

The main aim of the exercises is to make people think about the underlying physics. All the necessary input not provided here was given during the lectures or can be found in the notes online (with the exception of the gravitational wave emission from a rotating non-spherical pulsar, which I'm sure you are familiar with or you can easily find).

1. By making use of the stellar structure equations, and playing with the homology relations, derive the  $M - L$ ,  $M - R$ ,  $T - L$  relations for main sequence stars. To derive the latter equation, consider  $L \propto T^4$  and  $L \propto T^{18}$  for stars burning H and He respectively.  $M$  is the stellar mass,  $R$  is the radius,  $L$  is the luminosity and  $T$  is the temperature, (Pettini, lecture 10).
2. Derive the relation between  $P$ ,  $\rho$  and  $M$  in the same way. You can interpret  $P$  as the central pressure of the star. Use the obtained relation together with the equation of state of gas in the different regimes (radiation dominated, gas dominated, degeneracy pressure dominated) to find qualitative stellar evolutionary tracks on the  $T - \rho$  diagram (i.e. figure 8.2 in Pols lectures).
3. By using the heat transport equation in the radiative regime (stellar structure equations), the radiation pressure formula for a blackbody, and the condition of hydrostatic equilibrium, can you derive the Eddington luminosity? What is its physical interpretation?
4. What is the braking index of a pulsar assuming the spin-down is due to gravitational wave emission due to an asphericity  $\epsilon$ ? What would be the time needed to spin the Crab pulsar down from an initial period of, say, 1ms to its current observed value (33ms), assuming  $\epsilon = 10^{-6}$ ?
5. Consider a stellar mass function of the form  $dN/dM \propto M^{-\alpha}$ , with  $\alpha = 1$  for  $0.1 < (M/M_\odot) < 0.5$  and  $\alpha = 2.35$  for  $(M/M_\odot) > 0.5$  (this is approximately a Kroupa mass function). How many WD, NS and BHs do we expect in the MW? With the simplifying approximation that stars formed at a steady rate in the MW in the past 10Gyr, what is the expected supernova rate in the MW?
6. With the minimal assumptions that: (i) 50% of the stars form in binaries; (ii) binaries are equal mass; (iii) remnants formed in binary coalesce in  $T \ll T_H$  (where  $T_H$  is the Hubble time), what is the expected NS-NS and BH-BH rate of aLIGO events? Assume 400Mpc horizon for NS-NS and 3Gpc horizon for BH-BH.