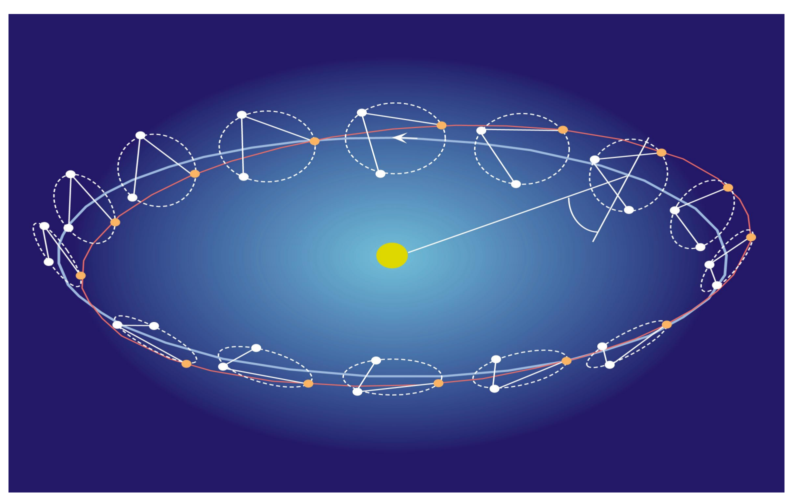
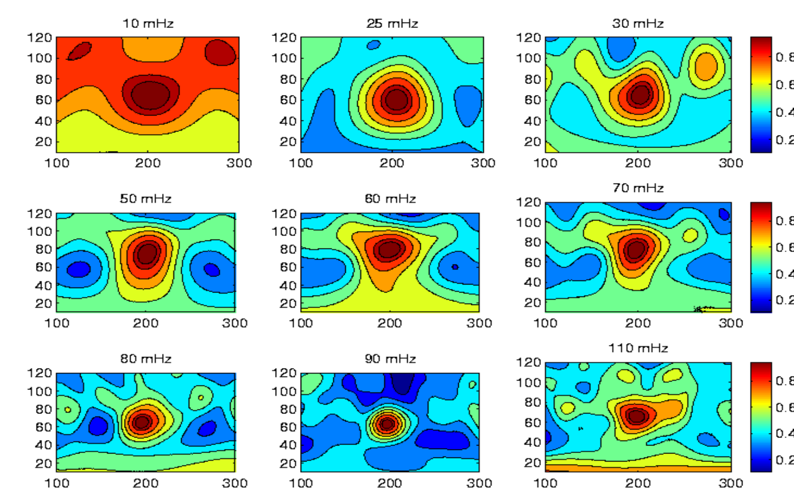
Source Optimal Tracking and LISA 'beamforming'





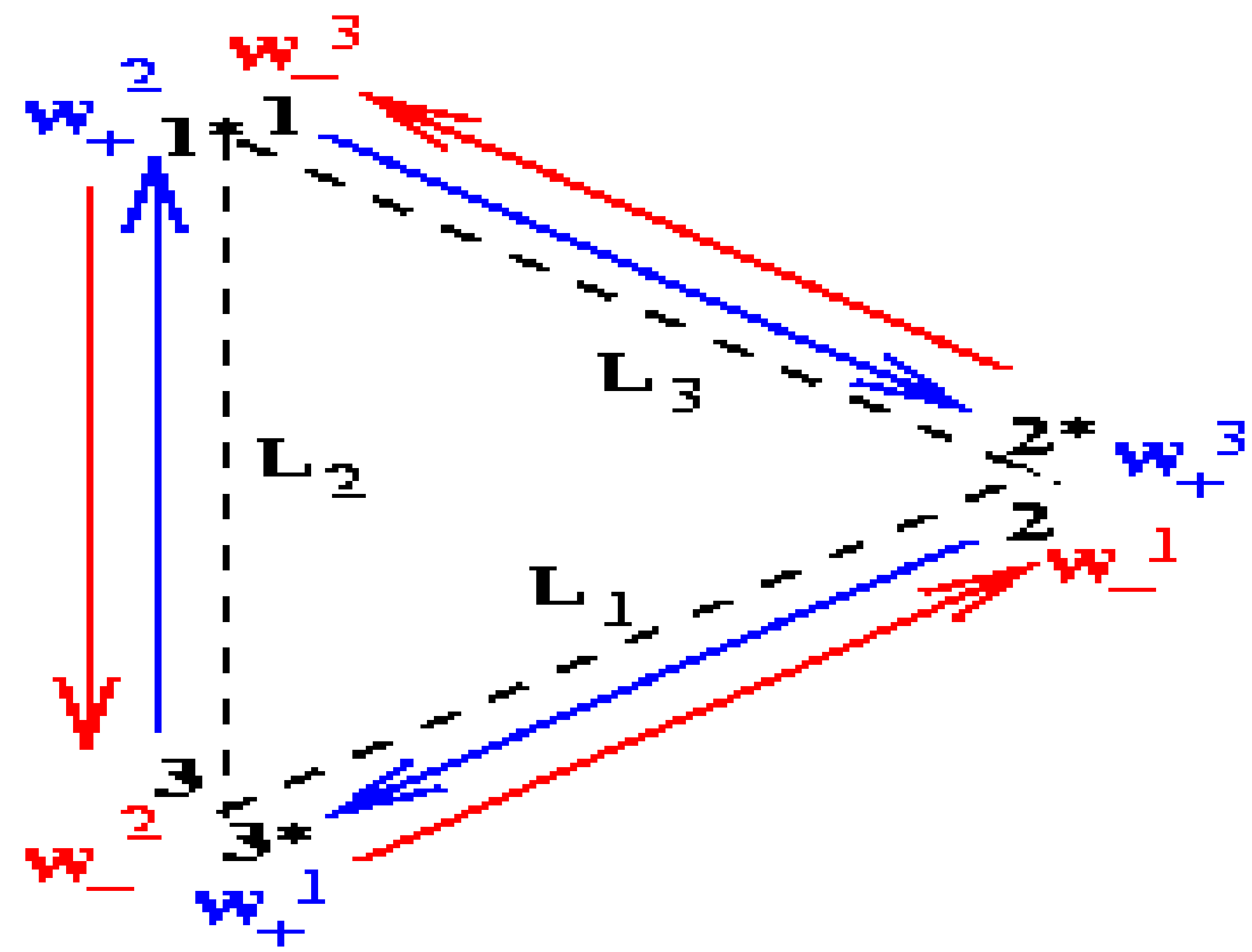
Archana Pai at Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)

LISA Astro-GR Meeting at AEI, Sept 2006

Recall from Bernard Schutz's talk

- 3 Space-Crafts in an equilateral triangle configuration in a heliocentric orbit
- Rotation period equals orbital period around sun (1 year)
- Arm-length: $5 \ \mathrm{million} \ \mathrm{km}$, Frequency band : 0.1 mHz - 1 Hz
- Noise sources :-
  optical path noise (e.g. shot noise ($\sim f$), laser frequency noise),
  acceleration noise ($\sim 1/f^2$).
  Dominant noise source – Laser frequency noise – TDI
- 6 Doppler data streams —- LISA is a network of interferometric detectors
- A,E,T – Noise uncorrelated TDI streams
- Issue: Network problem with TDI streams
  What is the optimal strategy to track a particular direction?

LISA: Doppler Data Streams



Doppler Data Streams: Beat incoming laser beam with the on-board laser beam

Clockwise — W_+^1, W_+^2, W_+^3 Anti-Clockwise — W_-^1, W_-^2, W_-^3

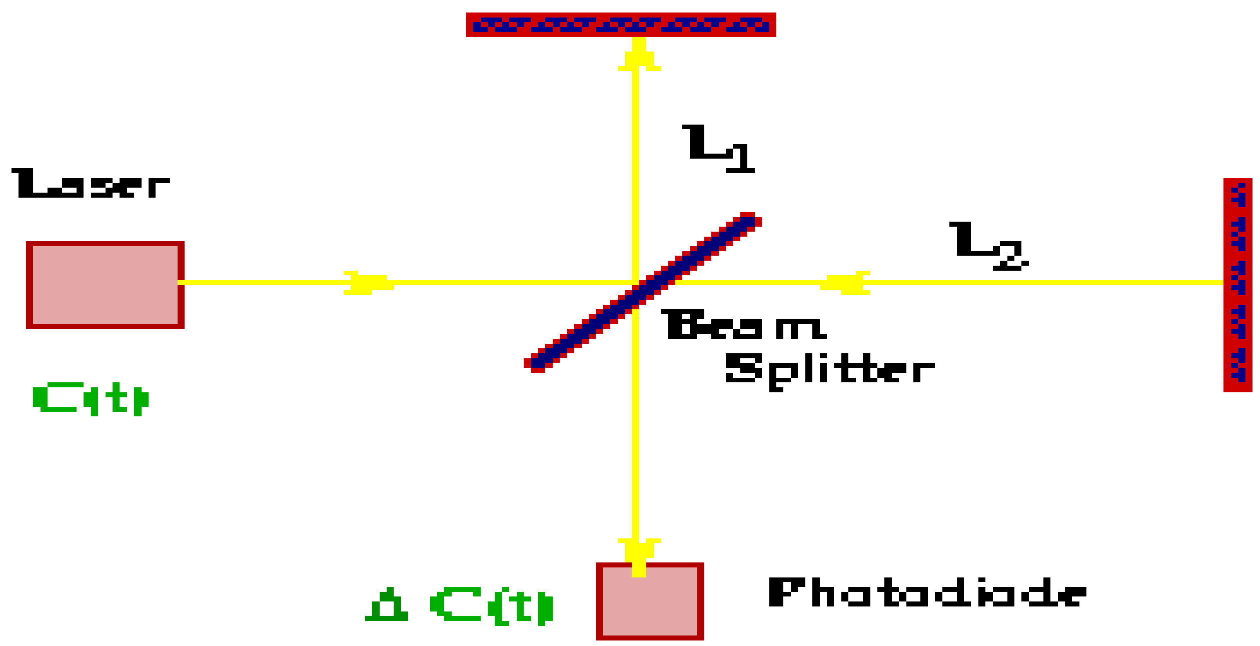
Example: Data Streams Generated on Space-craft 1,

$C_3(t - L_2) - C_1^* == W_+^2$ $C_2^*(t - L_3) - C_1 == W_-^3$

Laser Frequency Noise : Dominant Noise Source

Laser Frequency Noise: Fluctuation in laser frequency $C(t) = \delta\nu(t)/\nu_0$

$\Longrightarrow$ Fakes fluctuations in the optical path length (== mimics GW)



Michelson Interferometer:

Laser frequency noise at photodiode

$\Delta C(t) = C(t - 2L_1) - C(t - 2L_2)$

$|\Delta\tilde{C}(f)| \sim 4\pi f|\tilde{C}(f)\Delta L|$

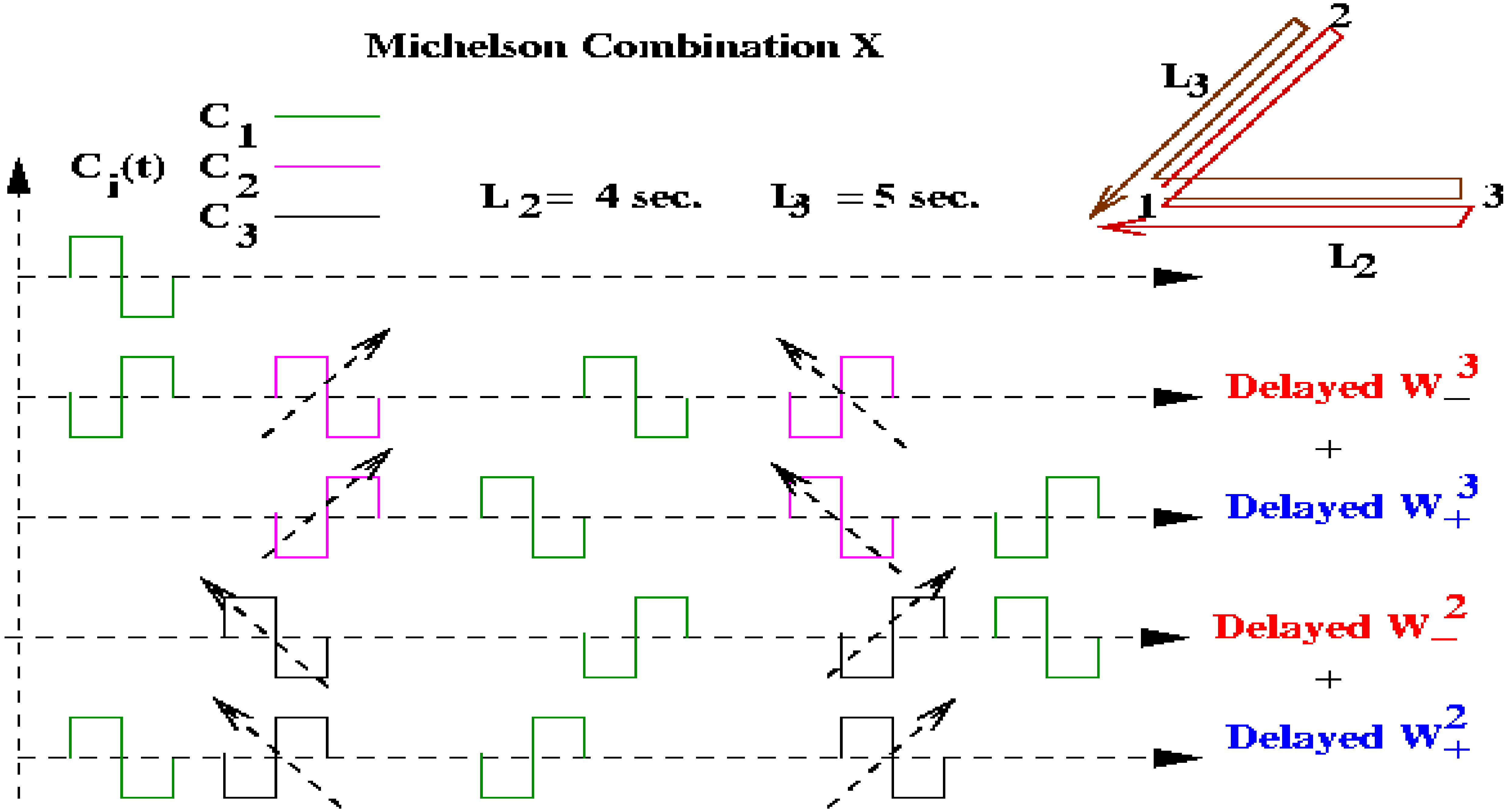
- Michelson with equal arms ($\Delta L = 0$): Light travels equal path $\Longrightarrow$ No laser freq. noise
- Michelson with unequal arms ($\Delta L \neq 0$) $\Longrightarrow$ Laser frequency noise.
- Estimation of Laser Frequency Noise for LISA:
  Nd-YAG laser: $\tilde{\delta\nu} \sim 30Hz/\sqrt{Hz}$ and $\nu_0 = 10^{14}\mathrm{Hz} \Longrightarrow |\tilde{C}(f)| \sim 10^{-13}/\sqrt{\mathrm{Hz}}$
  Assume $|\Delta L| \sim 10\% L \sim 1.6\mathrm{sec} \Longrightarrow |\Delta\tilde{C}(f)| \sim 10^{-16}/\sqrt{\mathrm{Hz}}$ at $1\mathrm{mHz}$
- LISA requirement: $\mathbf{h \sim 10^{-21} - 10^{-22}}/\sqrt{\mathrm{Hz}}$ at $1\mathrm{mHz}$
  $\Longrightarrow$ Need to suppress Laser Frequency Noise

LISA: Time Delay Interferometry (TDI)

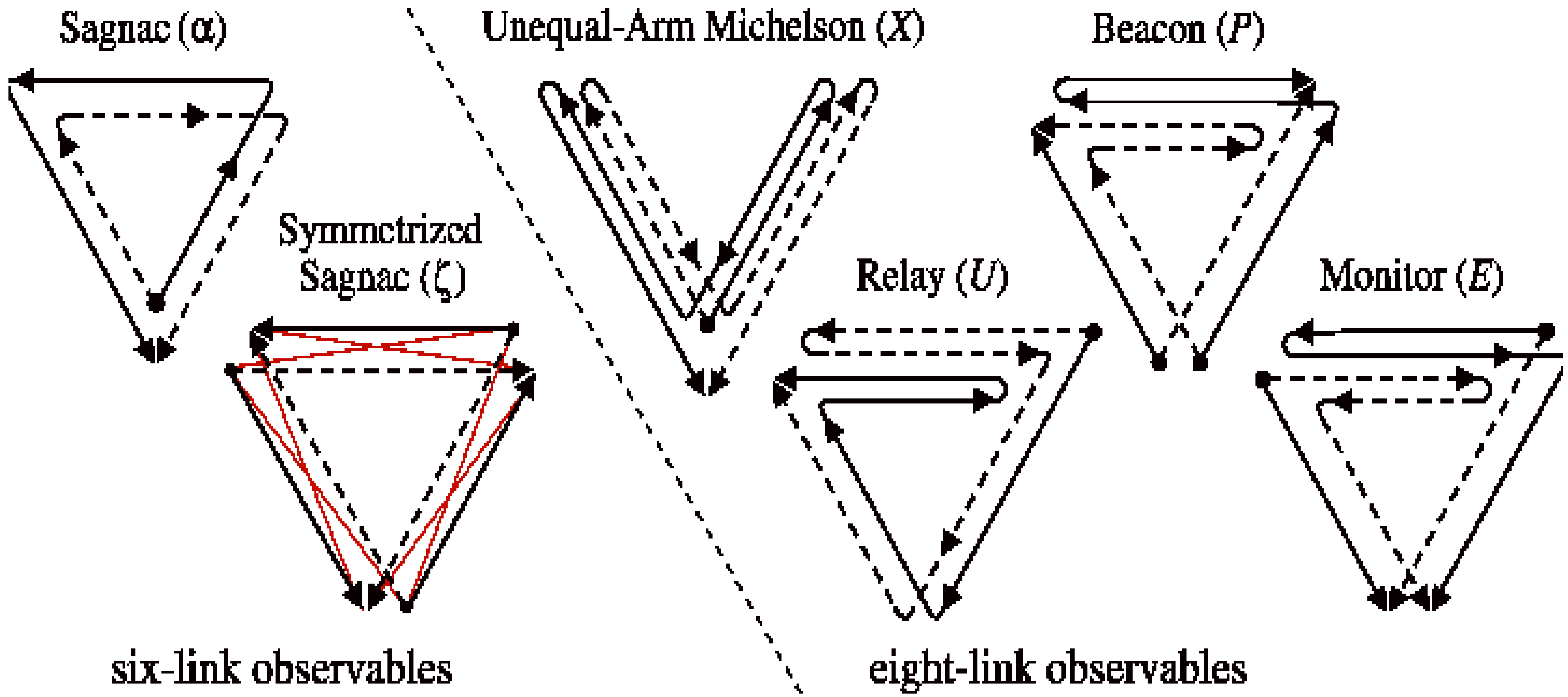
Principle: Make the interfering beams travel through the same light travel time.

How to: Combine Doppler Data streams with appropriate time-delay and suppress the laser frequency noise [Ap. J. 527,814 (1999); PRD 59, 102003 (1999)] $E_i X(t) = X(t - L_i)$

Michelson Combination:– $X = (E_2^2 - 1)(E_3 W_+^3 + W_-^3) - (E_3^2 - 1)(W_+^2 + E_2 W_-^2)$



Zoo of TDI combinations



Vallisnery: gr-qc/0504145

- A TDI generator set $\{\alpha, \beta, \gamma, \zeta\}$ gives all possible laser noise free data combinations e.g. Michelson $E_1 X = E_2 E_3 \alpha - E_2 \beta - E_3 \gamma + \zeta$
  [Review on TDI: MT,SVD Liv Rev.Rel 8, (2005),4]
- Drifting L_i with $\dot{L}_i(t) \sim 10\mathrm{m/s}$, residual noise is $30\times$ other noises $\rightarrow 2^{\mathrm{nd}}$ gen. TDI.

LISA TDI streams – Signal to Noise Ratio

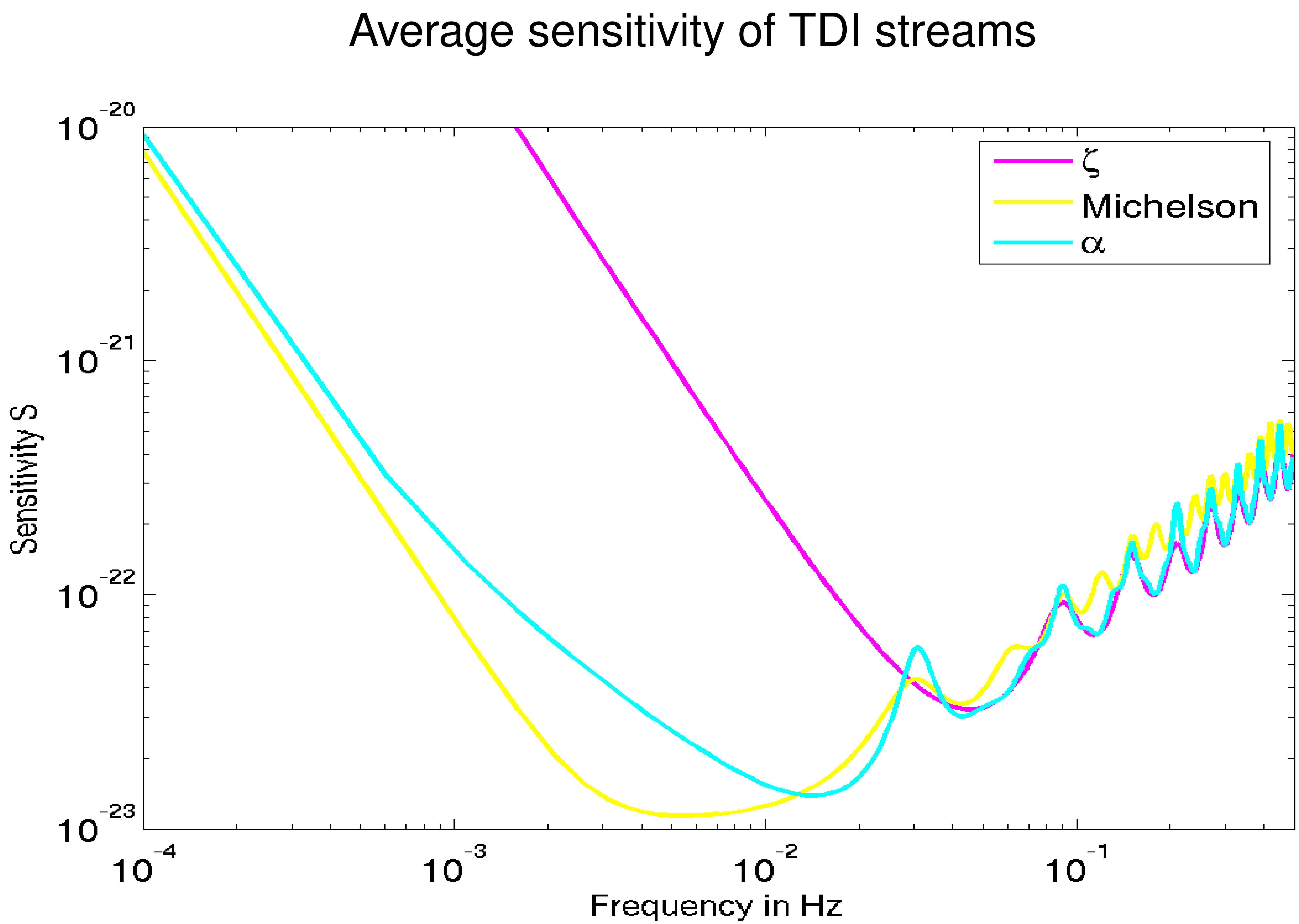
- GW polarizations $\longrightarrow \mathbf{h_+, h_\times}$
- Arbitrary TDI combination: $\mathbf{D(\Omega)} = \sum_{\mathbf{m=1}}^{\mathbf{3}} \sum_{\sigma=\pm} \rho_{\mathbf{m}\sigma}(\mathbf{\Omega})\mathbf{W}_\sigma^{\mathbf{m}}(\mathbf{\Omega})$
- GW response of $\mathbf{D}$;

$$\mathbf{h^{(D)}(\Omega) = F_+^{(D)}(\Omega)h_+(\Omega) + F_\times^{(D)}(\Omega)h_\times(\Omega)}$$

Antenna pattern functions $F_{+,\times}^{(D)}$ in terms of the transfer function of $\mathbf{W}_\sigma^{\mathbf{m}}(\mathbf{\Omega})$

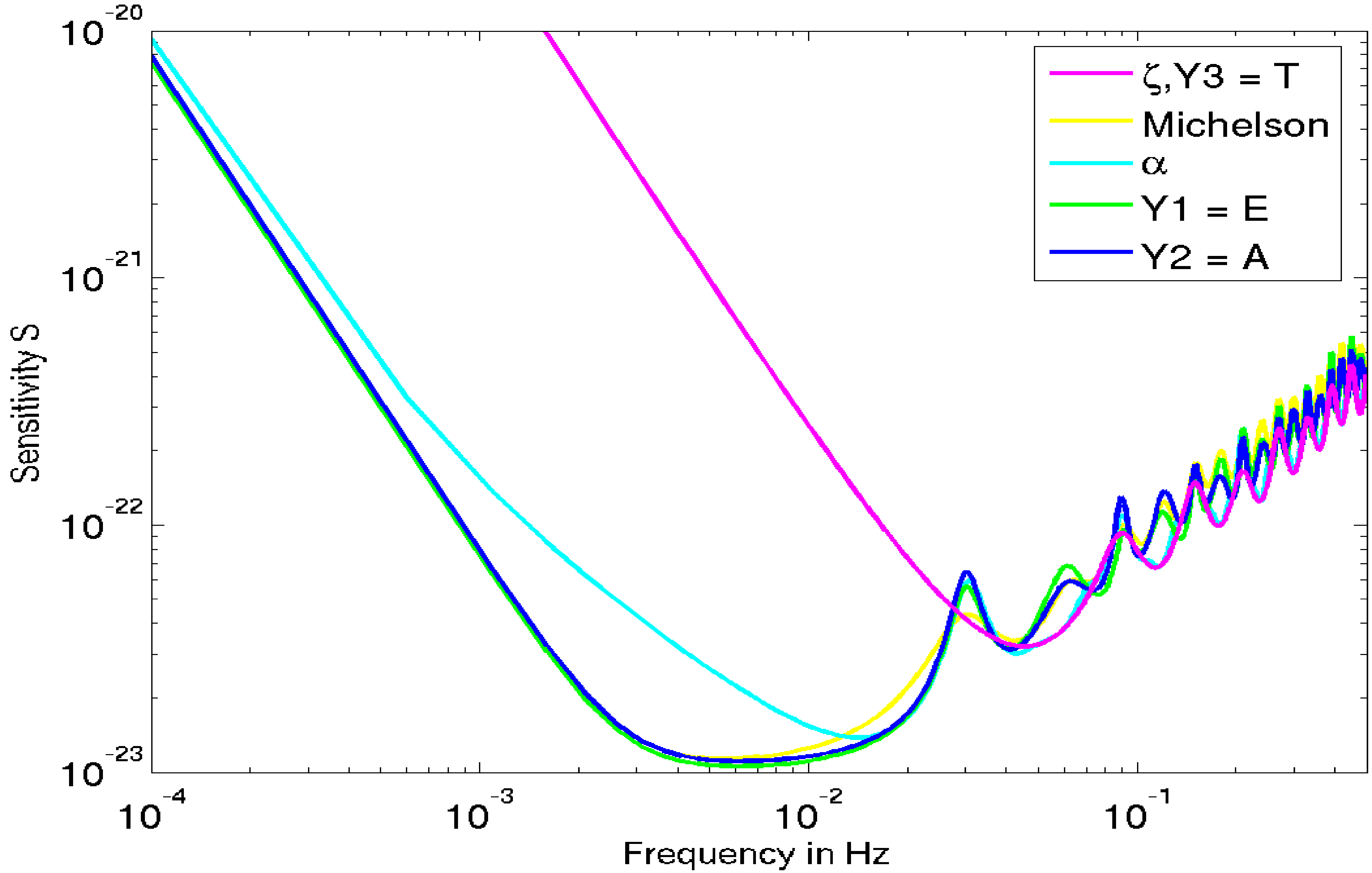
$$\mathbf{F}_{+,\times}^{\mathbf{(D)}}(\mathbf{\Omega}) = \sum_{\mathbf{m=1}}^{\mathbf{3}} \sum_{\sigma=\pm} \rho_{\mathbf{m}\sigma}^{\mathbf{(D)}} \mathbf{f}_{\mathbf{m}\sigma}(\mathbf{\Omega}, \hat{\mathbf{w}}, \hat{\mathbf{n}}_{\mathbf{m}}) \xi_{\mathbf{m};+,\times}(\hat{\mathbf{w}}, \hat{\mathbf{n}}_{\mathbf{m}})$$

- Noise vector for $\mathbf{D}$: $\mathbf{N^D}(\mathbf{\Omega};, \rho_{\mathbf{m}\sigma}, \mathbf{S^{pf}}, \mathbf{S^{opt}})$
  $\mathbf{S^{pf}}$: Proof Mass Noise PSD, $\mathbf{S^{opt}}$: Optical Path Noise PSD
- Signal-to-Noise ratio: $\mathbf{SNR(\Omega) = |h|(\Omega)/||N^X||(\Omega)}$
- Sensitivity; $\mathbf{S = 5}/\mathrm{SNR_{int}}$, for integration time of 1 year

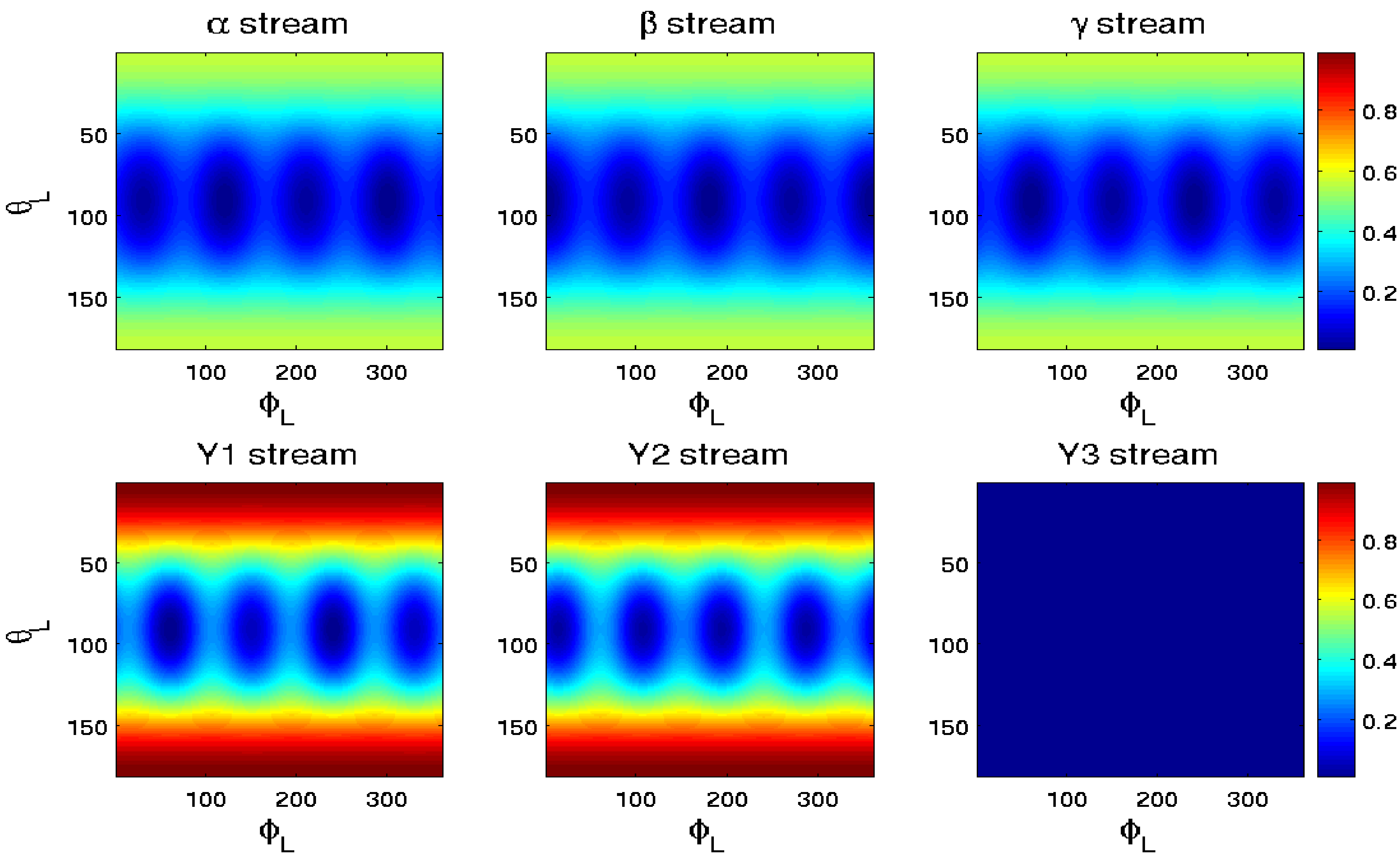


Average sensitivity of TDI streams
Sensitivity S
Frequency in Hz
ζ
Michelson
α
10^-20
10^-21
10^-22
10^-23
10^-4
10^-3
10^-2
10^-1

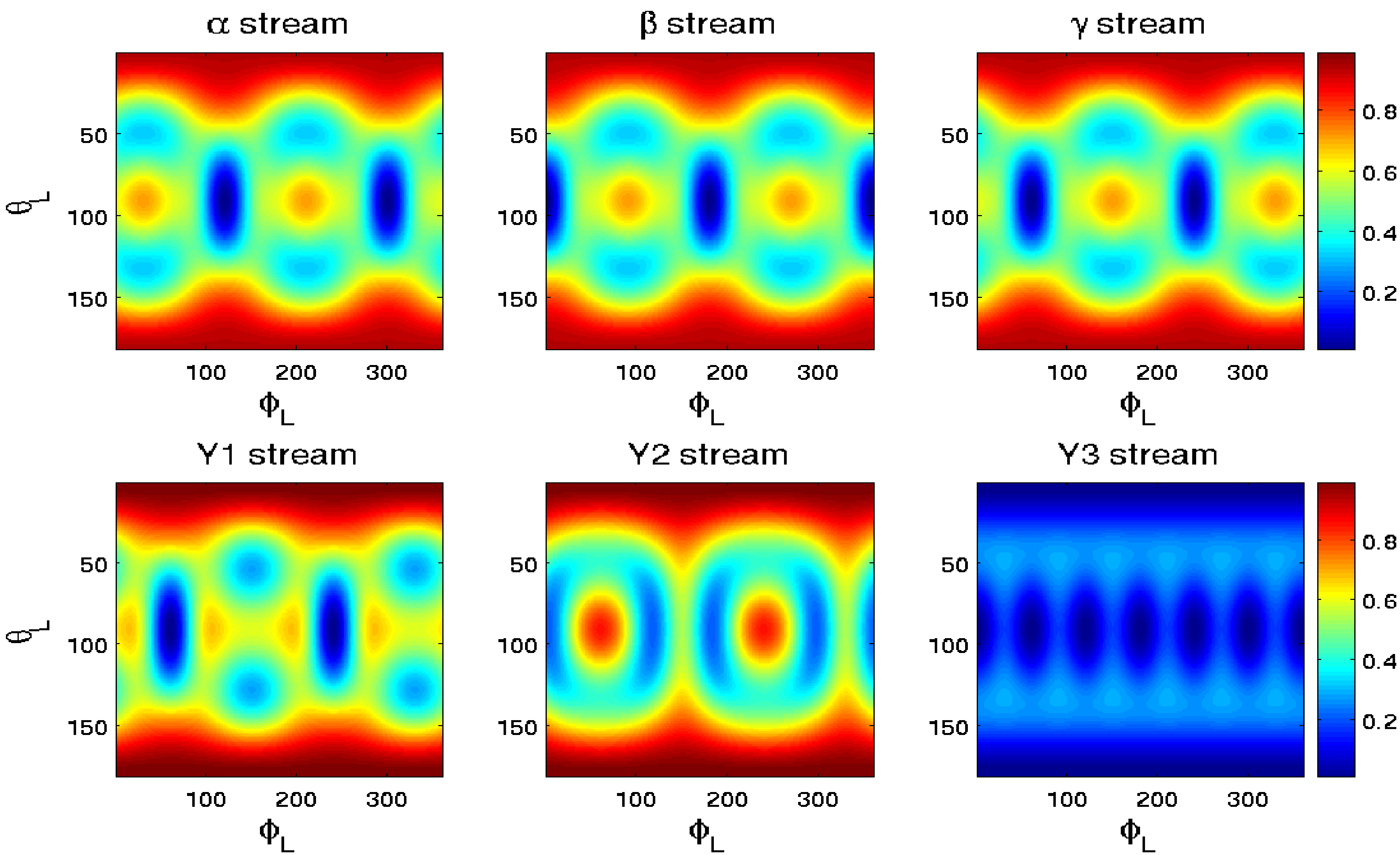
Noise Independent TDI streams – Y's



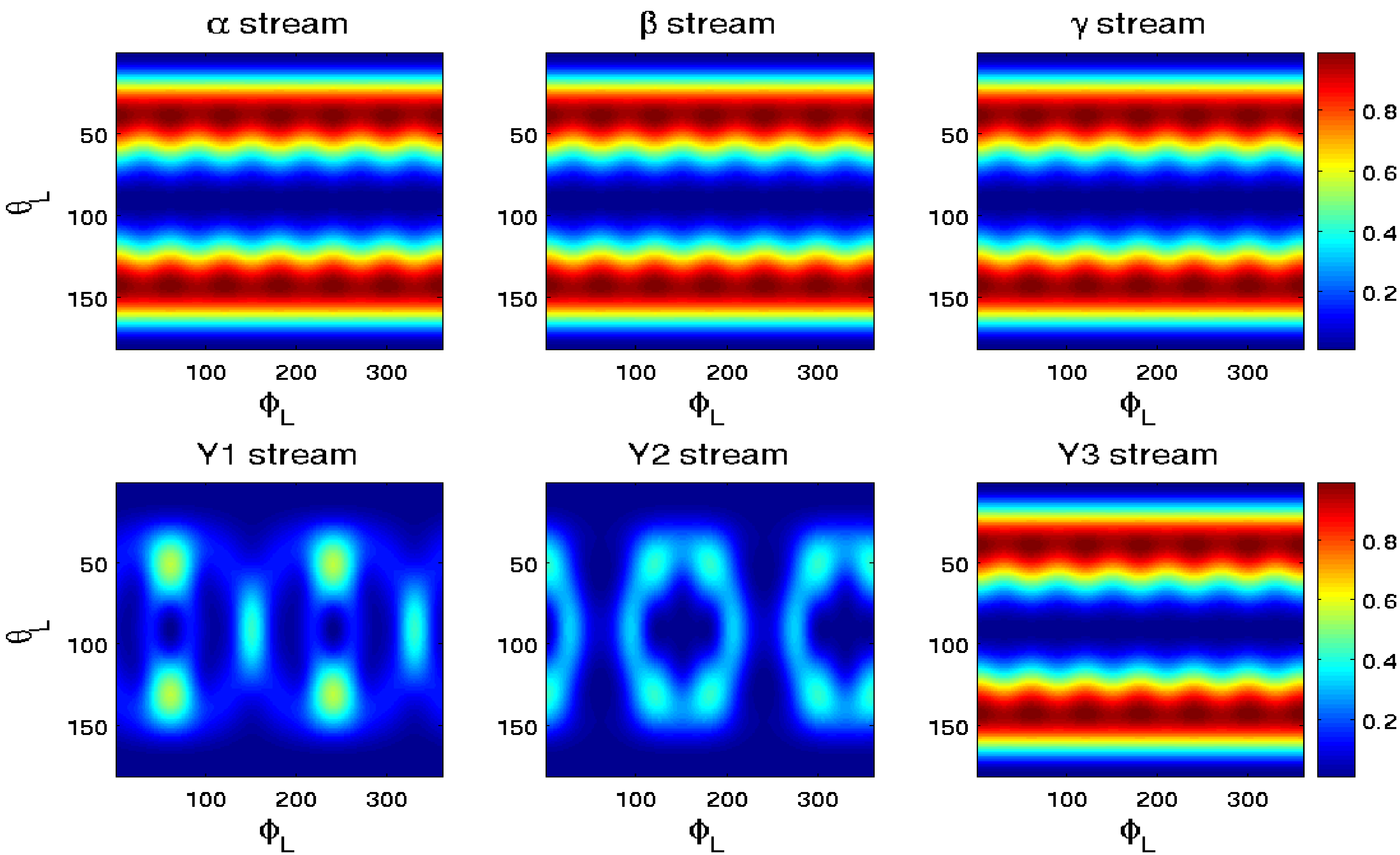
LISA TDI streams:directional SNR at $f = 10$ mHz



LISA TDI streams:directional SNR at $f = 25$ mHz



LISA TDI streams:directional SNR at $f = 50$ mHz



Response of LISA TDI streams: Summary

LISA is not a single interferometer but a 'network' of interferometric detectors.
Different TDI data streams have different frequency and angular response.

- At low frequency, $f < c/(2\pi L) \sim 10\mathrm{mHz}$
  α, β, γ have two arm interferometer antenna pattern.

- At high frequency, $f > c/(2\pi L) \sim 10\mathrm{mHz}$
  Angular response gets more complex due to higher multi-poles.
  Geometry of LISA is evident in the response.

- TDI data streams with uncorrelated noise – PRD 66 122002 (2002), CQG 20, 1217 (2003)

$$E \sim \alpha + \beta - 2\gamma = Y_1 \qquad A \sim \beta - \alpha = Y_2, \qquad T \sim \alpha + \beta + \gamma = Y_3$$

  Treat them as indepedent detectors for LISA data analysis.

- Are there data streams with directional information?

LISA TDI streams: Optimal Directionality PRD 68, 122001 (2003)

Optimal directional sensitivity with $Y1, Y2, Y3$ —

$$X = a_1(\Omega, \hat{w}_{L_0})\, Y_1 + a_2(\Omega, \hat{w}_{L_0})\, Y_2 + a_3(\Omega, \hat{w}_{L_0})\, Y_3$$

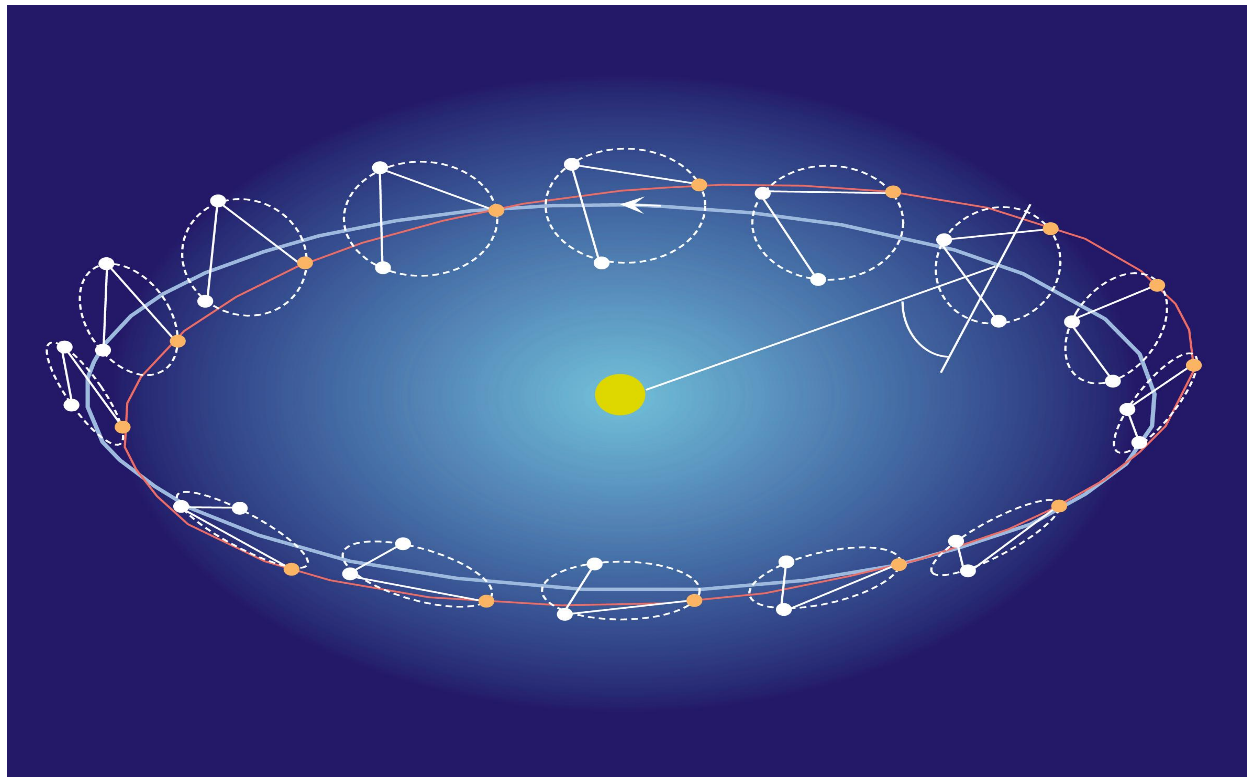
- Optimization of directional SNR gives 3 data streams

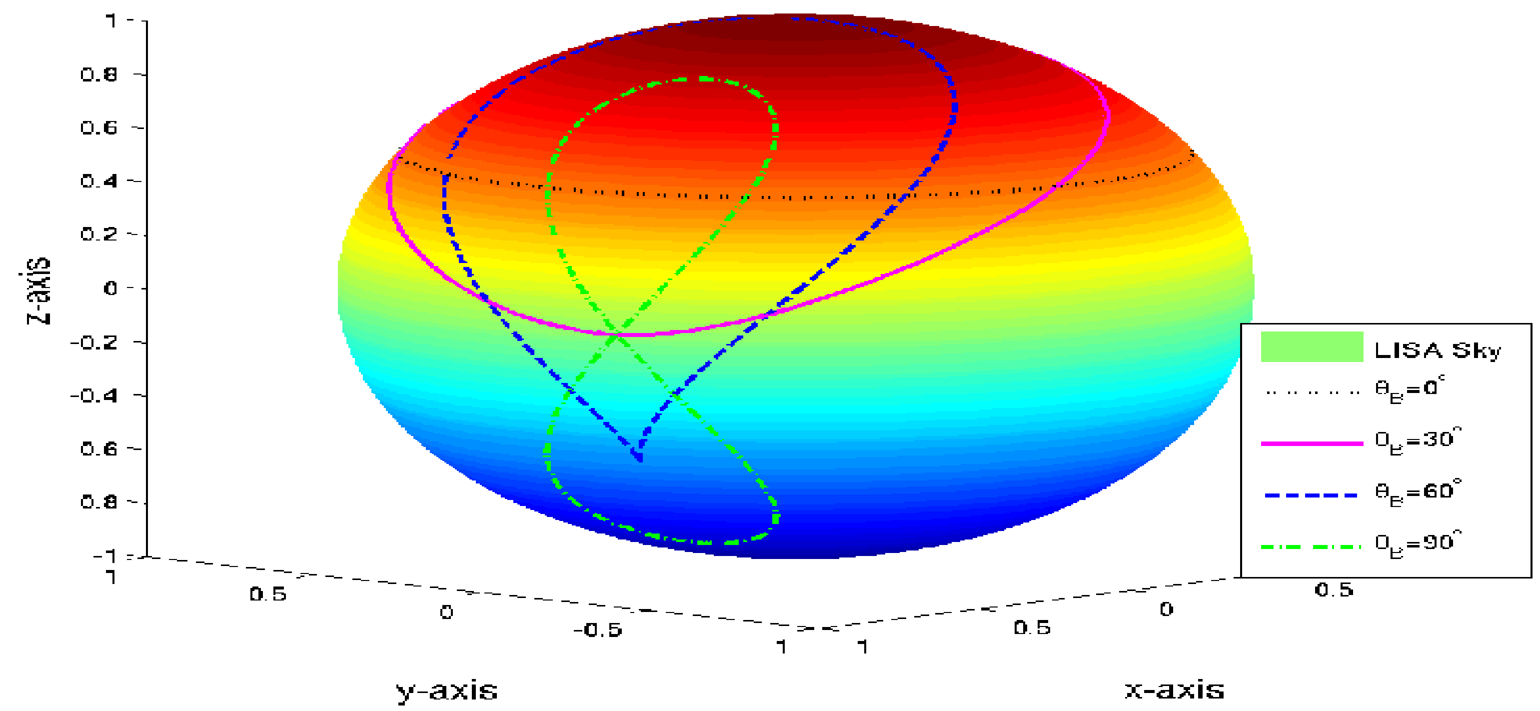
$$\vec{V}_+ = c_+ \vec{f}_+ + c_\times \vec{f}_\times\,, \quad \vec{V}_\times = c^*_\times \vec{f}_+ - c_+ \vec{f}_\times, \quad \vec{V}_0 = \vec{f}_+^{\,*} \times \vec{f}_\times^{\,*}\,,$$

  (1) maximum positive (2) minimum positive (3) null – zero directional SNR.

- Combine v_+ and $v_\times$ streams quadratically to improve SNR $\quad a_{(I)+,\times,0} = V^{(I)*}_{+,\times,0}/n_{(I)}$
- Complimentary: $\{v_+, v_\times\}$ gives network and v_0 gives zero directional SNR
- At low frequency: $c_\times = 0$, v_+ measures + polarisation and $v_\times$ measures $\times$.

LISA Motion and Source Tracking

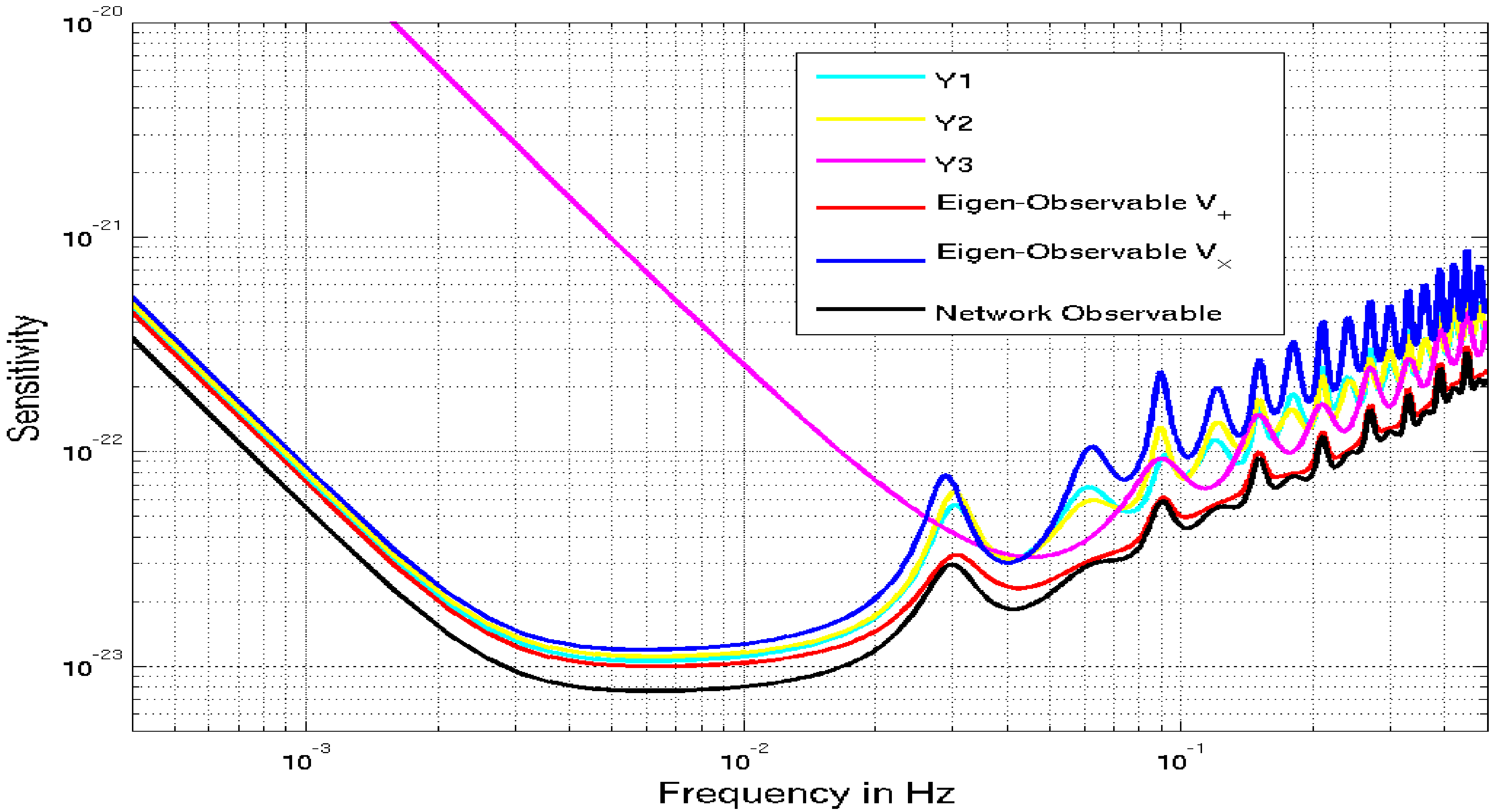




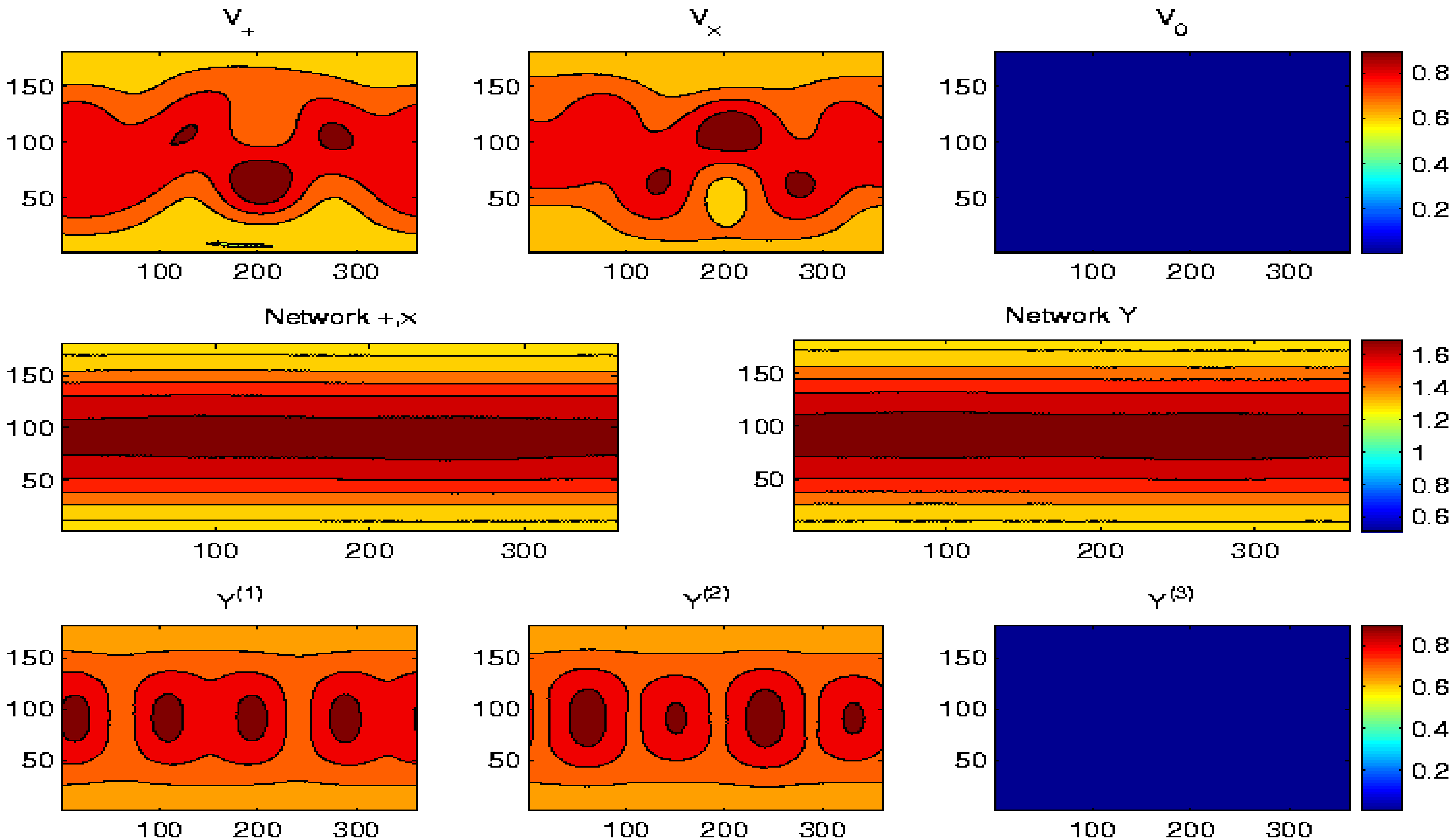
- A fixed source in the barycentric frame $\hat{w}_B$ makes a track in LISA's sky $(\theta_L(t;\hat{w}_B), \phi_L(t;\hat{w}_B))$.
  - AM – Antenna pattern depends on source position (which changes with time).
  - FM – Doppler modulation due to source motion relative to the detector.
- Optimal tracking of a direction: Switching to optimally directional streams i.e.$\{v_+(\theta_L(t;\hat{w}_B), \phi_L(t;\hat{w}_B)), v_\times(\theta_L(t;\hat{w}_B), \phi_L(t;\hat{w}_B))\}$.
- Optimal combinations use signal modulation due to LISA motion (via source's apparent motion).

Optimal Source Tracking: SNR Improvement

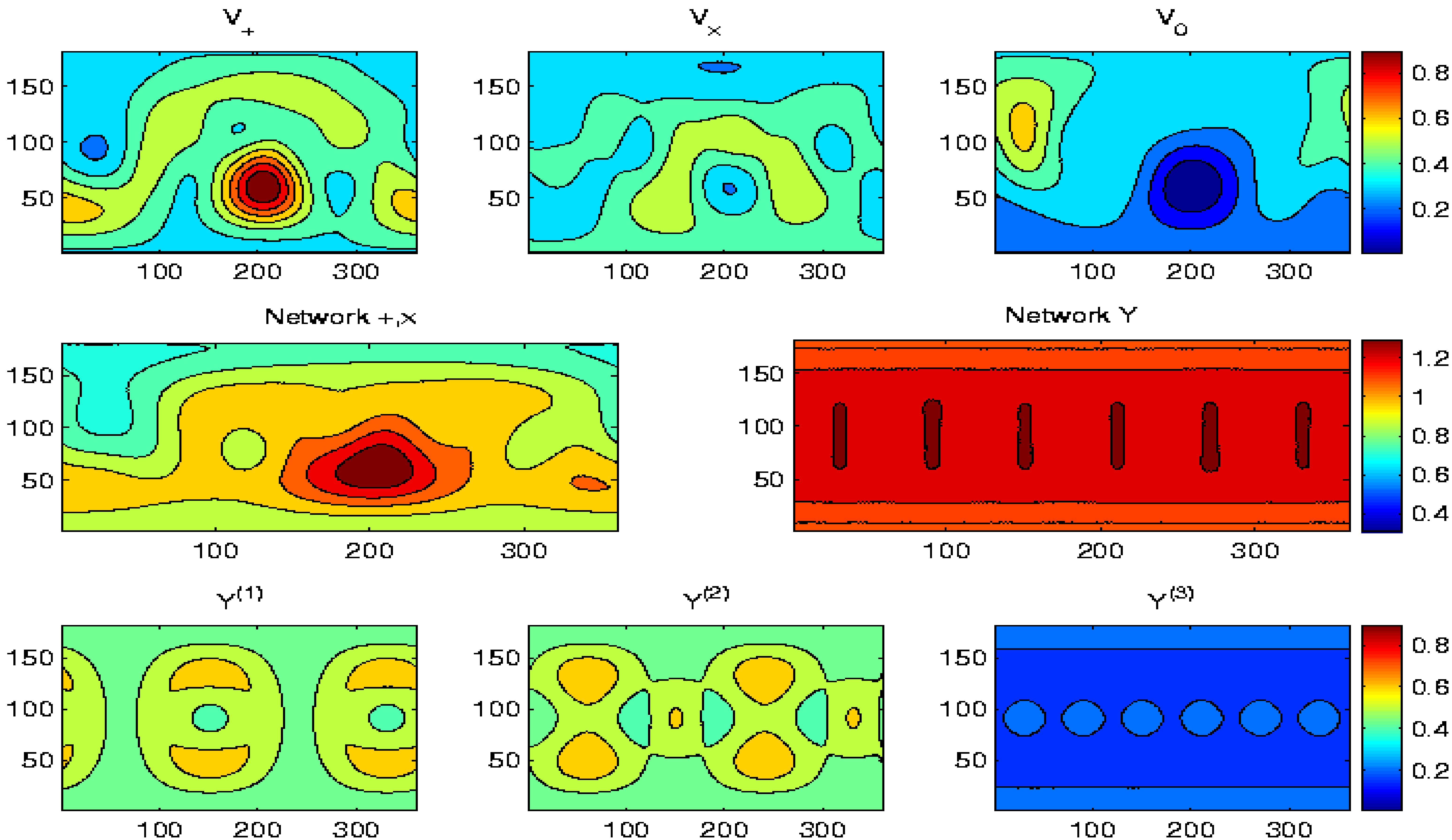
Improvement in SNR goes from 34% at low freq to 90% at 20 mHz



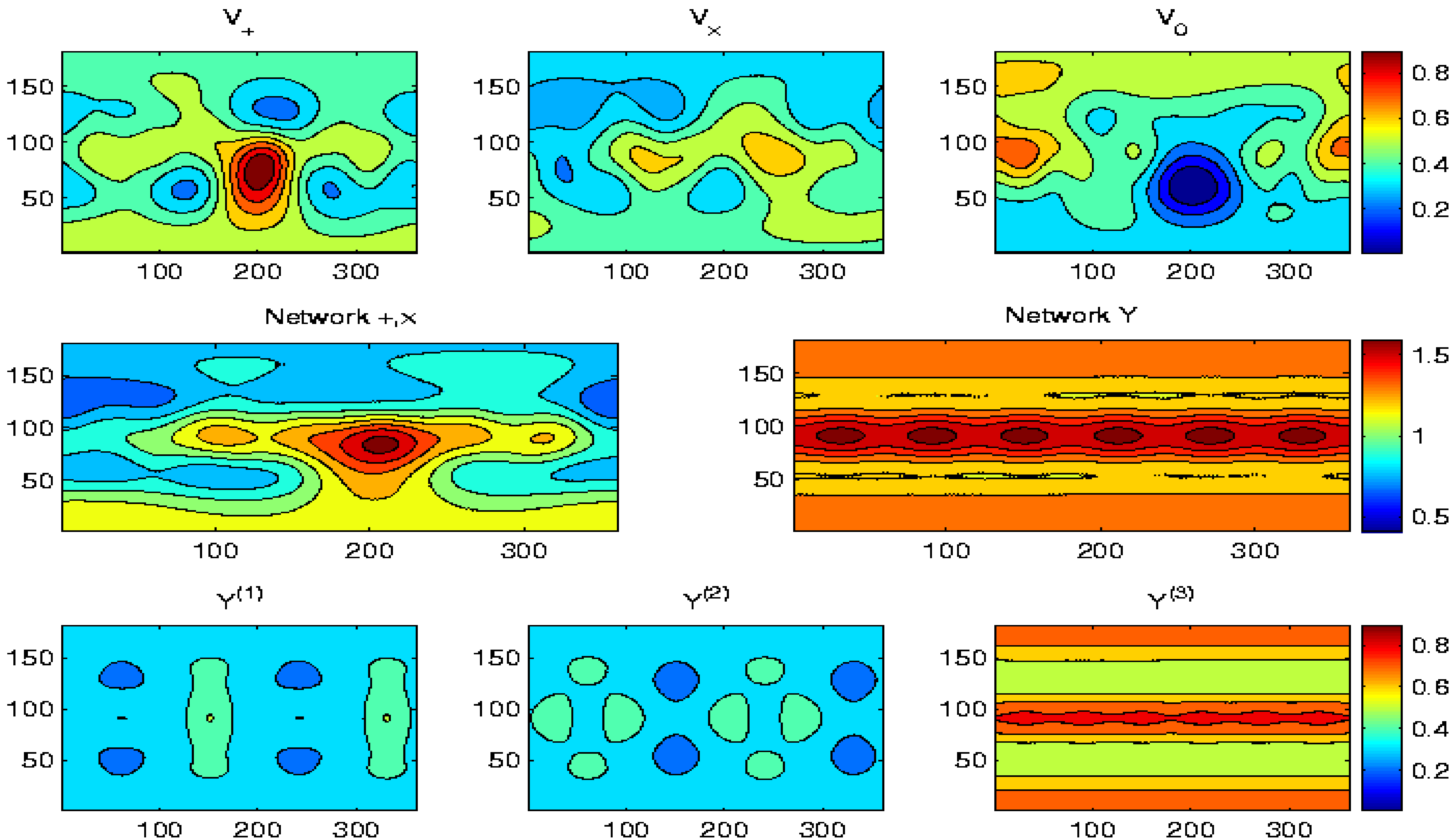
Optimal Source Tracking and Beamforming at $f = 10$ mHz gr-qc/0608138



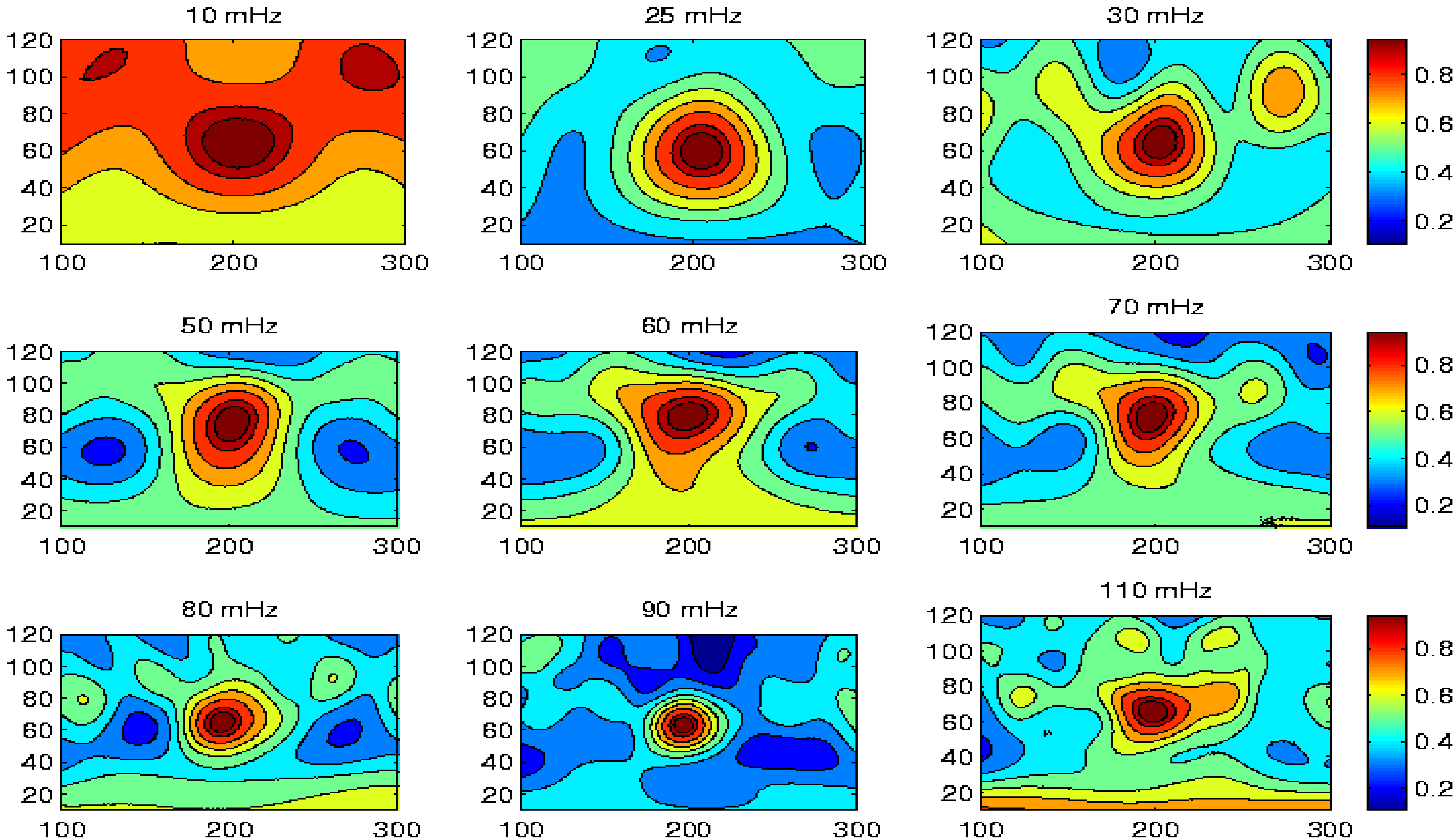
Optimal Source Tracking and Beamforming at $f = 25$ mHz



Optimal Source Tracking and Beamforming at $f = 60$ mHz



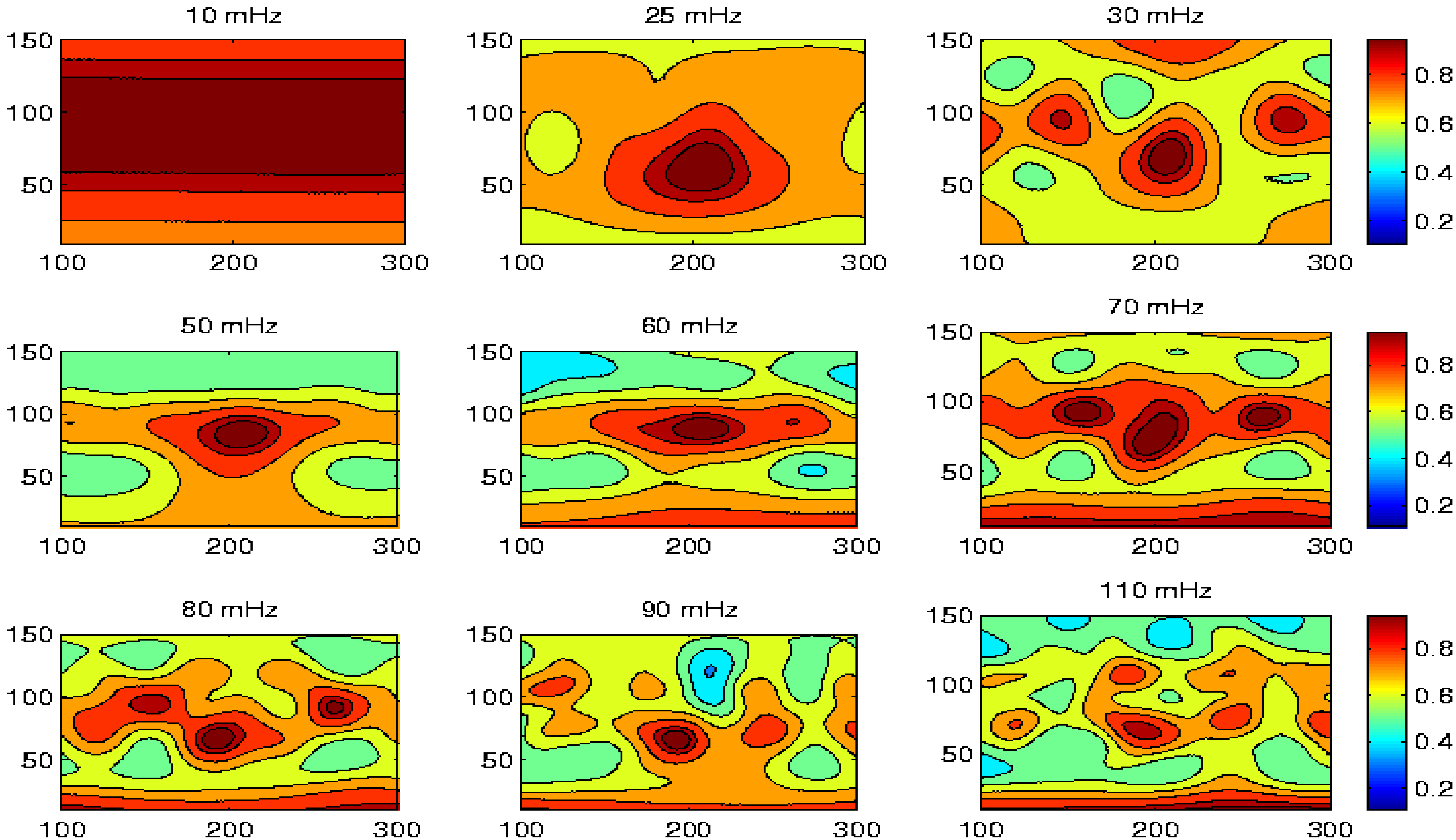
Optimal Source Tracking and Beamforming for v_+



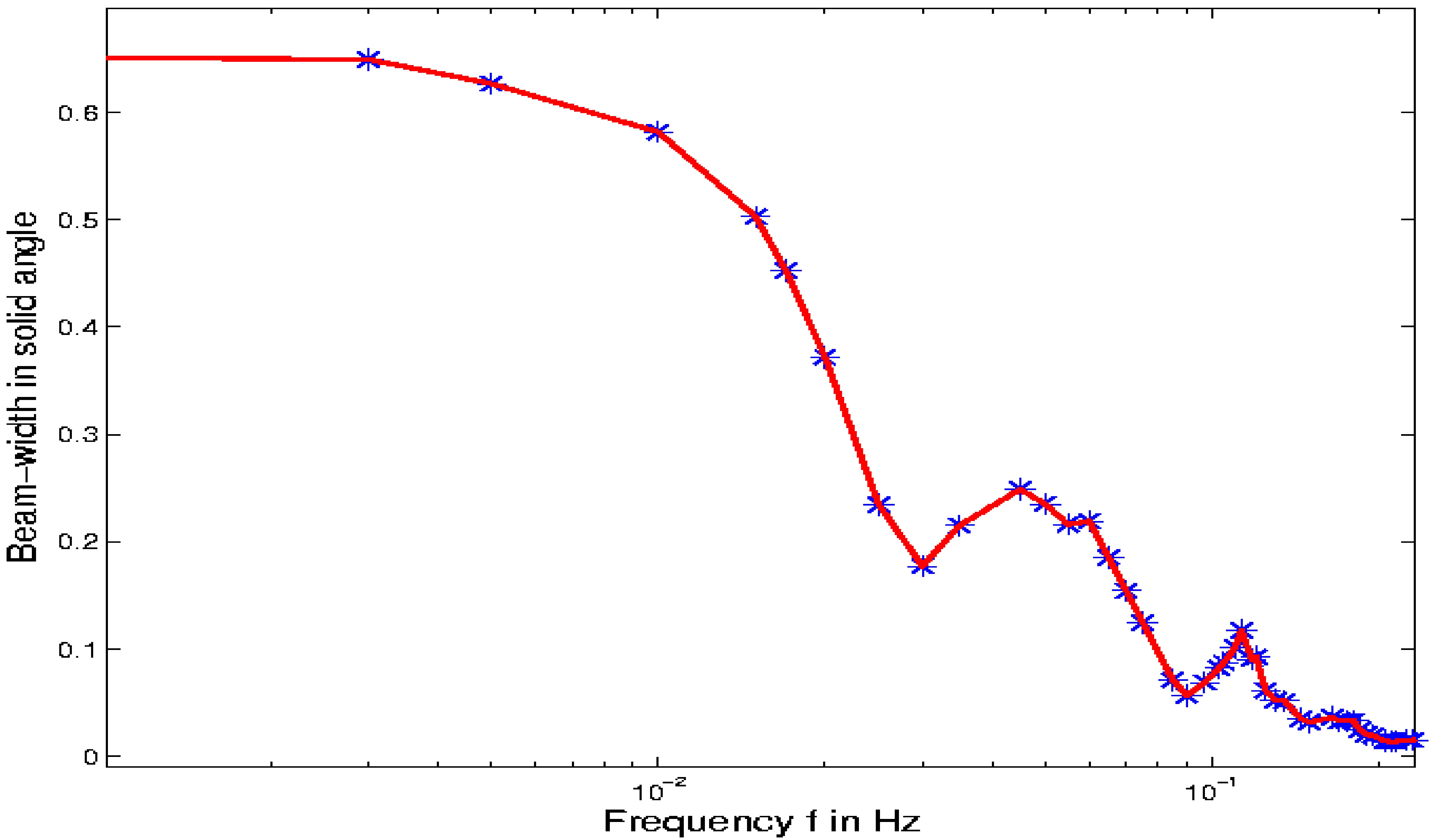
Discussion

- A fixed direction in barycenttric frame can be optimally tracked in LISA frame. $v_+, v_\times, v_0$ contains complete information of presence/abscence of signal in a direction.
- Network of $\{v_+, v_\times\}$ gives same SNR as that of network of A,E,T. However, $\mathrm{Net}_{+,\times}$ is beamed towards the tracking direction.
- Beaming improves towards high frequency.
- $\mathrm{Net}_{+,\times}$ is complimentary to v_0. – could be useful for consistency checks.
- It is best to work with time-dependent switched combinations at least for high frequencies.
- The scheme is straightforward to extend to frequency evolving sources.
- Possible application to E/IMRI ?:–
  Get switched data streams looking primarily in a particular direction.
  Apply detection algorithm to them. (under investigation with Stas Babak)

Optimal Source Tracking and Beamforming



Optimal Source Tracking and Beamforming



LISA TDI streams: open issues

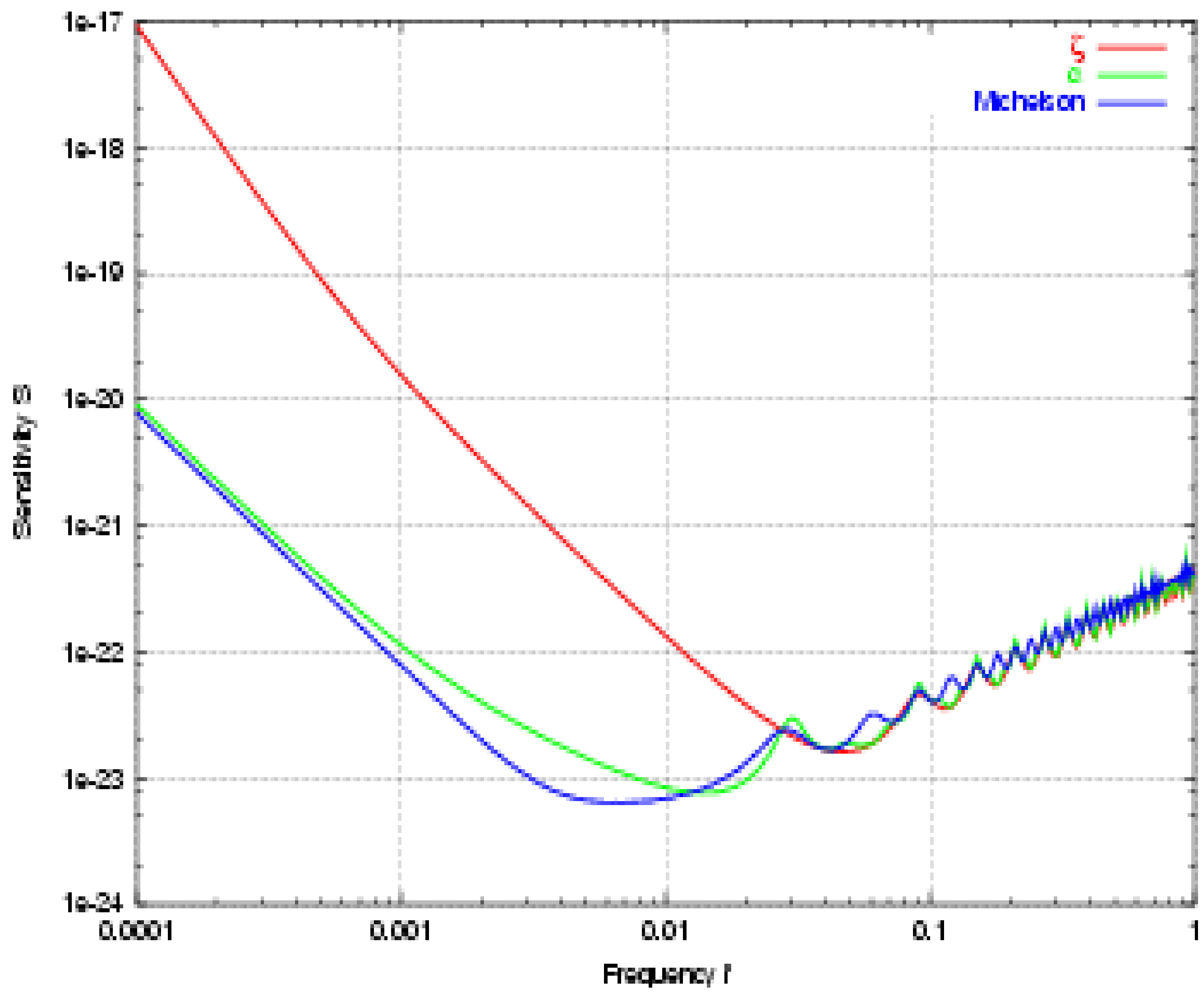
TDI directional optimally is an open issue for other sources

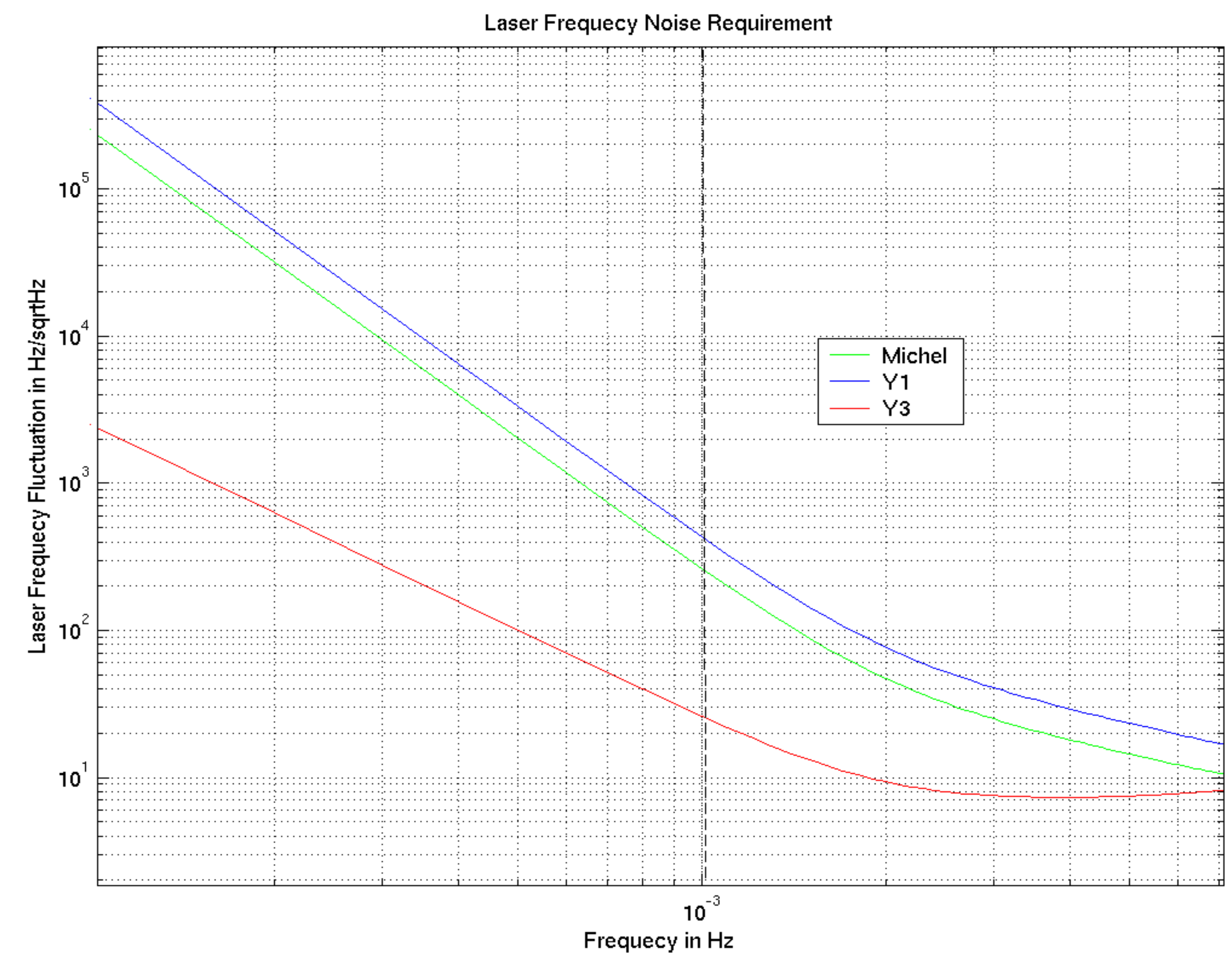
- Which TDI stream is optimal for MBH inspirals, mergers, ring-downs, EMRIs, stochastic background?
- Which TDI stream gives zero signal for MBH inspirals, meregers, ring-downs, EMRIs?
- When to switch the data combination as LISA moves?
- Can TDI tracking with data analysis algorithm be used for parameter estimation?
- What other TDI data streams are required for book-keeping?

Source related data analysis issues →

Average sensitivity of TDI streams

- Michelson combination can be expressed as $E_1 X = E_2 E_3 \alpha - E_2 \beta - E_3 \gamma + \zeta$





- If L_i is known, the $1^{\rm st}$ generation TDI cancels laser frequency noise.
  If $\delta L \sim 200$ m, laser stability req ($\propto \delta L^{-1}$) at mHz is $25Hz/\sqrt{Hz}$.
  Clock accuracy of ~ 100 ns, no high sampling, start the clock at req. delays.
  Drifting L_i with $\dot{L}_i(t) \sim 10{\rm m/s}$, residual noise is $30\times$ other noises $\rightarrow 2^{\rm nd}$ gen. TDI.
  [Review on TDI: MT,SVD Liv Rev.Rel 8, (2005),4].

LISA Signal Modulation

LISA's motion (1) rotational motion (2) orbital motion around the sun modulates the incoming GW signal

- AM – detector's motion sweeping the antenna pattern across the sky,
- FM – Motion of the detector relative to the source (doppler modulation).
- PM – Different responses to each polarization.

Frequency Modulation and resolution

- Orbital motion: Doppler modulation $f_0 \to f_0(1 \pm v/c) \quad v/c \sim 10^{-4}$
- Rotational motion spreads the signal frequency f_0 in the frequency bin of $1/T$
- At $f_0 = 1\mathrm{mHz}$, $f_0 v/c \sim 1/T$. No Doppler modulation for $f < 1\mathrm{mHz}$
- Angular resolution: $\theta \sim \lambda/R \sim 10^{-3}/f\mathrm{rad} \quad R = 2A.U.$
  For $f > 1mHz$ angular resolution decreases with f related to increase in doppler modulation with f. Source location from doppler modulation (like pulsar detection in ground based detectors)
  For $f < 1mHz$, source location is relied on amplitude modulation as LISA sweeps, does not depend on f at low freq.