tales of the $\mathcal{N}=8$ quartic vertex

in light-cone superspace

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Electrodynamics in the light-cone gauge

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$A_\mu = (A_+ , A_- , A , \bar{A})$$

Set : $A_- = -A^+ = 0$

x^+ is light-cone time $\longrightarrow$ single constraint from $\partial_\mu F^{\mu\nu} = 0$

$$\mathcal{L} = \underset{-}{\bar{A}} \, \Box \, \underset{+}{A}$$

Symmetry-based approach

Free theory for massless fields in four dimensions

$$\mathcal{L} = \int d^3x \; \bar{\phi} \, \Box \, \phi \qquad\qquad \mathcal{H} = \int d^3x \, \partial^+ \bar{\phi} \, \delta_{\mathcal{H}} \, \phi$$

$$\delta_{\mathcal{H}}\phi \equiv \partial^- \phi = \{\phi, H\} = \frac{\partial\bar{\partial}}{\partial^+} \, \phi \qquad\qquad \frac{1}{\partial^+}\phi(x^-) \equiv \int dy^- \; \epsilon(y^- - x^-) \, \phi(y^-)$$

The Hamiltonian picks up corrections when interactions are switched on

Symmetry-based approach

Start with an ansatz for the Hamiltonian

$$\delta_{\mathcal{H}}^{g}\phi = g\,K\ \partial^{+\mu}\left[\bar{\partial}^{B}\,\partial^{C}\,\partial^{+\rho}\,\phi\ \bar{\partial}^{D}\,\partial^{E}\,\partial^{+\sigma}\,\phi\right]$$

and use the requirement of Lorentz invariance to fix its form

$$\delta_J\,\phi = (x\bar{\partial} - \bar{x}\partial - \lambda)\,\phi$$

$$[\,\delta_J\,,\,\delta_{\mathcal{H}}^{g}\,]\,\phi = 0 \quad\Longrightarrow\quad B + D - C - E \,=\, \lambda$$

Light-cone Poincaré algebra

$$[\mathcal{H}, J^{+-}] \;=\; -i\mathcal{H} \;, \qquad [\mathcal{H}, J^{+}] = -iP \;, \quad [\mathcal{H}, \bar{J}^{+}] \;=\; -i\bar{P}$$

$$[P^{+}, J^{+-}] \;=\; iP^{+} \;, \qquad [P^{+}, J^{-}] \;=\; -iP \;, \quad [P^{+}, \bar{J}^{-}] \;=\; -i\bar{P}$$

$$[P, \bar{J}^{-}] \;=\; -i\mathcal{H} \;, \qquad [P, \bar{J}^{+}] \;=\; -iP^{+} \;, \quad [P, J] \;=\; P$$

$$[\bar{P}, J^{-}] \;=\; -i\mathcal{H} \;, \qquad [\bar{P}, J^{+}] \;=\; -iP^{+} \;, \quad [\bar{P}, J] \;=\; -\bar{P}$$

$$[J^{-}, J^{+-}] \;=\; -iJ^{-} \;, \qquad [J^{-}, \bar{J}^{+}] \;=\; iJ^{+-} + J \;, \quad [J^{-}, J] \;=\; J^{-}$$

$$[\bar{J}^{-}, J^{+-}] \;=\; -i\bar{J}^{-} \;, \qquad [\bar{J}^{-}, J^{+}] \;=\; iJ^{+-} - J \;, \quad [\bar{J}^{-}, J] \;=\; -\bar{J}^{-}$$

$$[J^{+-}, J^{+}] \;=\; -iJ^{+} \;, \qquad [J^{+-}, \bar{J}^{+}] \;=\; -i\bar{J}^{+} \;,$$

$$[J^{+}, J] \;=\; J^{+} \;, \qquad [\bar{J}^{+}, J] \;=\; -\bar{J}^{+}$$

Yang-Mills theory in the light-cone gauge

Cubic vertex for spin 1 (self-interactions)

$$\delta^{g}_{\mathcal{H}}\,\phi = \left[\,\frac{\bar{\partial}}{\partial^{+}}\phi\,\phi - \phi\,\frac{\bar{\partial}}{\partial^{+}}\phi\right] \,=\, 0$$

requires introduction of anti-symmetric constant f^{abc}

Bengtsson-Bengtsson-Brink, 1983

Algebra closure at next order (quartic vertices) requires that

$$f^{abc}\,f^{bde} + f^{abd}\,f^{bec} + f^{abe}\,f^{bcd} \,=\, 0$$

Ananth-Kar-Majumdar-Shah, 2017

Light-cone superspace

$$\theta^m \quad ; \quad \bar{\theta}_m \quad ; \quad m = 1 \ldots 4$$ SU(4)

$$d^m = -\frac{\partial}{\partial \bar{\theta}_m} - \frac{i}{\sqrt{2}} \theta^m \, \partial^+ \quad ; \quad \bar{d}_n = \frac{\partial}{\partial \theta^n} + \frac{i}{\sqrt{2}} \bar{\theta}_n \partial^+$$

$$y = \left(x, \, \bar{x}, \, x^+, \, y^- \equiv x^- - \frac{i}{\sqrt{2}} \theta^m \, \bar{\theta}_m \right)$$

symmetry algebra entirely determines the action for $\mathcal{N} = 4$ Yang-Mills theory

Ananth-Brink-Kim-Ramond, 2004

$\mathcal{N}=4$ Yang-Mills theory

The $\mathcal{N}=4$ superfield

$$\phi\,(y) \;=\; \frac{1}{\partial^+}\,A\,(y) + \frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) \;+\; \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\bar{C}_{mn}\,(y)$$

$$+\frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y) \;+\; \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)$$

Bengtsson-Bengtsson-Brink, 1983

$$C^{m\,4} = \frac{1}{\sqrt{2}}(A_{m+3} + i\,A_{m+6})$$

chiral | anti-chiral | inside-out

$$d^m\phi(y) = 0 \quad ; \quad \bar{d}_n\bar{\phi}(y) = 0 \quad ; \quad \bar{\phi} \;\sim\; (\bar{d})^4\,\phi$$

$\mathcal{N}=4$ Yang-Mills theory

Lagrangian

$$\mathcal{L} \quad = -\bar{\phi}\,\frac{\Box}{\partial^{+2}}\,\phi \;+\; \frac{4g}{3}\,f^{abc}\left(\frac{1}{\partial^{+}}\,\bar{\phi}^{a}\,\phi^{b}\,\bar{\partial}\,\phi^{c} + \mathrm{cc}\right)$$
$$-g^{2}f^{abc}\,f^{ade}\left(\frac{1}{\partial^{+}}(\phi^{b}\,\partial^{+}\phi^{c})\frac{1}{\partial^{+}}\,(\bar{\phi}^{d}\,\partial^{+}\,\bar{\phi}^{e}) + \frac{1}{2}\,\phi^{b}\bar{\phi}^{c}\,\phi^{d}\,\bar{\phi}^{e}\right)$$

In this formalism, both the $\mathcal{N}=4$ supersymmetry and the $SU(4)$ R-symmetry are manifest but Lorentz invariance is not

This formalism was key to the proof of finiteness for this theory

$\mathcal{N}=4$ Yang-Mills theory as a quadratic form

Non-linear terms in dynamical supersymmetry

$$\bar{Q}_m\,\phi^a \;=\; \mathcal{W}^a \;=\; \frac{\partial}{\partial^+}\bar{q}_+\phi^a - g\,f^{abc}\frac{1}{\partial^+}(\bar{d}\phi^b\partial^+\phi^c)$$

The Hamiltonian is a Quadratic Form in structure

$$H=\frac{1}{2\sqrt{2}}\,(\,\mathcal{W}^a\,,\,\mathcal{W}^a\,) \qquad\qquad (\,\phi\,,\,\xi\,)\;\equiv\;2i\int d^4x\,d^4\theta\,d^4\bar{\theta}\;\bar{\phi}\,\frac{1}{\partial^+}\xi$$

Ananth-Brink-Kim-Ramond, 2004 ; Ananth-Brink-Mali, 2015

Not equivalent to: Hamiltonian can be expressed as an anti-commutator

Special to maximally supersymmetric theories

$$H \sim \quad Q\,\bar{\phi}\,\bar{Q}\,\phi$$

$$= \quad \frac{1}{2}\,Q\,\bar{\phi}\,\bar{Q}\,\phi + \frac{1}{2}\,Q\,\bar{\phi}\,\bar{Q}\,\phi$$

$$\bar{\phi} \sim (\bar{d})^4\,\phi$$

$$\sim \quad \frac{1}{2}\,Q\,\bar{\phi}\,\bar{Q}\,\phi + \frac{1}{2}\,Q\,\phi\,\bar{Q}\,\bar{\phi}$$

$H = \dfrac{1}{2\sqrt{2}}\,(\,\mathcal{W}^a\,,\,\mathcal{W}^a\,)$ a little reminiscent of the Nicolai map…

The sound of symmetry ?

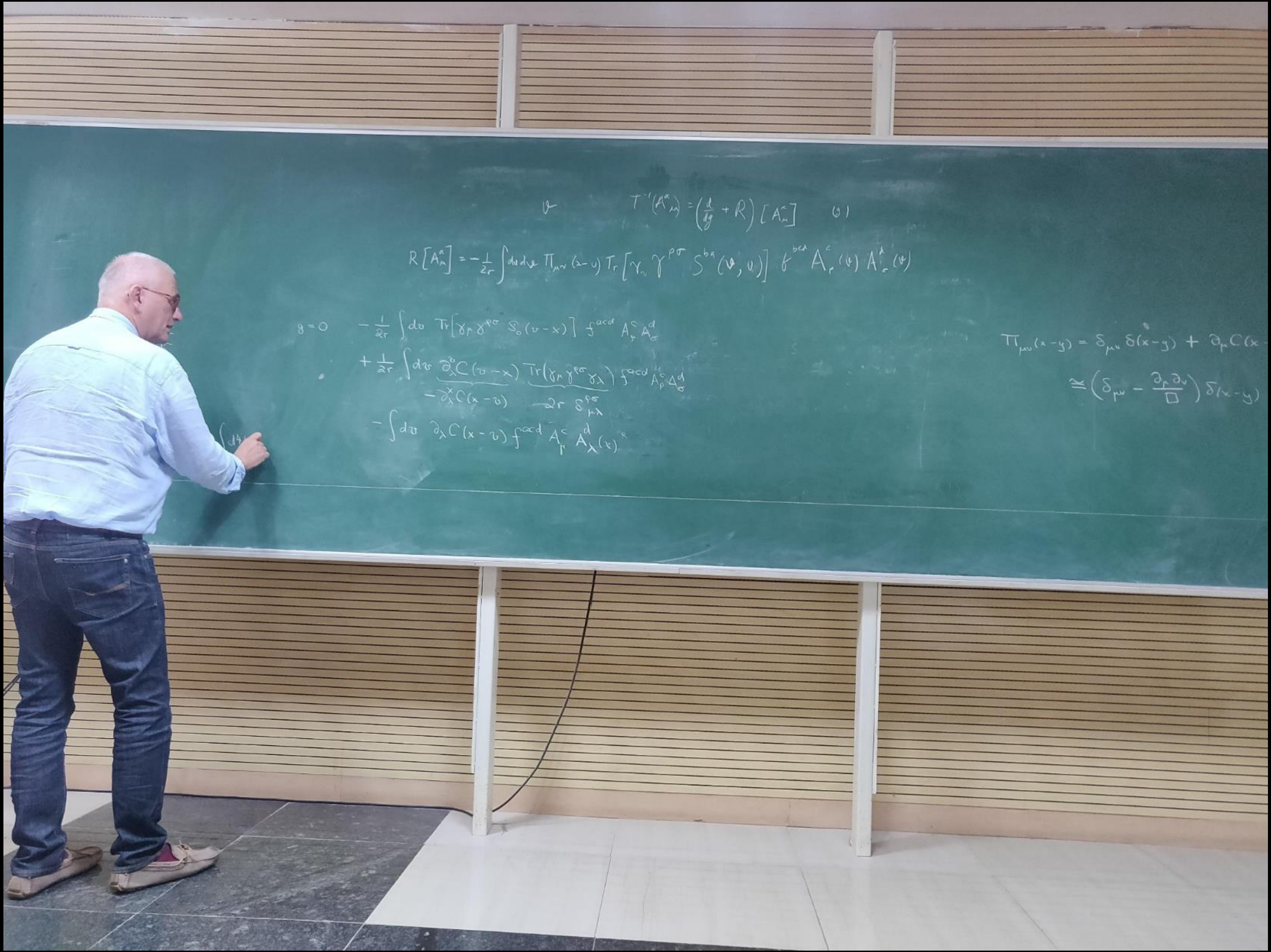
2007 Junior Scientist at the AEI

2018 Visit to the AEI to discuss QFs

The focus shifted from these forms
to the Map itself

Hermann visited Pune in 2019

पुणे महानगरपालिका
व
असोसिएशन ऑफ फ्रेन्डस् ऑफ जपान यांच्या सहकार्याने तयार करण्यात आलेल्या
पुणे - ओकायामा मैत्री उद्यानाचे उद्घाटन
-: शुभहस्ते :-
ना. विलासराव देशमुख
मुख्यमंत्री, महाराष्ट्र राज्य
-: प्रमुख पाहुणे :-
मा. मासाहिरो ईशी
राज्यपाल - ओकायामा प्रांत (जपान)
मा. सुरेश कलमाडी
खासदार व अध्यक्ष भारतीय ऑलिंपिक संघटना
ना. अजितदादा पवार
जलसंपदा मंत्री व पालकमंत्री पुणे जिल्हा
मा. वसंत चव्हाण
खासदार
ना. नारायण राणे
महसूल मंत्री, महाराष्ट्र राज्य
मा. उध्दवजी ठाकरे
कार्याध्यक्ष - शिवसेना
मा. नितीन गडकरी
अध्यक्ष - भारतीय जनता पक्ष, महाराष्ट्र प्रदेश
मा. गोपीनाथजी मुंडे
माजी उप-मुख्यमंत्री व आमदार
मा. रमेश बागवे
आमदार व सभागृहनेते
मा. शरद रणपिसे
आमदार
व


$T^{-1}(A^a_\mu) = \left(\frac{d}{dg} + R\right)[A^a_\mu]$
$R[A^a_\mu] = -\frac{1}{2r}\int du\, dv\, \Pi_{\mu\nu}(x-u)\, \mathrm{Tr}\left[\gamma_\nu \gamma^{\rho\sigma} S^{ba}(v,u)\right] f^{bcd} A^c_\rho(v) A^d_\sigma(v)$
$g=0 \quad -\frac{1}{2r}\int dv\, \mathrm{Tr}\left[\gamma_\mu \gamma^{\rho\sigma} S_0(v-x)\right] f^{acd} A^c_\rho A^d_\sigma$
$+\frac{1}{2r}\int dv\, \underbrace{\partial^v_\lambda C(v-x)}_{-\partial^x_\lambda C(x-v)}\, \underbrace{\mathrm{Tr}(\gamma_\mu \gamma^{\rho\sigma}\gamma_\lambda)}_{-2r\,\delta^{\rho\sigma}_{\mu\lambda}} f^{acd} A^c_\rho A^d_\sigma$
$-\int dv\, \partial_\lambda C(x-v) f^{acd} A^c_\mu A^d_\lambda(v)$
$\Pi_{\mu\nu}(x-y) = \delta_{\mu\nu}\,\delta(x-y) + \partial_\mu C(x$
$\cong \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}\right)\delta(x-y)$

Supersymmetric Yang–Mills theories:not quite the usual perspective,

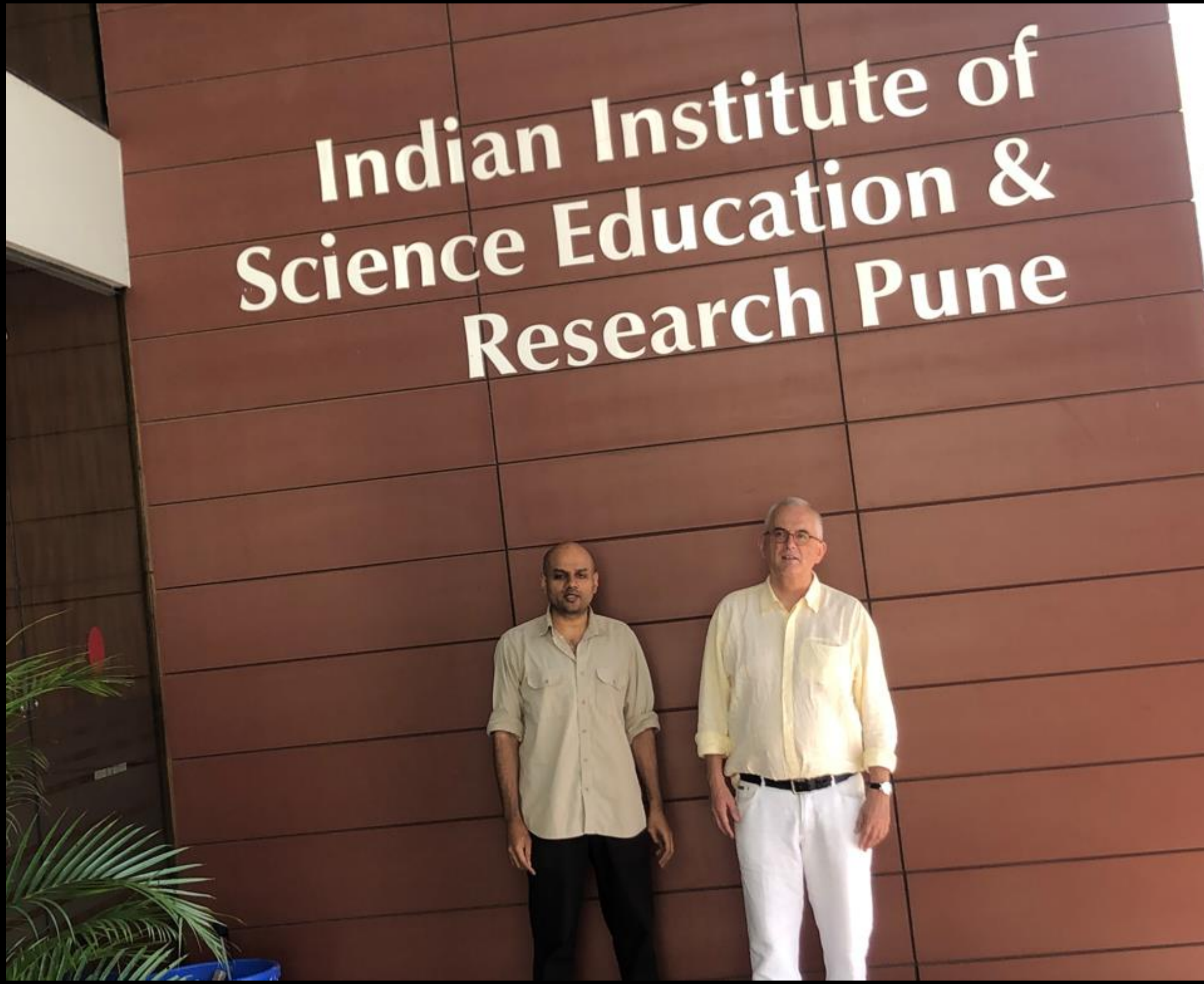
Sudarshan Ananth, Hermann Nicolai, Chetan Pandey and Saurabh Pant,

J. Phys. A 53 (2020) 17, 17

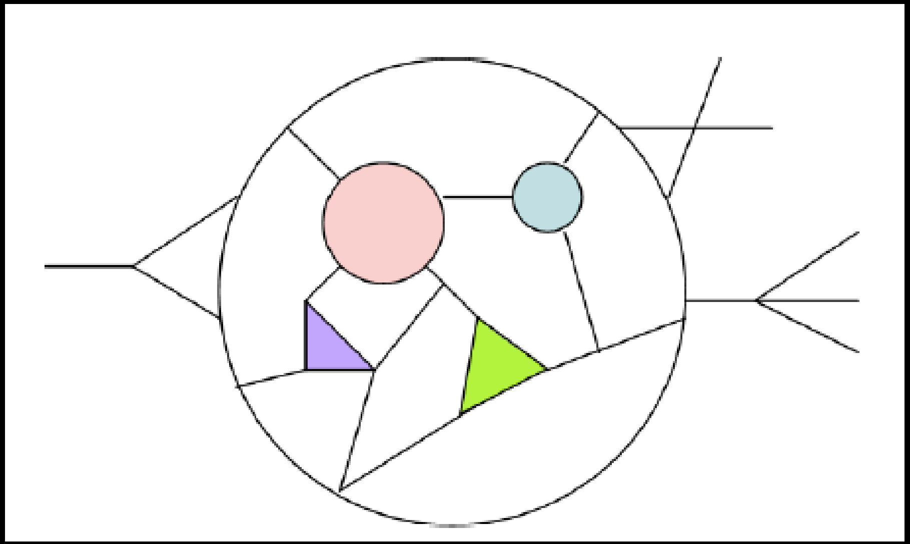
Perturbative linearization of supersymmetric Yang-Mills theory,

Sudarshan Ananth, Olaf Lechtenfeld, Hannes Malcha, Hermann Nicolai, Chetan Pandey and Saurabh Pant,

JHEP 10 (2020) 199



Proof of finiteness



The superficial degree of divergence of all supergraphs is zero using a version of the GRS power counting methods

Grisaru-Rocek-Siegel, 1979

This assumes that all momenta in a supergraph contribute to the loop integral and hence provides only a preliminary estimate

Brink-Lindgren-Nilsson, 1982; Mandelstam 1983

Proof of finiteness

Next distinguish between internal and external momenta and focus on vertices attached to external legs

Manipulate the chiral derivatives to make the superficial degree of divergence negative

Same analysis can be applied to prove that all subgraphs also have negative superficial degree of divergence

Ananth-Kovacs-Shimada, 2006

Finiteness of all Green functions follows from Weinberg's theorem

Weinberg, 1960

Can we apply a similar analysis to the N=8 theory ?

Limitations:

Unlikely to work the same way – dimensionful coupling constant

NO

Quartic or higher interaction vertices in this formalism (in any use-able form)

can ask questions about a subset of diagrams constructed only using cubic vertices…

Gravity in light-cone gauge

$$\mathcal{L} \;=\; \frac{1}{2\kappa^2}\,\sqrt{-g}\,R$$

Light-cone gauge: $$g_{--} \;=\; g_{-i} \;=\; 0 \qquad ; \quad i=1,2$$

$$g_{+-} \;=\; -\,e^{\frac{\psi}{2}} \quad ; \qquad g_{ij} \;=\; e^{\psi}\,\gamma_{ij}$$

$$R_{--} \;=\; 0 \quad \Longrightarrow \quad \psi = \psi(\gamma) \qquad\qquad \gamma_{ij} \;=\; \left(\,e^{\kappa\,H}\,\right)_{ij}$$

$$\mathcal{L} = \mathcal{L}(\gamma)$$

Scherk-Schwarz, 1974 ; Bengtsson-Cederwall-Lindgren, 1983

Light-cone gravity

$$\begin{aligned}\mathcal{H} \;=\;& \partial\bar{h}\,\bar{\partial}h \,-\, 2\,\kappa\,\bar{h}\,\partial_-{}^2\left\{-h\,\frac{\bar{\partial}^2}{\partial_-{}^2}\,h \,+\, \frac{\bar{\partial}}{\partial_-}h\,\frac{\bar{\partial}}{\partial_-}h\right\} \,-\, 2\,\kappa\,h\,\partial_-{}^2\left\{-\bar{h}\,\frac{\partial^2}{\partial_-{}^2}\,\bar{h} \,+\, \frac{\partial}{\partial_-}\bar{h}\,\frac{\partial}{\partial_-}\bar{h}\right\}\\
&-4\kappa^2\left\{-2\,\frac{1}{\partial_-^2}\left(\frac{\bar{\partial}}{\partial_-}\,h\,\partial_-^3\,\bar{h} \,-\, h\,\partial_-^2\,\bar{\partial}\bar{h}\right)\frac{1}{\partial_-^2}\left(\frac{\partial}{\partial_-}\,\bar{h}\,\partial_-^3\,h \,-\, \bar{h}\,\partial_-^2\,\partial h\right)\right.\\
&+\frac{1}{\partial_-^2}(\bar{\partial}h\,\partial_-^2\,\bar{h} \,-\, \partial_-h\,\partial_-\bar{\partial}\bar{h})\,\frac{1}{\partial_-^2}\,(\partial\bar{h}\,\partial_-^2\,h \,-\, \partial_-\bar{h}\,\partial_-\partial h) \,-\, 3\,\frac{1}{\partial_-}(\bar{\partial}h\,\partial_-\bar{h})\,\frac{1}{\partial_-}\,(\partial_-h\,\partial\bar{h})\\
&+\frac{1}{\partial_-}(\bar{\partial}h\,\partial_-\bar{h} \,-\, \partial_-h\,\bar{\partial}\bar{h})\,\frac{1}{\partial_-}\,(\partial\bar{h}\,\partial_-h \,-\, \partial_-\bar{h}\,\partial h) \,+\, 3\,\frac{1}{\partial_-}\,(\partial_-h\,\partial_-\bar{h})\,\frac{1}{\partial_-}\,(\bar{\partial}h\,\partial\bar{h})\\
&\left.+\left[\frac{1}{\partial_-^2}(\partial_-h\,\partial_-\bar{h}) \,-\, h\,\bar{h}\right](\bar{\partial}h\,\partial\bar{h} \,+\, \partial h\,\bar{\partial}\,\bar{h} \,-\, \partial_-h\,\frac{\partial\,\bar{\partial}}{\partial_-}\,\bar{h} \,-\, \partial_-\bar{h}\,\frac{\partial\,\bar{\partial}}{\partial_-}\,h)\right\}\end{aligned}$$

Bengtsson-Cederwall-Lindgren, 1983 ; Ananth-Brink-Heise-Svendsen, 2006

Extended to 5- and 6-point interaction vertices – both MHV and non-MHV structures

Ananth 2008 ; Ananth-Bhave-Raj, 2022

$\mathcal{N}=8$ light-cone superspace

$$\theta^m \quad ; \quad \bar{\theta}_m \quad ; \quad m = 1 \ldots 8 \qquad \text{SU(8)}$$

$$\begin{aligned}\phi(y) = & \frac{1}{\partial^{+2}} h(y) + i\theta^m \frac{1}{\partial^{+2}} \bar{\psi}_m(y) + \frac{i}{2}\theta^m \theta^n \frac{1}{\partial^+} \bar{A}_{mn}(y) \\ & - \frac{1}{3!} \theta^m \theta^n \theta^p \frac{1}{\partial^+} \bar{\chi}_{mnp}(y) - \frac{1}{4!}\theta^m \theta^n \theta^p \theta^q \bar{C}_{mnpq}(y) \\ & + \ldots + \frac{1}{7!}\theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \epsilon_{mnpqrstu} \partial^+ \psi^u(y) \\ & + \frac{4}{8!} \theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \theta^u \epsilon_{mnpqrstu} \partial^{+2} \bar{h}(y)\end{aligned}$$

Bengtsson-Bengtsson-Brink, 1983

Cremmer-Julia, 1978 ; 1979 ; de Wit-Nicolai, 1982 ; 1984 ; 1985

$\mathcal{N}=8$ supergravity

$$\phi = \frac{1}{4}\,\frac{(d)^8}{\partial^{+4}}\,\bar{\phi} \qquad\qquad (d)^8 \equiv d^1 d^2 \ldots d^8$$

Dynamical supersymmetry at order κ

$$\bar{Q}_m\,\phi = \frac{1}{\partial^+}\big(\,\bar{\partial}\bar{q}_{+\,m}\phi\,\partial^{+2}\phi - \partial^+\bar{q}_{+\,m}\phi\,\partial^+\bar{\partial}\phi\,\big)$$

Lagrangian

$$\mathcal{L} = -\bar{\phi}\,\frac{\Box}{\partial^{+4}}\,\phi \;-\; 2\,\kappa\,\Big(\,\frac{1}{\partial^{+2}}\,\bar{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\phi + \;\frac{1}{\partial^{+2}}\,\phi\,\partial\,\overline{\phi}\,\partial\,\overline{\phi}\Big)$$

Proof of finiteness – what is needed

Superspace finiteness analysis: always fall short by two powers of momenta

The sound of symmetry

New symmetry principle that excludes / eliminates divergent pieces

Residual reparameterization symmetry and consequences

Supergravity – Exceptional symmetries and beyond?

Damour-Henneaux-Nicolai 2002 ; Kleinschmidt-Nicolai 2006

$\mathcal{N}=8$ supergravity

The Hamiltonian is again a Quadratic form

$$\mathcal{H} = \frac{1}{4\sqrt{2}}\,(\,\mathcal{W}\,,\,\mathcal{W}\,) \;=\; \frac{2\,i}{4\sqrt{2}}\int d^4x\, d^8\theta\, d^8\,\bar{\theta}\;\overline{\mathcal{W}}\,\frac{1}{\partial^{+3}}\,\mathcal{W}$$

Ananth-Brink-Heise-Svendsen, 2006

$$E_{7(7)} = SU(8) \quad \times \quad \frac{E_{7(7)}}{SU(8)}$$

$\uparrow$ 63 linear ; $\uparrow$ 70 non-linear

de Wit-Freedman, 1977 ; Cremmer-Julia, 1978 ; 1979

$$T^m_{\;n} \;=\; \frac{i}{2\sqrt{2}\partial^+}\left(q^m\,\bar{q}_n \;-\; \frac{1}{8}\,\delta^m_n\,q^p\,\bar{q}_p\right) \quad ; \quad [\,T^m_{\;n}\,,\,T^p_{\;q}\,] \;=\; \delta^p_n\,T^m_{\;q} - \delta^m_q\,T^p_{\;n}$$

$$L_{A,1} = \frac{3}{4}\,\frac{1}{\partial^{+3}}\Big(\partial^+\phi\;\partial^+\bar{q}_+\phi\Big)\partial^{+4}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-3}$$

$$L_{A,2} = \frac{53}{12}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\frac{1}{\partial^+}\bar{q}_+\phi\Big)\;\partial^{+6}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-4}$$

$$+\frac{265}{12}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\bar{q}_+\phi\Big)\;\partial^{+5}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-5}$$

$$+\frac{265}{6}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\partial^+\bar{q}_+\phi\Big)\;\partial^{+4}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-6}$$

$$+\frac{265}{6}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\partial^{+2}\bar{q}_+\phi\Big)\;\partial^{+3}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-7}$$

$$+\frac{265}{12}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\partial^{+3}\bar{q}_+\phi\Big)\;\partial^{+2}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-8}$$

$$+\frac{53}{12}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\phi\;\partial^{+4}\bar{q}_+\phi\Big)\;\partial^+\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-9}$$

$$+\frac{119}{12}\,\frac{1}{\partial^+}\Big(\phi\;\frac{1}{\partial^+}\bar{q}_+\phi\Big)\;\partial^{+5}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-10}$$

$$+\frac{358}{9}\,\frac{1}{\partial^+}\Big(\phi\;\bar{q}_+\phi\Big)\;\partial^{+4}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-11}$$

$$+\frac{553}{9}\,\frac{1}{\partial^+}\Big(\phi\;\partial^+\bar{q}_+\phi\Big)\;\partial^{+3}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-12}$$

$$+\frac{121}{3}\,\frac{1}{\partial^+}\Big(\phi\;\partial^{+2}\bar{q}_+\phi\Big)\;\partial^{+2}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-13}$$

$$+\frac{373}{36}\,\frac{1}{\partial^+}\Big(\phi\;\partial^{+3}\bar{q}_+\phi\Big)\;\partial^+\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-14}$$

$$+\frac{1}{9}\,\frac{1}{\partial^+}\Big(\phi\;\partial^{+4}\bar{q}_+\phi\Big)\;\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-15}$$

$$+\frac{193}{36}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\frac{1}{\partial^+}\bar{q}_+\phi\Big)\;\partial^{+4}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-16}$$

$$+\frac{305}{18}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\bar{q}_+\phi\Big)\;\partial^{+3}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-17}$$

$$+\frac{67}{4}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\partial^+\bar{q}_+\phi\Big)\;\partial^{+2}\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-18}$$

$$+\frac{107}{18}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\partial^{+2}\bar{q}_+\phi\Big)\;\partial^+\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-19}$$

$$+\frac{1}{9}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\partial^{+3}\bar{q}_+\phi\Big)\;\partial\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-20}$$

$$L_{B,1} = -\frac{3}{2}\,\frac{1}{\partial^{+3}}\Big(\partial^+\bar{q}_+\phi\;\partial\phi\Big)\partial^{+5}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-21}$$

$$-\frac{1}{\partial^{+3}}\Big(\bar{q}_+\phi\;\partial^+\partial\phi\Big)\partial^{+5}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-22}$$

$$-\frac{1}{2}\,\frac{1}{\partial^{+3}}\Big(\partial^+\bar{q}_+\phi\;\partial^+\partial\phi\Big)\partial^{+4}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-23}$$

$$-\frac{1}{\partial^{+3}}\Big(\partial^{+2}\bar{q}_+\phi\;\partial^+\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-24}$$

$$-\frac{1}{\partial^{+3}}\Big(\partial^+\bar{q}_+\phi\;\partial^{+2}\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-25}$$

$$L_{B,2} = -\frac{1}{2}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\bar{q}_+\phi\;\partial^{+2}\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-26}$$

$$-\frac{1}{2}\,\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\bar{q}_+\phi\;\partial^{+3}\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-27}$$

$$+\frac{3}{2}\,\frac{1}{\partial^+}\Big(\bar{q}_+\phi\;\partial^+\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-28}$$

$$-\frac{1}{2}\,\frac{1}{\partial^+}\Big(\bar{q}_+\phi\;\partial^{+2}\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-29}$$

$$-\frac{1}{2}\,\frac{1}{\partial^+}\Big(\partial^+\bar{q}_+\phi\;\partial^+\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-30}$$

$$+\frac{1}{\partial^+}\Big(\frac{1}{\partial^+}\bar{q}_+\phi\;\partial\phi\Big)\partial^{+5}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-31}$$

$$-\frac{221}{18}\,\frac{1}{\partial^+}\Big(\partial^{+4}\phi\;\bar{q}_+\partial\phi\Big)\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-46}$$

$$-\frac{53}{12}\,\frac{1}{\partial^+}\Big(\partial^{+4}\phi\;\partial^+\bar{q}_+\partial\phi\Big)\frac{1}{\partial^+}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-47}$$

$$L_{C,1} = -\frac{3}{2}\,\frac{1}{\partial^{+3}}\Big(\phi\;\partial^+\bar{q}_+\partial\phi\Big)\partial^{+5}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-32}$$

$$-\frac{11}{4}\,\frac{1}{\partial^{+3}}\Big(\partial^+\phi\;\partial\bar{q}_+\phi\Big)\partial^{+5}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-33}$$

$$-\frac{1}{2}\,\frac{1}{\partial^{+3}}\Big(\partial^+\phi\;\partial^+\bar{q}_+\partial\phi\Big)\partial^{+4}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-34}$$

$$-\frac{1}{\partial^{+3}}\Big(\partial^+\phi\;\partial^{+2}\bar{q}_+\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-35}$$

$$-\frac{1}{\partial^{+3}}\Big(\partial^{+2}\phi\;\partial^+\bar{q}_+\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-36}$$

$$L_{C,2} = -\frac{1}{6}\,\frac{1}{\partial^+}\Big(\partial^+\phi\;\partial^+\bar{q}_+\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-37}$$

$$-\frac{13}{6}\,\frac{1}{\partial^+}\Big(\partial^{+2}\phi\;\frac{1}{\partial^+}\bar{q}_+\partial\phi\Big)\partial^{+3}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-38}$$

$$+\frac{1}{6}\,\frac{1}{\partial^+}\Big(\partial^{+2}\phi\;\bar{q}_+\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-39}$$

$$-\frac{1}{6}\,\frac{1}{\partial^+}\Big(\partial^{+2}\phi\;\partial^+\bar{q}_+\partial\phi\Big)\partial^+\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-40}$$

$$-\frac{179}{18}\,\frac{1}{\partial^+}\Big(\partial^{+3}\phi\;\frac{1}{\partial^+}\bar{q}_+\partial\phi\Big)\partial^{+2}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-41}$$

$$-\frac{407}{18}\,\frac{1}{\partial^+}\Big(\partial^{+3}\phi\;\bar{q}_+\partial\phi\Big)\partial^+\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-42}$$

$$-\frac{199}{12}\,\frac{1}{\partial^+}\Big(\partial^{+3}\phi\;\partial^+\bar{q}_+\partial\phi\Big)\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-43}$$

$$-\frac{53}{12}\,\frac{1}{\partial^+}\Big(\partial^{+3}\phi\;\partial^{+2}\bar{q}_+\partial\phi\Big)\frac{1}{\partial^+}\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-44}$$

$$-\frac{191}{18}\,\frac{1}{\partial^+}\Big(\partial^{+4}\phi\;\frac{1}{\partial^+}\bar{q}_+\partial\phi\Big)\partial^+\bar{\phi}\;\frac{1}{\partial^{+4}}q_+\bar{\partial}\bar{\phi} \tag{D-45}$$

Ananth-Brink-Mali, 2015

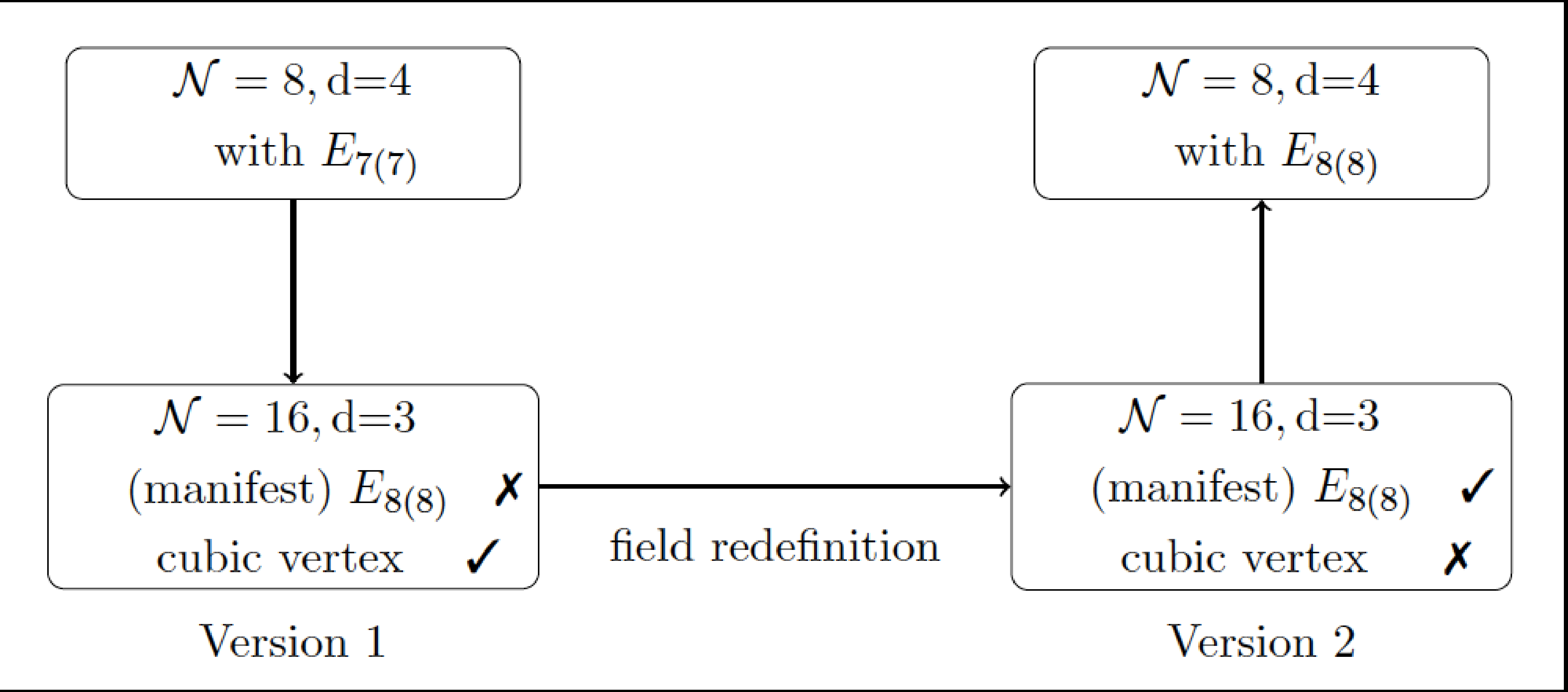
Mismatch in Dynkin indices at order 8

One loop in light-cone ✓ ; Formula at two loops?

Index	$G_{\mu\nu}$	$A_{\mu\nu\rho}$	Ψ_{μ}
$I^{(0)}$	44	84	128
$I^{(2)}$	88	168	256
$I^{(4)}$	232	408	640
$I^{(6)}$	712	1080	1792
$I^{(8)}$	2440	3000	5248

Curtright, 1981

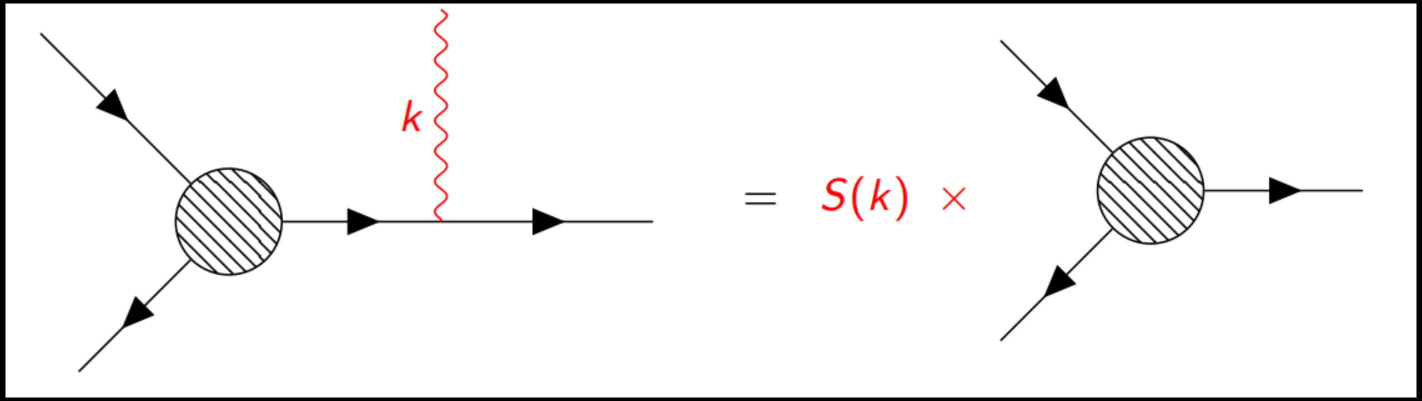
Exceptional symmetry enhancement



Working out the consequences for amplitudes is extremely involved

Ananth-Brink-Majumdar, 2021

Inverse Soft approach to the quartic vertex



Weinberg, 1965

$$M_{n+1}(p_1, p_2, \ldots p_n, k) = S(p_1, p_n, k) \;\times\; M_n(p_1', p_2, \ldots p_n')$$

Arkani Hamed-Cachazo-Cheung-Kaplan, 2010

Inverse Soft approach to the quartic vertex

Ananth-Pandey-Pant, work in progress

From the super Yang-Mills action

$$S = \int d^4x \; d^4\theta \; d^4\bar{\theta} \left\{ \frac{1}{4}\, \bar{\phi}^a \frac{\Box}{\partial^{+\,2}}\, \phi^a + \frac{2}{3}\, g f^{abc} \left(\frac{1}{\partial^{+\,2}}\, \bar{\phi}^a\, \bar{\partial}\phi^b\, \partial^+ \phi^c + c.c \right) + \mathcal{O}(g^2) \right\}$$

We derive the soft factor in this formalism for the N=4 theory

$$s_{\mathcal{N}=4} = \sum_i \frac{k^+}{\langle p_i k \rangle} \sqrt{\frac{p_i^+}{k^+}} = \frac{\langle pl \rangle}{\langle pk \rangle \langle kl \rangle}\; k^+$$

Inverse Soft approach to the quartic vertex

Ananth-Pandey-Pant, work in progress

From the supergravity cubic vertex we find

$$s_{\mathcal{N}=8} = \sum_i \frac{[p_i k]}{\langle p_i k\rangle} \frac{\langle p_i l\rangle^2}{\langle lk\rangle^2} k^{+\,2} = \sum_i \frac{[p_i k]}{\langle p_i k\rangle} \frac{\langle p_i l\rangle \langle q p_i\rangle}{\langle lk\rangle \langle qk\rangle} k^{+\,2}$$

Using which we can inverse-soft our way to a quartic vertex

$$\mathcal{L}_4^{\mathcal{N}=8} = \frac{\langle pl\rangle^8\,[pl]}{\langle pl\rangle\langle pq\rangle\langle lq\rangle\langle pk\rangle\langle lk\rangle\langle kq\rangle^2} \left\{ \frac{p^{+\,2}\,l^{+\,2}\,q^{+\,2}\,k^{+\,2}}{(p^+ + l^+)^4\,(q^+ + k^+)^4} + \frac{p^{+\,2}\,l^{+\,2}\,q^{+\,2}\,k^{+\,2}}{(p^+ + q^+)^4\,(l^+ + k^+)^4} + \frac{p^{+\,2}\,l^{+\,2}\,q^{+\,2}\,k^{+\,2}}{(p^+ + k^+)^4\,(l^+ + q^+)^4} \right\}$$

More updates at the next meeting…

Max Planck Partner Group meeting

Pune, 2011

The director

Diversity

Fashion

Indian Nationals Now Have to Wait for a German Schengen Visa Appointment Until November

August 5, 2022 Subscribe to our daily news digest

Happy Birthday Hermann!