Mapping spacetimes with LISA

Recent progress and future challenges

Kostas Glampedakis

University of Southampton, UK

(in collaboration with Stas Babak)

Setting the stage

- “Mapping of Kerr geometry”, “holiodesy” : usual highlights in LISA review talks.
- Key questions: To what extend is this a realistic goal, what issues do we need to address and what are the available tools ?
- Basic idea: use EMRI observations to deduce the properties of the massive object’s exterior gravitational field.

  ★ An alternative scheme: use ringdown signal following massive BHs mergers (Not discussed here, see talk by E. Berti).

- Possible deviations from the exact Kerr spacetime:

  (I) A “dirty” Kerr black hole (= Kerr BH + environment: accretion disk, galactic potential, etc). Not discussed here (see talk by E. Barausse).

  (II) An intrinsic deviation from the Kerr metric:

  ★ GR is the correct theory ⇒ object not a Kerr BH, but some other exotic body (This talk, and all past and present work, see also talk by J. Gair).

  ★ Or: something is wrong with GR itself (object could be a BH solution in a non-GR theory).

Non-Kerr metrics

- The metric exterior to any stationary and axisymmetric 'source' can be written in terms of mass and current multipole moments M_ℓ, S_ℓ (*Geroch, Hansen, 1970s*). Symbolically :

$$g_{ab} \sim \sum_{n=0}^{\infty} \frac{M_{2n}}{r^{2n+1}} \mathcal{P}_1(\theta) + \sum_{n=1}^{\infty} \frac{S_{2n-1}}{r^{2n+1}} \mathcal{P}_2(\theta)$$

- The Kerr spacetime is very special: the lowest two moments (mass $M \equiv M_0$ and spin $J \equiv S_1 = aM$) determine all higher ones:

$$M_\ell + iS_\ell = M(ia)^\ell, \quad \ell = 0, 1, 2, 3, \; \ldots$$

- "Spacetime mapping", first suggested by F. Ryan (1995):

  ★ Use EMRI GW observations to extract a number of multipole moments.

  ★ Measuring just the first three moments M, J and M_2 is sufficient for identifying a non-Kerr geometry.

- Note : Current forms of multipole-expanded metrics make use of vacuum GR field equations $\Rightarrow$ not suitable for "testing GR" !

Non-Kerr EMRIs: technical problems

- Unlike Kerr, the general axi-stationary spacetime is no longer Petrov-type $\mathcal{D}$. Many of the "miracles" of the Kerr spacetime are lost (*Carter 1968*).

- Point-particle motion:

  No "third" integral (Carter constant) available $\Rightarrow$ off the equatorial plane orbital motion complicated, typically chaotic (see talk by I. Mandel).

  ★ Example: which axisymmetric potentials allow circular orbits $(dr/dt = 0)$?

  Newtonian gravity: $V(r,\theta) = V_0(r) + V_1(\theta)/r^2$ ($V_0,\ V_1$ arbitrary)

- Wave-dynamics:

  ★ No Teukolsky-like wave equation since the perturbative equations for the Weyl ψ scalars do not decouple. Need to solve 10 PDEs for metric perturbations.

  ★ A serious weakness: agnostic about the "inner boundary" (hard/soft surface or horizon ?) since gravitating source is not specified.

  ★ No available work, consequently no rigorous waveforms or fluxes available.

- A cheap (but quite accurate) alternative is to use "kludge" waveforms (combination of "quadrupole formula" with exact geodesic motion) and Post-Newtonian fluxes.

Multipole moment expansions

- Is there an “optimal” way to expand in multipole moments ?
- Example of Kerr metric:

  ★ Expansion in $1/r$:

$$g_{rr} = 1 + \frac{2M}{r} + \frac{4M^2 - a^2 \sin^2\theta}{r^2} + \frac{8M^3 - 4Ma^2 + 2Ma^2\cos^2\theta}{r^3} + \mathcal{O}(\frac{1}{r^4})$$

  ★ Expansion around Schwarzschild metric:

$$g_{rr} = g_{rr}^{\mathrm{Schw}} - \frac{a^2\sin^2\theta}{r^2} + \frac{2Ma^2\cos^2\theta - 4Ma^2}{r^3} + \mathcal{O}(\frac{a^2}{r^4})$$

  This includes the exact contribution of the monopole moment (mass).
- The Schwarzschild portion of the metric is common for any arbitrary axisymmetric-stationary metric.
  Suggestion: re-formulate multipole moment expansion with respect to the Schwarzschild metric.

A less detailed map: the 'quasi-Kerr' metric

- Basic idea: Instead of modelling an arbitrary multipole-structured spacetime, assume a small deviation from the known Kerr metric (*Collins & Hughes 2004, KG & Babak 2005)*. In other words, this 'quasi-Kerr' metric would have:

$$M, J, \quad M_\ell = M_\ell^{\rm Kerr} + \delta M_\ell, \quad S_\ell = S_\ell^{\rm Kerr} + \delta S_\ell$$

- We only consider a deviation in the quadrupole moment:

$$\delta M_2 = -\epsilon M^3 \quad (\epsilon \ll 1), \quad \delta M_\ell = \delta S_\ell = 0 \quad \text{for} \quad \ell \geq 3$$

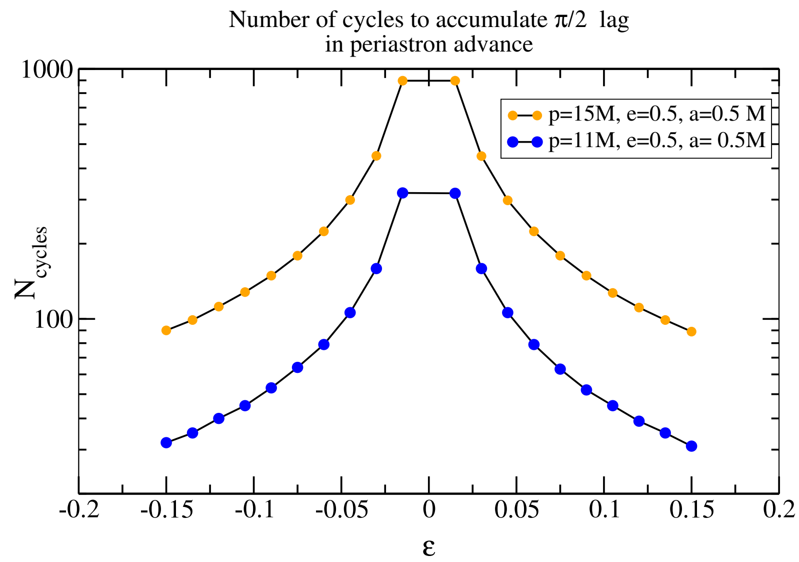
- A quasi-Kerr metric would look like:

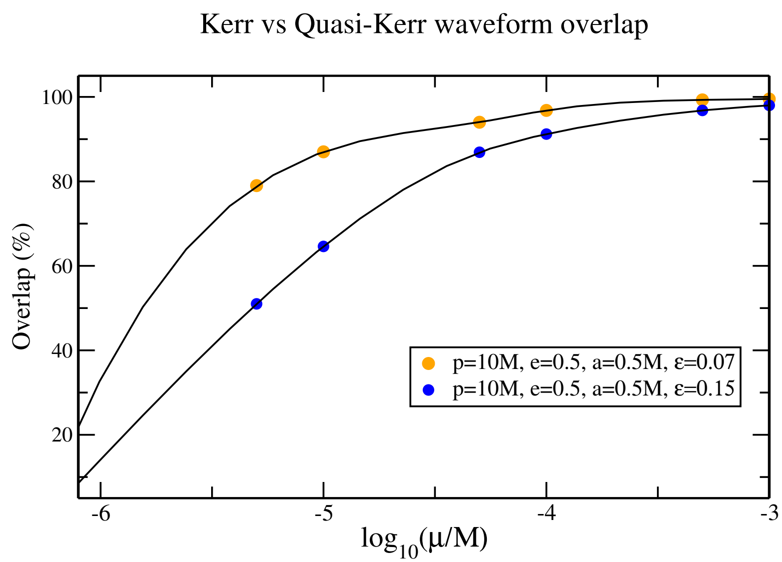
$$g_{ab} = g_{ab}^{\rm Kerr} \quad + \quad \underbrace{\epsilon\, h_{ab}(r,\theta)}_{\text{Hartle-Thorne metric}} \quad + \quad \mathcal{O}(\delta M_{\ell\geq 4}, \delta S_{\ell \geq 3})$$

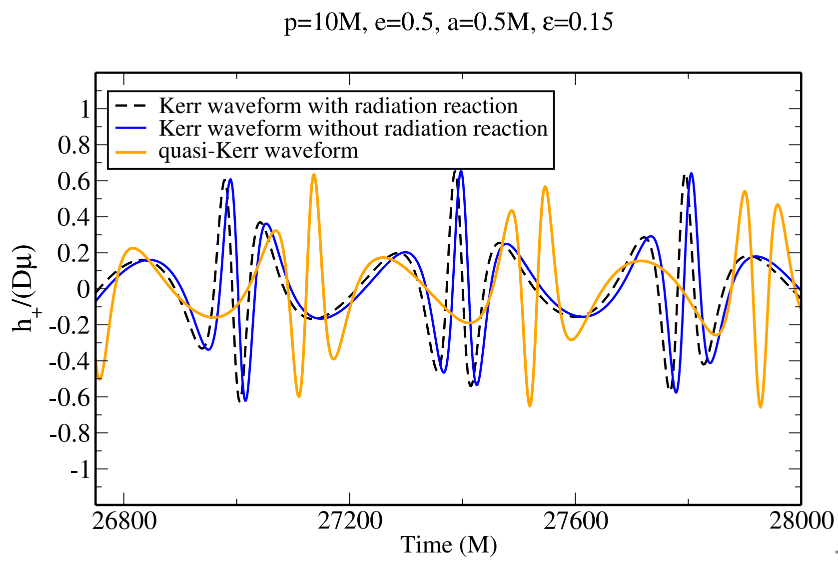
- Main advantage: not a '$1/r$' expansion, suitable for strong-field dynamics. 'Almost' type-$\mathcal{D}$ $\Rightarrow$ integrable geodesic motion as in Kerr, with $\mathcal{O}(\epsilon)$ modified orbital frequencies $\{\Omega_r, \Omega_\phi, \Omega_\theta\}$.

Kerr vs Quasi-Kerr

- **Case study:**
  Equatorial orbits in Kerr and quasi-Kerr with the same initial orbital parameters.
- For a $\sim 10\%$ fractional change in M_2 the difference in periastron shift accumulates to π after only ~ 200 orbits.
- Waveform overlap rapidly decreases in the "dephasing" timescale
  $T_{\rm deph} \sim M\sqrt{M/\mu}$







A "confusion problem" ?

- For identical orbital parameters $\{p, e\}$ and spin a/M:
  $\Rightarrow$ Overlaps $\sim 50 - 70\%$ after $t \sim T_{\rm deph}$.

- **Emerging question:** Is it possible to match a Kerr waveform to a quasi-Kerr waveform (overlap $\gtrsim 95\%$) by choosing different set of parameters $\{a/M, p, e\}$?

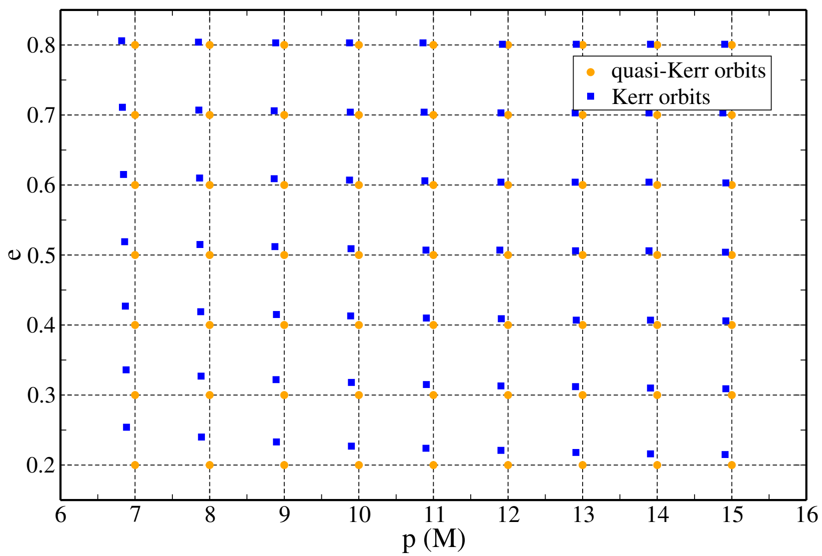
  ★ If yes, then LISA could confuse a true non-Kerr object with a Kerr BH !

- Suggestion: fix the orbital frequencies $\Omega_r, \Omega_\phi, \Omega_\theta$, which results in small shifts $\delta p, \delta e, \delta\iota \sim \mathcal{O}(\epsilon)$.

  $\Rightarrow$ overlaps $> 98\%$

  $\Rightarrow$ waveform confusion !

  ★ Radiation reaction has been ignored here.



Not (so much) confusion with radiation reaction.

- Inclusion of radiation reaction could ameliorate the confusion problem.
- Example: circular equatorial inspiral.

$$\Phi(t) = \int_0^t dt' \Omega(t'), \qquad \Omega(t) = \Omega_0 + t \left[\frac{d\Omega}{dE} \dot{E} \right] + \ldots$$

$$\dot{E} = \dot{E}_{\rm Kerr} - \frac{64}{5} \left(\frac{\mu}{M} \right)^2 \epsilon \, v^{14}$$

- The phase-shift between Kerr and non-Kerr inspirals,

$$\Delta\Phi = \Phi_{\rm nonKerr} - \Phi_{\rm Kerr} \approx -\frac{5}{2} t^2 \epsilon v^4 \left(\frac{d\Omega}{dE} \right)_0 \dot{E}_{\rm Kerr}$$

- Define a “deconfusion” timescale $T_{\rm deconf}$: time required to accumulate $|\Delta\Phi| = 2\pi$.

$$T_{\rm deconf} \approx \left(\frac{\pi}{24} \right)^{1/2} v^{-7} \epsilon^{-1/2} M \left(\frac{M}{\mu} \right)^{1/2} \Rightarrow$$

$$T_{\rm deph} \ll T_{\rm deconf} \ll T_{\rm RR}$$

Concluding remarks

- A decade after Ryan's work: Non-Kerr EMRI problem still in its initial stage.
- Multipole expansion scheme: attractive, but plagued with technical problems (complicated geodesic motion, no Teukolsky formalism, wave dynamics via metric perturbations).
- Current non-Kerr metrics not suitable for "testing GR", instead could be suitable for identifying massive objects, alternative to Kerr BHs.
- Our quasi-Kerr metric: a practical tool, admits simple geodesic motion, but computing accurate waveforms could still be a difficult task.
- Small deviations from the Kerr metric leave clear imprints in waveform phasing.
- LISA data analysts: Beware of possible confusion between Kerr and non-Kerr waveforms.
- Set upper limits on possible environmental influence before attempting to "map the Kerr geometry".