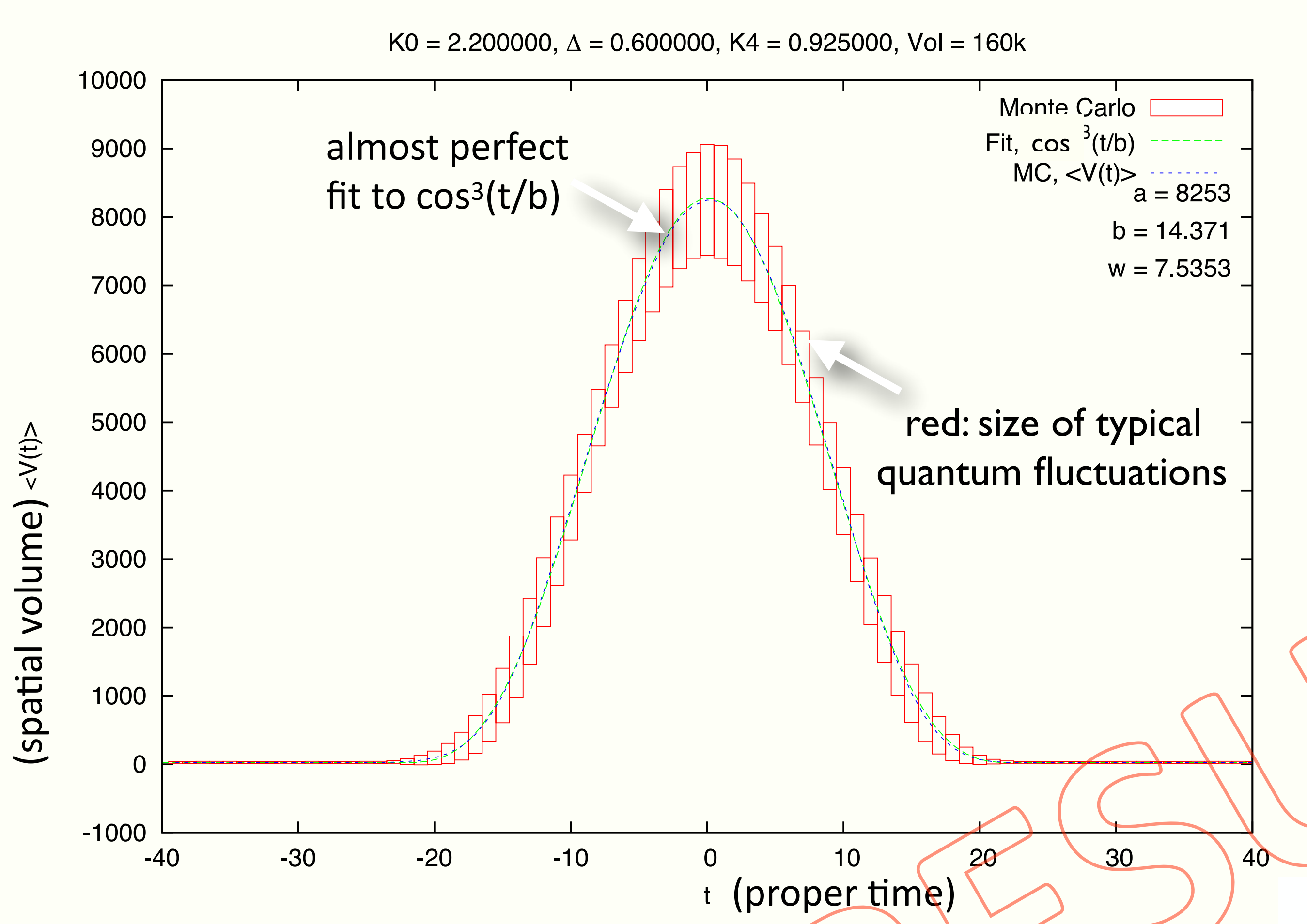




What about the ***local geometry*** of this quantum universe?

Can we attribute ***local curvature*** to a non-smooth metric space? $R^{\kappa}{}_{\lambda\mu\nu}(x) = [g, \partial g, \partial^2 g] = ?$ Recently, we defined, tested and measured a well-defined notion of ***quantum Ricci curvature*** applicable in a Planckian regime. **N. Klitgaard & RL, PRD 97 (2018) 0460008 and 106017, Eur. Phys. J. C80 (2020) 990, J. Brunekreef & RL, PRD 103 (2021) 026019**

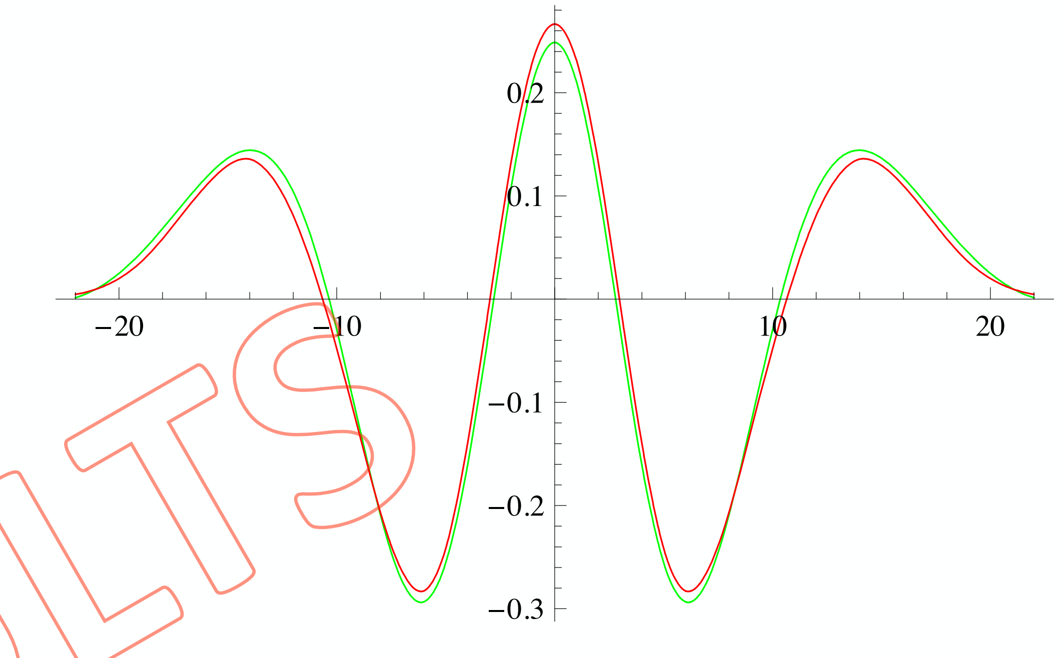
The universe is de Sitter-shaped



almost perfect fit to $\cos^3(t/b)$

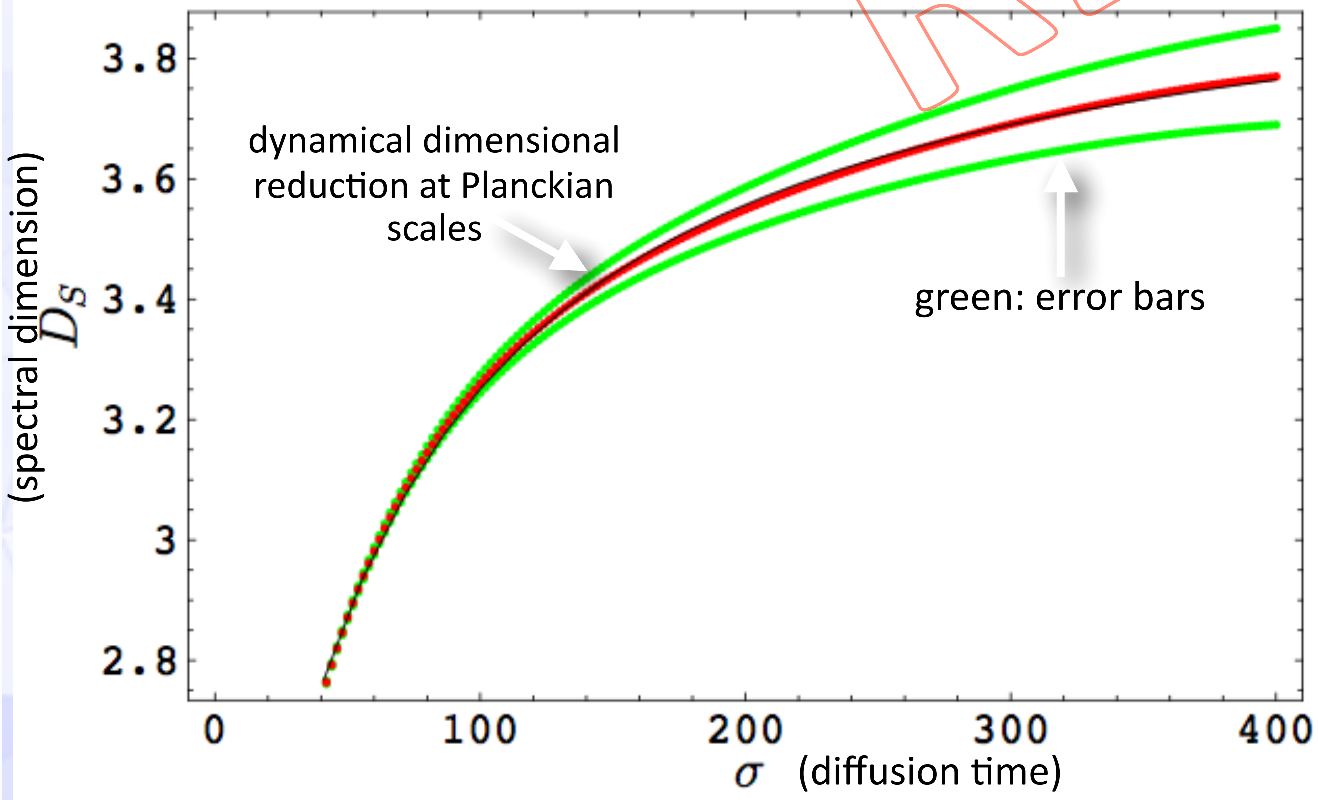
red: size of typical quantum fluctuations

Volume fluctuations around de Sitter



(low-lying eigenmode matches with semiclassical expectation)

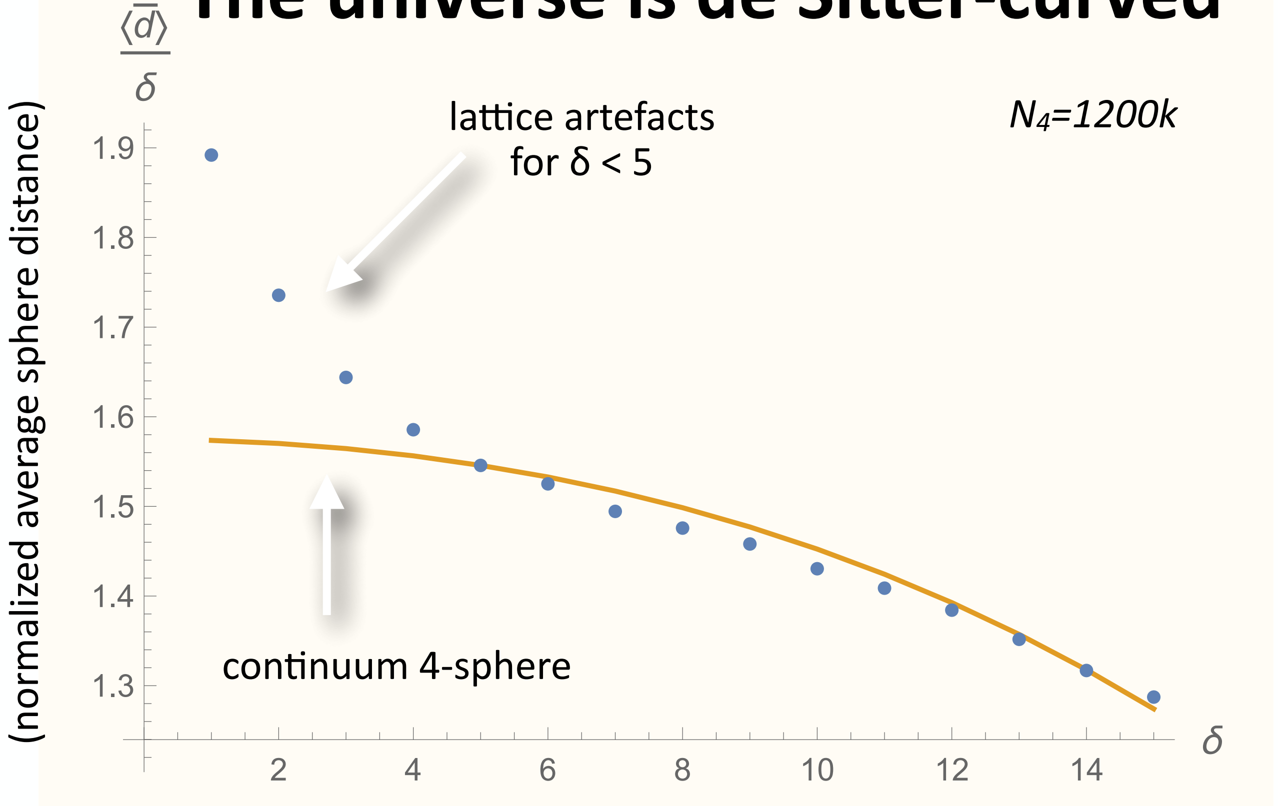
Spectral dimension of the universe



dynamical dimensional reduction at Planckian scales

green: error bars

The universe is de Sitter-curved



lattice artefacts for δ < 5

continuum 4-sphere

(quantum Ricci curvature of the dynamically generated dS universe)

Relation to our actual universe

CDT predicts a universe **with positive Λ**, which on large scales is extended and four-dimensional, and whose **shape and average curvature** are compatible with those **of a de Sitter space**, matching our current understanding of the very early universe.

Remarkably, these properties have been derived *from first principles* in the full quantum theory; we also have in principle access to (diffeomorphism-invariant) correlation functions.

At what scales and how does gravity interact with matter?

$$Z = \int\limits_{\mathcal{G}(M)} \mathcal{D}g \int\limits_{\Phi} \mathcal{D}\phi \; \mathrm{e}^{i(S_{\mathrm{grav}}[g]+S_{\mathrm{matter}}[g,\phi])}$$

Investigations of CDT coupled to matter fields have not found a significant impact on the geometry ⇒ "matter doesn't matter at ℓ_{Pl}"?

Current ambitions and prospects

CDT QG is in a position to reap the benefits of a nonperturbative framework that can produce "numbers" (= expectation values of quantum observables) without relying on ad-hoc assumptions. The art is to ***identify (more) observables*** that can be reliably measured inside the available scale window, while yielding interesting physics.

The new ***quantum Ricci curvature*** Y. Ollivier, J. Funct. Anal. 256 (2009) 810 opens exciting avenues towards a relation with early-universe physics:

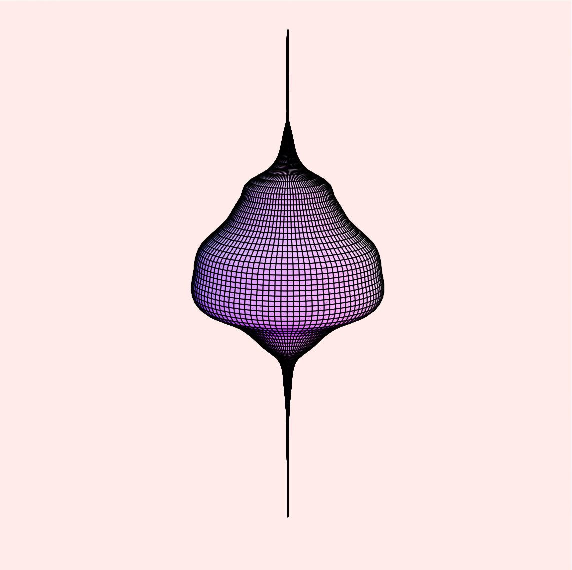
- use it to quantify the *local* effect of a massive Planckian particle and compare with a semiclassical expectation G. Clemente, R.L., w.i.p.
- use it to examine the string-like singularity spontaneously forming in the bifurcation phase of CDT, as a possible *candidate of early-universe structure formation (a primordial black hole?)*
- compute its *two-point function,* compare with QFT on dS space

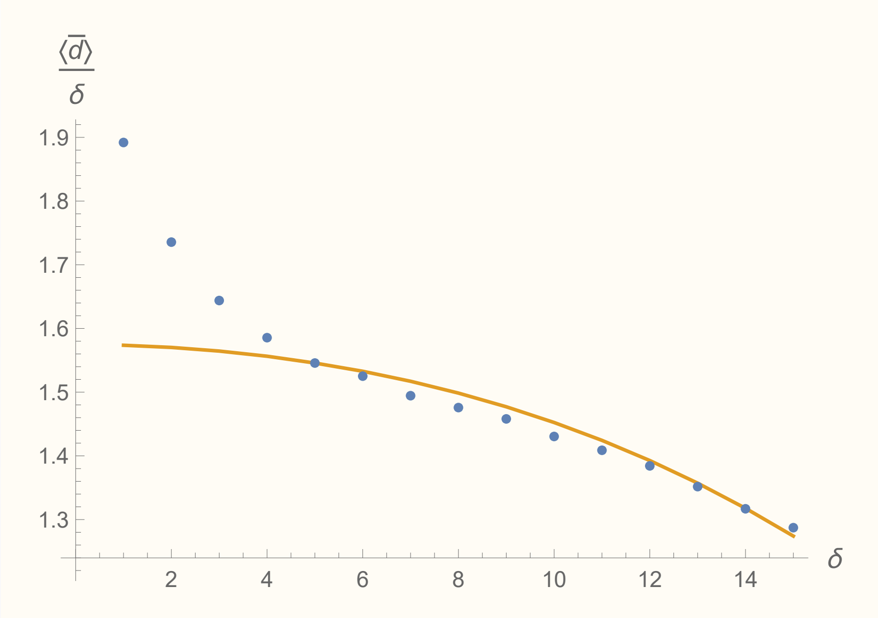
Outlook

- ***genuine progress*** in nonperturbative quantum gravity: instead of comparing "approaches", started to compare observables/results, e.g. with those obtained by functional RG methods **F. Saueressig et al.**
- CDT quantum gravity is a rare example of ***spacetime emergence***
- ***work in progress***: quantum measures of homogeneity & isotropy **A. Silva, R.L.**; extend earlier RG flow analysis **J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, R.L., Front. in Phys. 8 (2020) 247** and look for independent evidence of asymptotic safety; measure curvature correlators, starting in 2D **J. van der Duin, R.L.**
- ***challenge:*** match nonperturbative and perturbative observables

⇒ watch this space!

CDT reviews: J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., Phys. Rep. 519 (2012) 127, arXiv: 1203.3591; R.L., Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669




$\frac{\langle \bar{d} \rangle}{\delta}$
1.9
1.8
1.7
1.6
1.5
1.4
1.3
2
4
6
8
10
12
14
δ

Thank you!