# -ERRATA-

# **Eisenstein Series and Automorphic Representations**

with Application in String Theory

Cambridge University Press (2018): ISBN: 9781107189928

Philipp Fleig, Henrik P. A. Gustafsson, Axel Kleinschmidt, Daniel Persson

July 25, 2023

# **Chapter 4: Automorphic Forms**

- p62, 13: 'co-called' should be replaced by 'so-called' [J. Gerken, 30/7/2018]
- p63, (4.21): The standard notation (c, d) = 1 on the sum means that c and d are co-prime: gcd(c, d) = 1. [J. Gerken, 30/7/2018]
- p65, (4.33): The standard notation (m, n) = 1 on the sum means that m and n are co-prime: gcd(m, n) = 1. [J. Gerken, 30/7/2018]
- p66, (4.35): The correct formula is

$$\left(\phi|_{k,m}\gamma\right)(\tau,z) = (c\tau+d)^{-k} e^{2\pi i m \left[-\frac{c(z+\lambda\tau+\mu)^2}{c\tau+d} + \lambda^2\tau + 2\lambda z + \lambda\mu\right]} \phi\left(\frac{a\tau+b}{c\tau+d}, \frac{z+\lambda\tau+\mu}{c\tau+d}\right).$$
(4.35)

The  $+\lambda\mu$  in the phase is only relevant when considering covers of the Jacobi group. For  $\lambda, \mu \in \mathbb{Z}$  it can also be removed since *m* is an integer. [J. Gerken, 30/7/2018]

p66, (4.39): The argument of  $\phi$  on the left-hand side of the first line should be the matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  acting on  $\tau$  to give the first line

$$\phi\left(\frac{a\tau+b}{c\tau+d},\frac{z}{c\tau+d}\right) = (c\tau+d)^k e^{2\pi i m \frac{cz^2}{c\tau+d}} \phi(\tau,z) .$$
(4.39)

[J. Gerken, 30/7/2018]

p73, (4.74): The correct functional relation (appearing in (1.25) and other places in the book) is

$$\xi(2s)E(s,z) = \xi(2(1-s))E(1-s,z).$$
(4.74)

[J. Gerken, 30/7/2018]

p74, (4.82): The standard notation (c, d) = 1 on the sum means that c and d are co-prime: gcd(c, d) = 1. [J. Gerken, 30/7/2018]

#### **Chapter 5: Automorphic Representations and Eisenstein Series**

p117, (5.130): The formula and its preceding text should be replaced by: ...functions in Ind  ${}^{G}_{\bar{B}}\sigma$  transform as

$$\left(\pi(g)f\right)(x) = (cx+a)^{-2s} f\left(\frac{dx+b}{cx+a}\right)$$
(5.130)

### **Chapter 6: Whittaker Functions and Fourier Coefficients**

- p135, Prop. 6.20: The map  $\varphi = f \circ \phi^{-1} : G(\mathbb{Q}) \setminus G(\mathbb{A}) / K_f \to \mathbb{C}$  is missing its target.
- p151, Table 6.2: The entry for  $\pi_{ntm}$  for  $E_{6(6)}$  can be expanded as follows. Next-to-minimal representations for  $E_{6(6)}$  are found for *generic*  $s_1$  as well as for *generic*  $s_6$ .  $s_5 = 1$  is also next-to-minimal as already stated.

#### **Chapter 7: Fourier Coefficients of Eisenstein Series on** $SL(2, \mathbb{A})$

p166, (7.64ff): The argument leading from (7.64) to (7.67) is incorrectly separated into local factors. A global argument giving (7.67) is as follows. Since  $\psi$  is non-degenerate, there is a  $t \in N(\mathbb{A})$  such that  $\psi(t) \neq 1$ . But since

$$\int_{N(\mathbb{Q})\setminus N(\mathbb{A})} \overline{\psi(n)} dn = \int_{N(\mathbb{Q})\setminus N(\mathbb{A})} \overline{\psi(nt)} dn = \overline{\psi(t)} \int_{N(\mathbb{Q})\setminus N(\mathbb{A})} \overline{\psi(n)} dn$$

by translation invariance, the integral (7.64) vanishes. [J. R. Love, 26/3/2021]

## **Chapter 9: Whittaker Coefficients of Eisenstein Series**

p203, (9.73): The sign of  $\rho$  in the exponents should be changed in the second and third line.

## **Chapter 10: Analysing Eisenstein Series and Small Representations**

p237, (10.67): Some entries contain too many squares. The correct table is

W	$M(w,\lambda)$	
1	1	
$w_1$	c(2s-1)	
$w_2 w_1$	c(2s-1)c(2s-2)	
<i>w</i> <sub>3</sub> <i>w</i> <sub>2</sub> <i>w</i> <sub>1</sub>	c(2s-1)c(2s-2)c(2s-3)	(10.67)
$w_4 w_2 w_1$	c(2s-1)c(2s-2)c(2s-3)	
$w_4 w_3 w_2 w_1$	$c(2s-1)c(2s-2)c(2s-3)^2$	
$w_2 w_4 w_3 w_2 w_1$	$c(2s-1)c(2s-2)c(2s-3)^2c(2s-4)$	
$w_1w_2w_4w_3w_2w_1$	$c(2s-1)c(2s-2)c(2s-3)^{2}c(2s-4)c(2s-5)$	

## **Chapter 11: Hecke Theory and Automorphic L-functions**

p262, (11.29): There is a typo in the argument of the modular form. The correct formula is

$$(T_n f)(z) = n^{k-1} \sum_{d|n} d^{-k} \sum_{b=0}^{d-1} f\left(\frac{\mathbf{n}z + bd}{d^2}\right) \qquad (f \text{ holomorphic of weight } k) \tag{11.29}$$

## **Chapter 13: Elements of String Theory**

p322, Table 13.2: The factor  $\mathbb{Z}_2$  in the discrete subgroup  $G(\mathbb{Z})$  in the row d = 1 should be removed.

#### **Chapter 14: Automorphic Scattering Amplitudes**

- p380, (14.25): Applying (10.89) to the Whittaker coefficient of Example 10.27 leads to a prefactor  $\frac{4\zeta(3)}{\xi(3)}$  instead of  $\frac{4\zeta(3)}{\xi(4)}$ .
- p381, (14.27): As in the correction to (14.25), the prefactor should be  $\frac{4\zeta(3)}{\xi(3)}$  instead of  $\frac{4\zeta(3)}{\xi(4)}$ .
- p381, (14.28): Working out the corrected prefactor in (14.27) leads to  $8\pi$  instead of  $\frac{180\zeta(3)}{\pi^2}$ .
- p391, (14.43): The sequence of differential operators is  $\mathcal{D}_{11}\mathcal{D}_{10}\cdots\mathcal{D}_{0}$ .