# Eisenstein Series and Automorphic Representations 

with Application in String Theory
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## Chapter 4: Automorphic Forms

p62, 13: 'co-called' should be replaced by 'so-called' [J. Gerken, 30/7/2018]
p63, (4.21): The standard notation $(c, d)=1$ on the sum means that $c$ and $d$ are co-prime: $\operatorname{gcd}(c, d)=1$. [J. Gerken, 30/7/2018]
p65, (4.33): The standard notation $(m, n)=1$ on the sum means that $m$ and $n$ are co-prime: $\operatorname{gcd}(m, n)=1$. [J. Gerken, 30/7/2018]
p66, (4.35): The correct formula is

$$
\begin{equation*}
\left(\left.\phi\right|_{k, m} \gamma\right)(\tau, z)=(c \tau+d)^{-k} e^{2 \pi i m\left[-\frac{c(z+\lambda \tau+\mu)^{2}}{c \tau+d}+\lambda^{2} \tau+2 \lambda z+\lambda \mu\right]_{\phi}}\left(\frac{a \tau+b}{c \tau+d}, \frac{z+\lambda \tau+\mu}{c \tau+d}\right) \tag{4.35}
\end{equation*}
$$

The $+\lambda \mu$ in the phase is only relevant when considering covers of the Jacobi group. For $\lambda, \mu \in \mathbb{Z}$ it can also be removed since $m$ is an integer. [J. Gerken, 30/7/2018]
p66, (4.39): The argument of $\phi$ on the left-hand side of the first line should be the matrix $\left(\begin{array}{ll}a & b \\ c & d\end{array}\right)$ acting on $\tau$ to give the first line

$$
\begin{equation*}
\phi\left(\frac{a \tau+b}{c \tau+d}, \frac{z}{c \tau+d}\right)=(c \tau+d)^{k} e^{2 \pi i m \frac{c z^{2}}{c \tau+d}} \phi(\tau, z) . \tag{4.39}
\end{equation*}
$$

[J. Gerken, 30/7/2018]
p 73 , (4.74): The correct functional relation (appearing in (1.25) and other places in the book) is

$$
\begin{equation*}
\xi(2 s) E(s, z)=\xi(\mathbf{2}(1-s)) E(1-s, z) \tag{4.74}
\end{equation*}
$$

[J. Gerken, 30/7/2018]
p74, (4.82): The standard notation $(c, d)=1$ on the sum means that $c$ and $d$ are co-prime: $\operatorname{gcd}(c, d)=1$. [J. Gerken, 30/7/2018]

## Chapter 5: Automorphic Representations and Eisenstein Series

p117, (5.130): The formula and its preceding text should be replaced by: ...functions in Ind ${ }_{\bar{B}}^{G} \sigma$ transform as

$$
\begin{equation*}
(\pi(g) f)(x)=(c x+a)^{-2 s} f\left(\frac{d x+b}{c x+a}\right) \tag{5.130}
\end{equation*}
$$

## Chapter 6: Whittaker Functions and Fourier Coefficients

p135, Prop. 6.20: The map $\varphi=f \circ \phi^{-1}: G(\mathbb{Q}) \backslash G(\mathbb{A}) / K_{f} \rightarrow \mathbb{C}$ is missing its target.
p151, Table 6.2: The entry for $\pi_{\mathrm{ntm}}$ for $E_{6(6)}$ can be expanded as follows. Next-to-minimal representations for $E_{6(6)}$ are found for generic $s_{1}$ as well as for generic $s_{6} . s_{5}=1$ is also next-to-minimal as already stated.

## Chapter 7: Fourier Coefficients of Eisenstein Series on $S L(2, \mathbb{A})$

p166, (7.64ff): The argument leading from (7.64) to (7.67) is incorrectly separated into local factors. A global argument giving (7.67) is as follows. Since $\psi$ is non-degenerate, there is a $t \in N(\mathbb{A})$ such that $\psi(t) \neq 1$. But since

$$
\int_{N(\mathbb{Q}) \backslash N(\mathbb{A})} \overline{\psi(n)} d n=\int_{N(\mathbb{Q}) \backslash N(\mathbb{A})} \overline{\psi(n t)} d n=\overline{\psi(t)} \int_{N(\mathbb{Q}) \backslash N(\mathbb{A})} \overline{\psi(n)} d n
$$

by translation invariance, the integral (7.64) vanishes. [J. R. Love, 26/3/2021]

## Chapter 9: Whittaker Coefficients of Eisenstein Series

p203, (9.73): The sign of $\rho$ in the exponents should be changed in the second and third line.

## Chapter 10: Analysing Eisenstein Series and Small Representations

p237, (10.67): Some entries contain too many squares. The correct table is

| $w$ | $M(w, \lambda)$ |
| ---: | :--- |
| $\mathbb{1}$ | 1 |
| $w_{1}$ | $c(2 s-1)$ |
| $w_{2} w_{1}$ | $c(2 s-1) c(2 s-2)$ |
| $w_{3} w_{2} w_{1}$ | $c(2 s-1) c(2 s-2) c(2 s-3)$ |
| $w_{4} w_{2} w_{1}$ | $c(2 s-1) c(2 s-2) c(2 s-3)$ |
| $w_{4} w_{3} w_{2} w_{1}$ | $c(2 s-1) c(2 s-2) c(2 s-3)^{2}$ |
| $w_{2} w_{4} w_{3} w_{2} w_{1}$ | $c(2 s-1) c(2 s-2) c(2 s-3)^{2} c(2 s-4)$ |
| $w_{1} w_{2} w_{4} w_{3} w_{2} w_{1}$ | $c(2 s-1) c(2 s-2) c(2 s-3)^{2} c(2 s-4) c(2 s-5)$ |

## Chapter 11: Hecke Theory and Automorphic L-functions

p262, (11.29): There is a typo in the argument of the modular form. The correct formula is

$$
\begin{equation*}
\left(T_{n} f\right)(z)=n^{k-1} \sum_{d \mid n} d^{-k} \sum_{b=0}^{d-1} f\left(\frac{\mathbf{n} z+b d}{d^{2}}\right) \quad(f \text { holomorphic of weight } k) \tag{11.29}
\end{equation*}
$$

## Chapter 13: Elements of String Theory

p322, Table 13.2: The factor $\mathbb{Z}_{2}$ in the discrete subgroup $G(\mathbb{Z})$ in the row $d=1$ should be removed.

## Chapter 14: Automorphic Scattering Amplitudes

p380, (14.25): Applying (10.89) to the Whittaker coefficient of Example 10.27 leads to a prefactor $\frac{4 \zeta(3)}{\xi(3)}$ instead of $\frac{4 \zeta(3)}{\xi(4)}$.
p381, (14.27): As in the correction to (14.25), the prefactor should be $\frac{4 \zeta(3)}{\xi(3)}$ instead of $\frac{4 \zeta(3)}{\xi(4)}$.
p381, (14.28): Working out the corrected prefactor in (14.27) leads to $8 \pi$ instead of $\frac{180 \zeta(3)}{\pi^{2}}$.
p391, (14.43): The sequence of differential operators is $\mathcal{D}_{11} \mathcal{D}_{10} \cdots \mathcal{D}_{0}$.

