HermannFest

QUANTUM GRAVITY PHENOMENOLOGY FROM THE THERMODYNAMICS OF SPACETIME

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MOTIVATION

- Understand the interface between gravitational dynamics and thermodynamics: One step further
- Role of entropies and connection among them
- On the search for quantum gravity...
  - Quantum phenomenological models to get insight into possible effects
- Look at the entropy: Quantum modifications of entropy
  - Same qualitative behavior in all the models
- Could we obtain phenomenological modified equations of motion from it?
  - General phenomenological quantum gravity dynamics
  - Quantum cosmology: Resolution of the singularity?

OUTLINE

- Einstein equations from thermodynamics: Basic concepts
- Modified entropy
- Quantum phenomenological equations of motion
- Interpretation of the modified dynamics and application to cosmology
- Discussion
- Further perspectives

EINSTEIN EQUATIONS FROM THERMODYNAMICS

- Gravitational dynamics $\longrightarrow$ equilibrium condition for maximal entropy

$$\boxed{\delta S = 0}$$

Quantum correlations across horizon + matter-energy crossing it

- Entanglement entropy: $S = \eta \mathcal{A}$ $\longrightarrow$ Bekenstein entropy $S_{BH} = \frac{k_B \mathcal{A}}{4 l_P^2}$

- Entropy of matter: in terms of entanglement for GLCD explicitly evaluated for small perturbations from vacuum

  Clausius entropy: Equal to leading order for conformal fields (without gravitational dynamics)

$\Rightarrow$ Einstein equations are derived $\longrightarrow$ Unimodular gravity

[T. Jabobson, PRL 116 (2016) 20, 201101]

- Unimodular gravity: Restricting full diffeomorphism invariance of GR
  - Local energy conservation must be assumed separately
  - Λ appears as an integration constant
  - Only conformally invariant part couples to gravity $\longrightarrow$ conformal invariance
  - GR and UG same classical dynamics
  - Introduction of quantum effects could indicate which theory is obtained

- Equivalences of entropy
  - Same result $\longrightarrow$ interchangeable at semiclassical level
  - Is this general?

GEODESIC LOCAL CAUSAL DIAMONDS

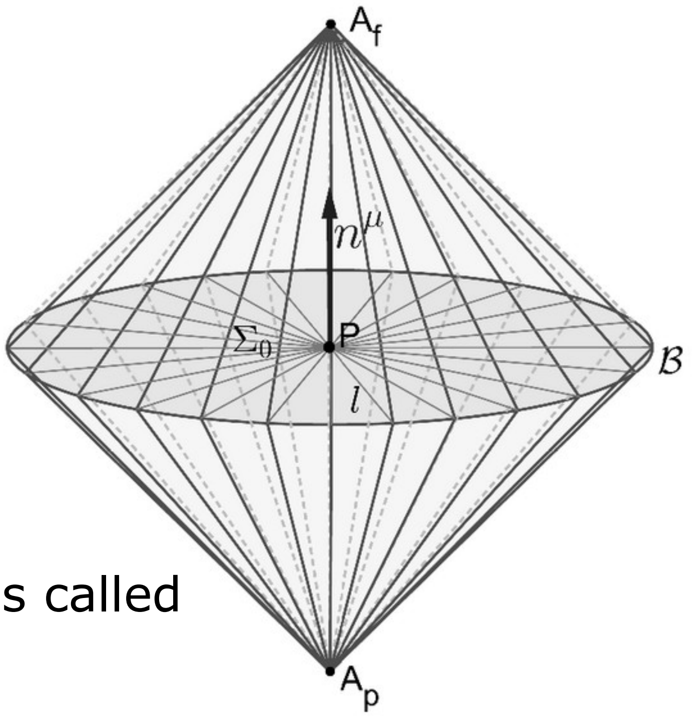
- In an arbitrary *P* choose any unit timelike vector n^μ and construct Riemann normal coordinates (RNC)
  - Metric expansion around *P*

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta}(P)\, x^\alpha x^\beta + O\left(x^3\right)$$

- Area of the 2-sphere

$$\mathcal{A} = 4\pi l^2 - \frac{4\pi}{9} l^4 G_{00}(P) + O\left(l^5\right)$$

- Spacetime region causally determined by this ball is called geodesic local causal diamond (GLCD)



- Conformal Killing vector $\longrightarrow$ conformal Killing horizon

MODIFIED ENTANGLEMENT ENTROPY

- Leading order quantum gravity modifications to Bekenstein entropy

$$S_{BH,q} = \frac{k_B \mathcal{A}}{4l_P^2} + \mathcal{C} k_B \ln\left(\frac{\mathcal{A}}{\mathcal{A}_0}\right) + O\left(\frac{k_B l_P^2}{\mathcal{A}}\right)$$

  - Predicted by different theories of quantum gravity and phenomenological approaches
  - Calculations entanglement entropy $\longrightarrow$ local causal horizons

QUANTUM PHENOMENOLOGICAL EQUATIONS OF MOTION

EQUATIONS OF MOTION FROM ENTANGLEMENT EQUILIBRIUM

[T. Jabobson, PRL 116 (2016) 20, 201101]

- Maximal vacuum entanglement hypothesis (MVEH):

  "When the geometry and quantum fields are simultaneously varied from maximal symmetry, the entanglement entropy in a small geodesic ball is maximal at fixed volume"

- Consider the GLCD carried out in a maximally symmetric spacetime (MSS), and we simultaneously vary geometry and state of quantum fields:

$$\delta S_e + \delta S_m = 0$$

  - On one side: δS_e with the logarithmic correction

$$S_{e,q} = \eta \mathcal{A} + k_B \mathcal{C} \ln \frac{\mathcal{A}}{\mathcal{A}_0} + O\left(\frac{k_B l_P^2}{\mathcal{A}}\right)$$

$$\Big\Downarrow \; \delta\mathcal{A}|_V$$

$$\delta S_{e,q} = S_{e,q} - S_{e,q}^{MSS} = \eta\delta\mathcal{A} + k_B \mathcal{C} \frac{\delta\mathcal{A}}{\mathcal{A}_{MSS}} - k_B \frac{\mathcal{C}}{2}\left(\frac{\delta\mathcal{A}}{\mathcal{A}_{MSS}}\right)^2 + O\left((\delta\mathcal{A})^3\right)$$

- On the other side: δS_m

  - Conformal Killing vector $\longrightarrow$ Conformal Killing horizon with T_{Unruh}

  - Quantum fields in the thermal state $\longrightarrow$ Minkowski vacuum (EEP)

  - Variation of entropy of quantum fields (Temperatures cancel out)

$$\delta S_m = \frac{2\pi k_B c}{\hbar}\frac{4\pi}{15}l^4\left(\delta\langle T_{00}\rangle + \delta X\right) - 4\psi l^2 C^2 \frac{2\pi k_B l_P^2}{\hbar c^3}\frac{4\pi}{15}l^4\left(\delta\langle T_{00}\rangle + \delta X\right) + O\left(l^5\right)$$

➢ Considering the MVEH: $\delta S_{e,q} + \delta S_m = 0$

Requirement of recovering Einstein equations for $C \to 0$ fixes $\eta = k_B/4l_P^2$ (SEP)

$\Rightarrow$ Due to EEP $\Rightarrow$ every spacetime point *P*

GLCD in *P* $\Rightarrow$ any arbitrary unit timelike vector n^μ

- We obtain:

$$S_{\mu\nu}\left(P\right)n^{\mu}n^{\nu}+\frac{Cl_{Pl}^{2}}{30\pi}S_{\alpha\beta}\left(P\right)n^{\alpha}n^{\beta}S_{\mu\nu}\left(P\right)n^{\mu}n^{\nu}-\Phi\left(P\right)=\frac{8\pi G}{c^{4}}\delta\langle T_{\mu\nu}\left(P\right)\rangle\left(P\right)n^{\mu}n^{\nu}$$

with: $S_{\mu\nu}=R_{\mu\nu}-Rg_{\mu\nu}/4$

We differentiate respect to n^{μ} getting a system of conditions $\longrightarrow \Phi$

Finally we obtain:

$$S_{\mu\nu}-\frac{Cl_{Pl}^{2}}{30\pi}S_{\mu\lambda}S^{\lambda}{}_{\nu}-\frac{8\pi G}{c^{4}}\delta\langle T_{\mu\nu}\rangle=-{}^{(0)}\Phi g_{\mu\nu}$$

Taking the trace

$$\boxed{S_{\mu\nu}-\frac{Cl_{Pl}^{2}}{30\pi}S_{\mu\lambda}S^{\lambda}{}_{\nu}+\frac{Cl_{Pl}^{2}}{120\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda}-\frac{1}{4}R^{2}\right)g_{\mu\nu}=\frac{8\pi G}{c^{4}}\left(\delta\langle T_{\mu\nu}\rangle-\frac{1}{4}\delta\langle T\rangle g_{\mu\nu}\right)}$$

(Classically Einstein equations recovered by imposing: $T_{\mu}{}^{\nu}{}_{;\nu}=0 \longrightarrow \Lambda$)

EQUATIONS OF MOTION FROM CLAUSIUS ENTROPY

- For arbitrary spacelike cross-section $\mathcal{S}$ of an arbitrary null surface in a curved spacetime, the Clausius entropy can be defined as:

[V. Baccetti, M. Visser, CQG 31(2014) 035009]

$$\frac{\mathrm{d}S_C(t)}{\mathrm{d}t} = \frac{2\pi k_B c}{\hbar} t \int_{\mathcal{S}(t)} T_{\mu\nu}\left(x\left(t,\theta,\phi\right)\right) k_{\pm}^{\mu} k_{\pm}^{\nu} \mathrm{d}^2\mathcal{A} + O\left(l^4\right) + O\left(\psi \frac{l_P^2 a^2}{c^4}\right) + O\left(\frac{1}{a^2}\right)$$

Quantum modifications $\longrightarrow$ new construction of Clausius entropy

- We focus on the specific case of GLCD

- Clausius entropy: δS_m

$$\Delta S_C = S_C\left(A_f\right) - S_C\left(\mathcal{B}\right) = -\frac{8\pi^2 k_B l^4}{9\hbar c}\left(T_{00}\left(P\right) + \frac{1}{4} T\left(P\right)\right) + O\left(l^5\right) + O\left(\psi \frac{l_P^2 a^2}{c^4}\right) + O\left(\frac{1}{a^2}\right)$$

$\Rightarrow$ Equivalence between Clausius and entanglement entropy

- The entanglement entropy

$$\Delta S_{e,q} = -4\pi\eta l^2 + \frac{4\pi\eta l^4}{9} G_{00}(P) - k_B \mathcal{C} \ln\left(\frac{4\pi\eta l^2 - \frac{4\pi\eta l^4}{9} G_{00}(P)}{\mathcal{A}_0}\right) + O\left(l^5\right)$$

Substract “equilibrium state contribution”: A MSS

- Balance equation (assuming equivalence of entropies):

$$\Delta S_e - \Delta S_{e\,MSS} + \Delta S_C = 0$$

- The entropy balance equation yields to a similar expression than previous procedure, and finally we obtain

$$\boxed{S_{\mu\nu} - \frac{\mathcal{C} l_P^2}{18\pi} S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{\mathcal{C} l_P^2}{72\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu}\right)}$$

INTERPRETATION OF THE EQUATIONS

$$S_{\mu\nu} - D l_P^2 S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{D l_P^2}{4}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu}\right)$$

- Equivalence of the derivations → Equivalence of entropies?
- Traceless equations → Unimodular Gravity
- General equations
- Nonlinear in second derivatives
- Consistency checks
- Diffeomorphism invariance?
- Equivalence principles and Lorentz invariance?

SKETCH OF A COSMOLOGICAL MODEL

- Consider a homogeneous, isotropic cosmological model (FLRW model) (k=0)

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a(t)^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right)$$

$\Downarrow$ Universe filled with dust

Raychaudhuri equation: $\dot{H} = -4\pi G\rho\left(1 - 4\pi D\dfrac{\rho}{\rho_P}\right)$

$\Downarrow$ Energy conservation

Friedmann equation: $H^2 = \dfrac{8\pi G\rho}{3}\left(1 - \dfrac{2\pi D\rho}{\rho_P}\right) + \tilde{\Lambda}$

- *D>0* Avoid singularity (similar to bounce in LQC)

DISCUSSION

- One step further in thermodynamics of spacetime
  - Equivalence of entropies
  - Unimodular Gravity
- Introducing quantum gravity effects via logarithmic corrections to entropy
  - General quantum phenomenological dynamics
  - Effects in a cosmological model

FUTURE PERSPECTIVES

- Understanding the issues of energy conservation and diffeomorphism invariance, equivalence principles, etc…
- Could we find the action which implies the equations of motion?
- Constraints on parameter *D*
- Equivalence of entropies and appearance of Unimodular Gravity
- Study of explicit solutions
- Noether charge formalism

[A A-S, L. J. Garay, M. Liška, PRD 106 (2022) 064024, 2204.08245 [gr-qc]]


Happy birthday, Hermann!