Testing General Relativity using Observations of EMRIs

Jonathan Gair, IoA Cambridge

LISA Astro-GR, September 19th 2006

Talk Outline

- Brief description of extreme mass ratio inspiral waveforms.
- “Testing relativity” using EMRIs.
- Ryan’s theorem on spacetime mapping.
- The possible imprints of excess multipole moments on orbital dynamics and gravitational waveforms.
- How might we detect black hole ‘hair’ in LISA data?
- How can we interpret the observations astrophysically?

Acknowledgments

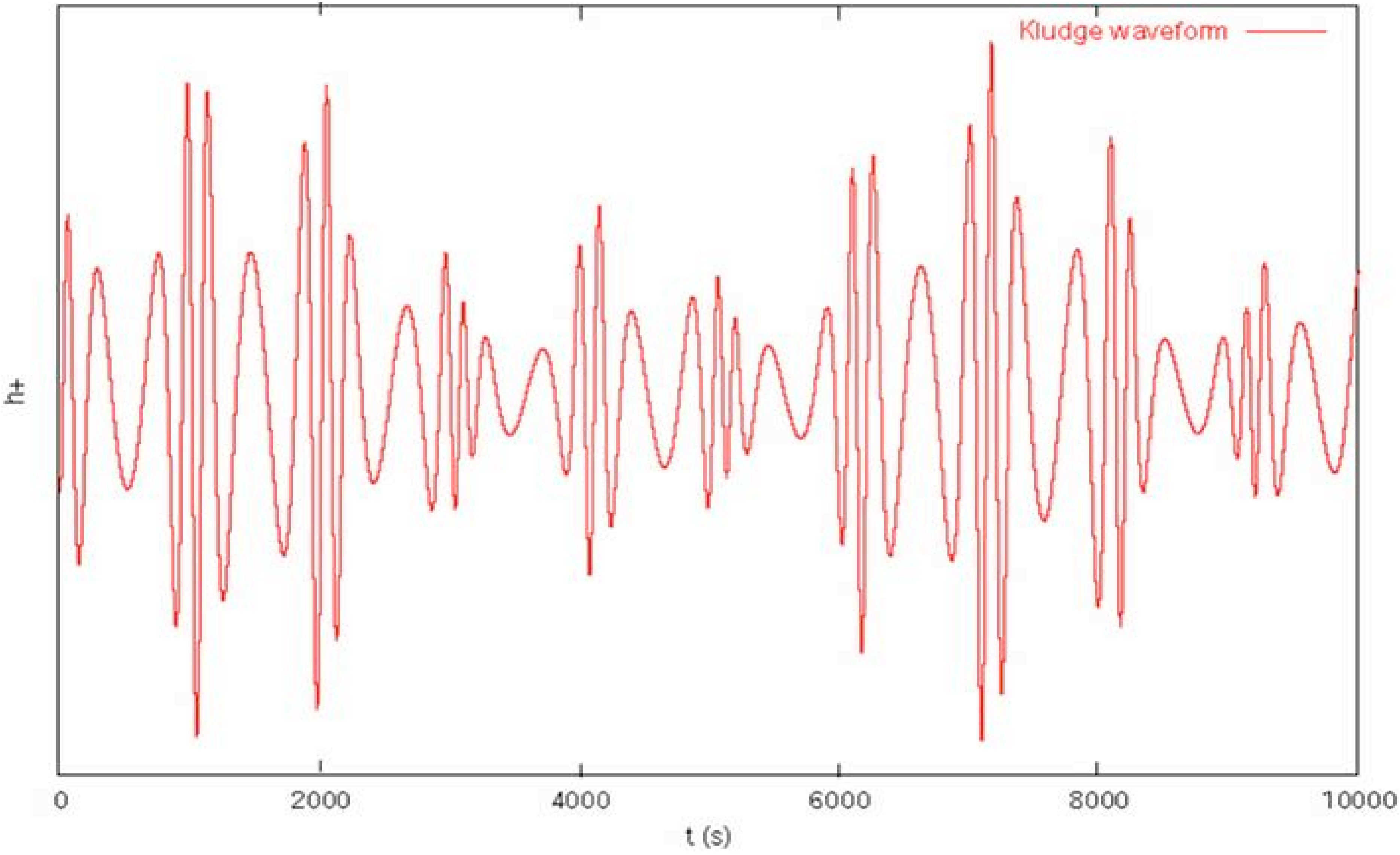
Many people are involved in this project (speakers in black):

Caltech:	Jeandrew Brink	Duncan Brown	Hua Fang
	Geoffrey Lovelace	Chao Li	Ilya Mandel
	Yi Pan	Kip Thorne	
AEI:	Stanislav Babak	Yanbei Chen	
So'ton:	Leor Barack	Kostas Glampedakis	
JPL:	Curt Cutler	Steve Drasco	
Others:	Enrico Barausse	Scott Hughes	

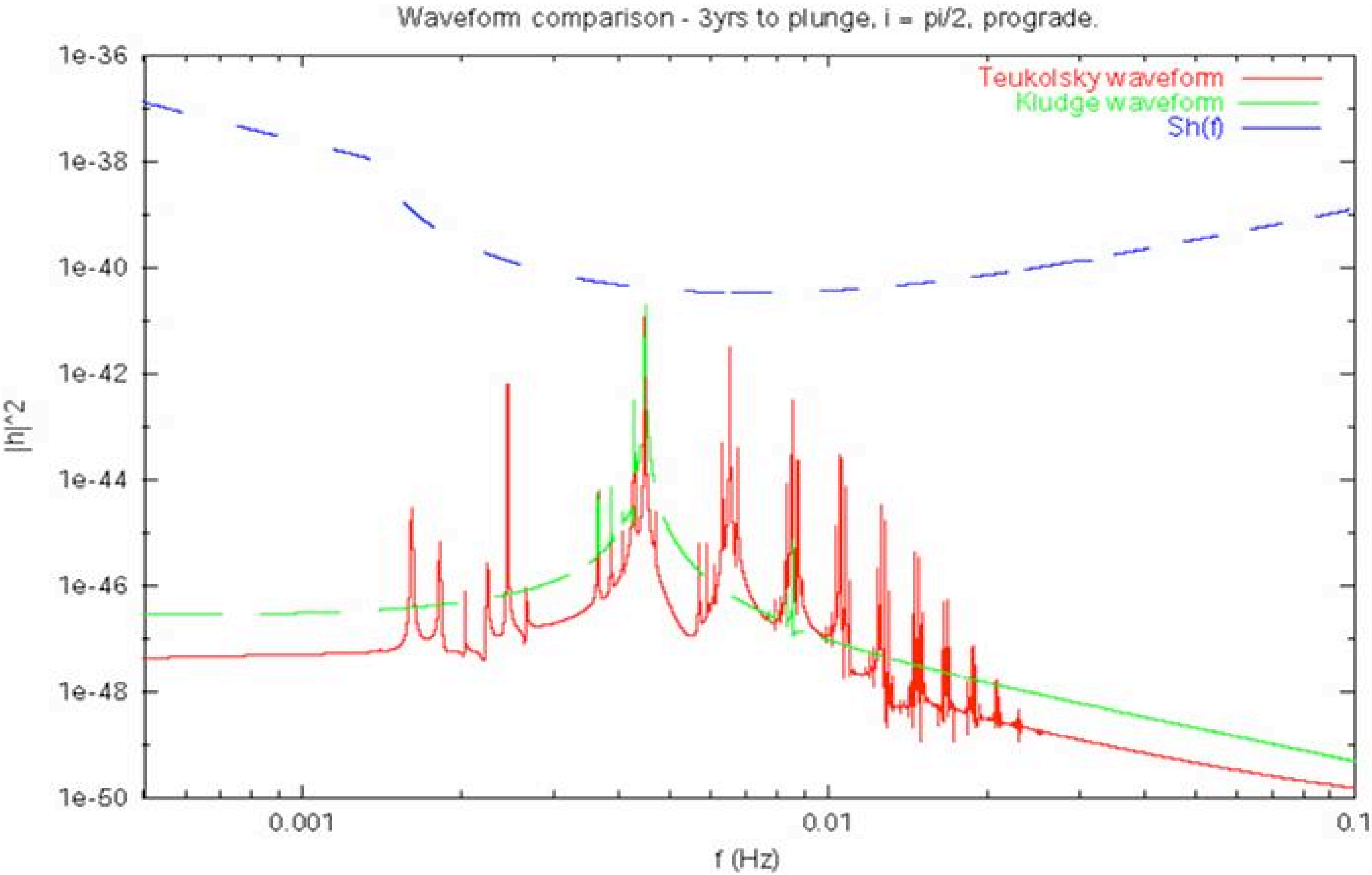
Introduction

- The extreme mass ratio ensures the inspiralling body acts like a test particle in the background spacetime of the central object.
- EMRI orbits are typically eccentric, inclined with respect to the spin axis of the central black hole and inspiral significantly over an observation – the inspiralling body extensively explores the black hole spacetime. Waveform traces the underlying orbit.
- A typical observation includes $\sim 10^5$ waveform cycles $\rightarrow$ lots of information for probing the spacetime.
- Geodesics in Kerr possess a complete set of integrals – energy, angular momentum and "Carter" constant (Carter 1968).
- The motion is therefore triperiodic – waveforms contain harmonics of the orbital frequency, the perihelion precession frequency and the spin induced orbital plane precession frequency.
- Complex "zoom-whirl" waveforms are strongly coloured by these effects.

Example Waveform



Example Waveform



Spacetime mapping

- **If the EMRI is generated by an inspiral into a Kerr black hole, these features in the waveforms provide measurements of the system parameters with unprecedented accuracy.**
- **However, if the GW was generated by an inspiral in a spacetime not described by the Kerr metric, the EMRI waveforms will in principle tell us this, since they encode a map of the spacetime structure near the black hole.**
- **Compare observed and theoretical waveforms to test whether central object is a Kerr black hole.**
- **Analogy with geodesy led to the term 'bothrodesy' from the greek βοθρος meaning 'sacrificial pit' (ancient Greek).**
- **Or 'cesspool' (modern Greek)! We prefer 'holiodesy'….**

“Testing General Relativity”

- **In principle, the EMRI observation gives us the Riemannian metric of the spacetime, $g_{\mu\nu}$. Any metric is consistent with Einstein’s equations, defining an energy-momentum tensor:**

$$T_{\mu\nu} = \frac{c^4}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)$$

- **Spacetime mapping is not really testing GR, but is testing aspects of relativity theory relating to massive compact objects:**
  - Test the energy conditions.
  - Test the black-hole “No-hair” Theorem and the Cosmic Censorship Hypothesis.
  - Explore the matter content in the vicinity of massive compact objects in the Universe.
  - Perhaps test GR, e.g., that observations are consistent with a Riemannian metric. Test GR against specific alternative theories.

Ryan's Theorem

- **Ryan (1995) demonstrated that, for nearly circular and equatorial orbits in an arbitrary axisymmetric spacetime, the multipole moments of the system are encoded in GW observables**
  - One can expand the energy spectrum, $\Delta E/\mu$, and precession rates, Ω_p/Ω and Ω_z/Ω, as functions of the orbital period, Ω.
  - The different spacetime multipole moments enter at different orders in this expansion and can therefore be read off.
- **For the Kerr metric, all multipole moments are determined by the monopole and quadrupole moments ("No-hair" theorem):**

$$M_l + \mathrm{i}S_l = M\left(\mathrm{i}a\right)^l$$

- **If measured multipole moments are inconsistent with the above, the system must deviate from the Kerr metric. Need 3 moments to rule out a Kerr BH, 4 to rule out a spinning boson star.**

Ryan's Theorem

- **Ryan considered only nearly circular, nearly equatorial orbits in spacetimes without a horizon**
  - Generalise to arbitrary orbits (even more information in principle).
  - Include tidal coupling (energy "lost" into the horizon).
  - See talk by Chao Li.
- **Ryan's theorem tells us that the GWs encode the multipoles, but not how to extract that information. Key observable is the number of cycles the fundamental frequency spends near f.**

$$\Delta\mathcal{N}(f) = \frac{f^2}{\mathrm{d}f/\mathrm{d}t} = f^2 \frac{\mathrm{d}E/\mathrm{d}f}{\mathrm{d}E/\mathrm{d}t}$$

- **To compute dE/dt, need to understand wave generation in an arbitrary spacetime. Non trivial!**
- **Multipole decomposition is an inconvenient way to characterize spacetimes, since need an infinite number to describe Kerr.**

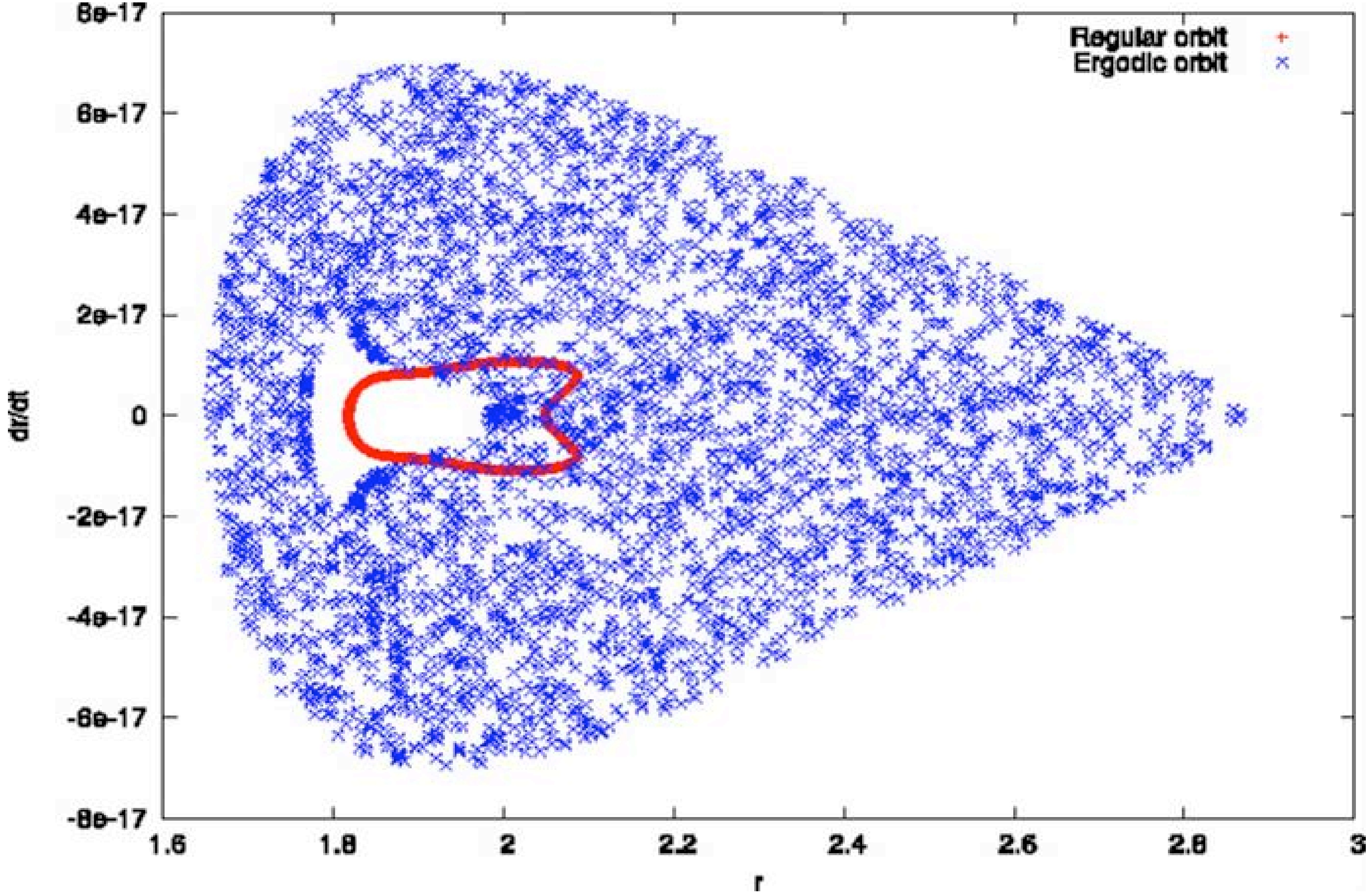
“Bumpy Black Holes”

- **Alternative to a multipole expansion is to consider spacetimes that are Kerr plus a small perturbation. Formulate an EMRI observation as a test of the “null hypothesis” that the spacetime is Kerr (Collins & Hughes 2004).**
- **Construct “bumpy” spacetimes in various ways**
  - Perturb Schwarzschild (Collins & Hughes).
  - Use exact solutions, e.g., Manko-Novikov spacetimes (see talk by Ilya Mandel).
  - Use “quasi-Kerr” solutions - perturb Kerr using the Hartle-Thorne metric (see talks by Kostas Glampedakis and Enrico Barausse). Either generic perturbations or astrophysically motivated ones, e.g., an accretion disc.
- **Investigate properties of geodesics and inspirals in these bumpy spacetimes to find possible observable effects.**
- **Have the same problem when it comes to computing accurate gravitational waveforms - use “kludges” to make progress.**

Orbital dynamics in perturbed spacetimes

- **It is remarkable that the equations separate for geodesic motion in the Kerr spacetime. Might not expect this to generalise.**
- **However, we find that most orbits in most “bumpy” spacetimes possess an apparent third integral and are tri-periodic to high accuracy. In principle, this allows application of Ryan’s theorem.**
- **In certain cases, the third integral can be lost. In general, this occurs in astrophysically uninteresting regions of parameter space, but if observed it is a ‘smoking gun’**
    - See loss of third integral in Poincaré map of orbits.

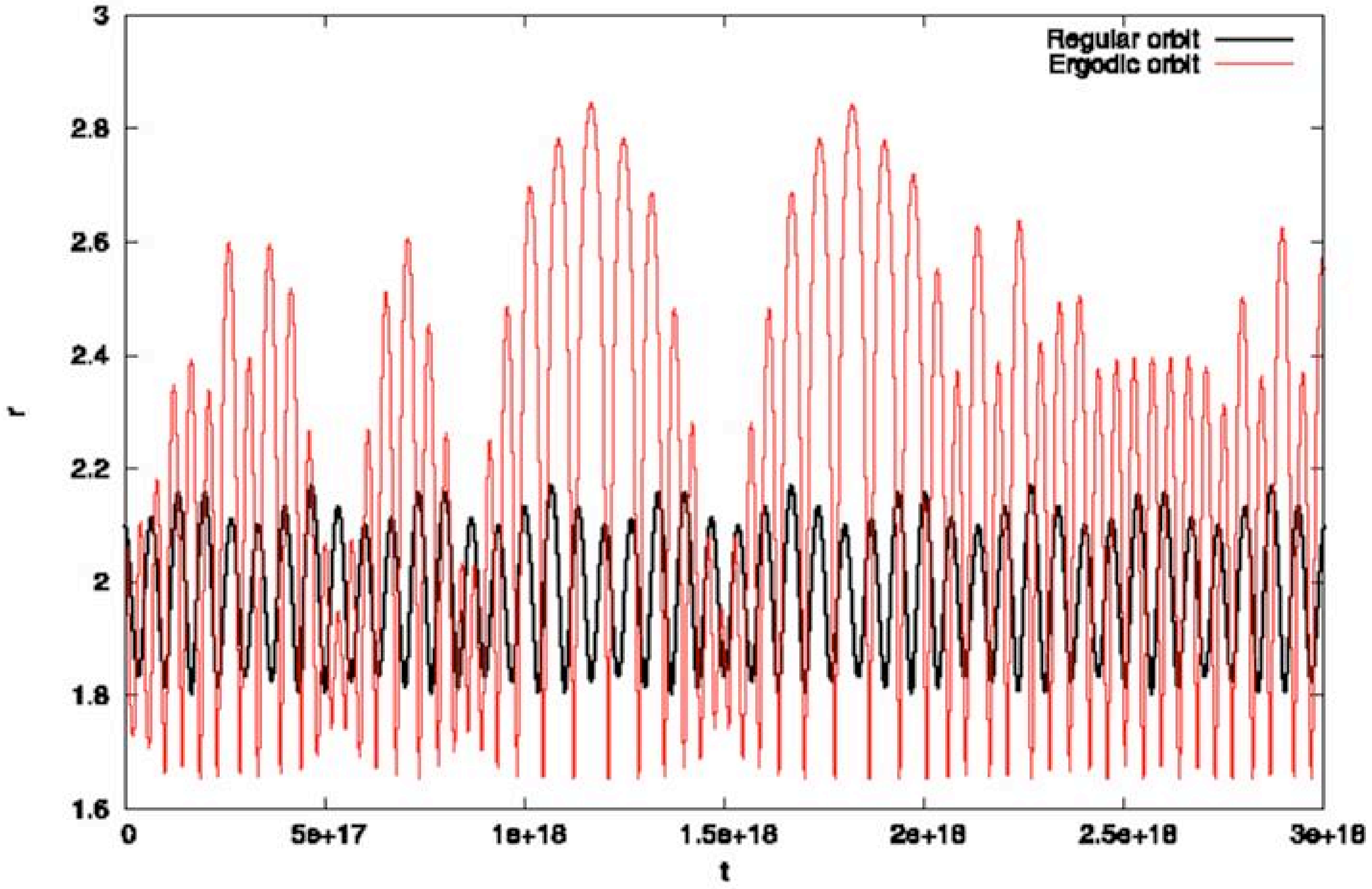
Orbital dynamics in perturbed spacetimes



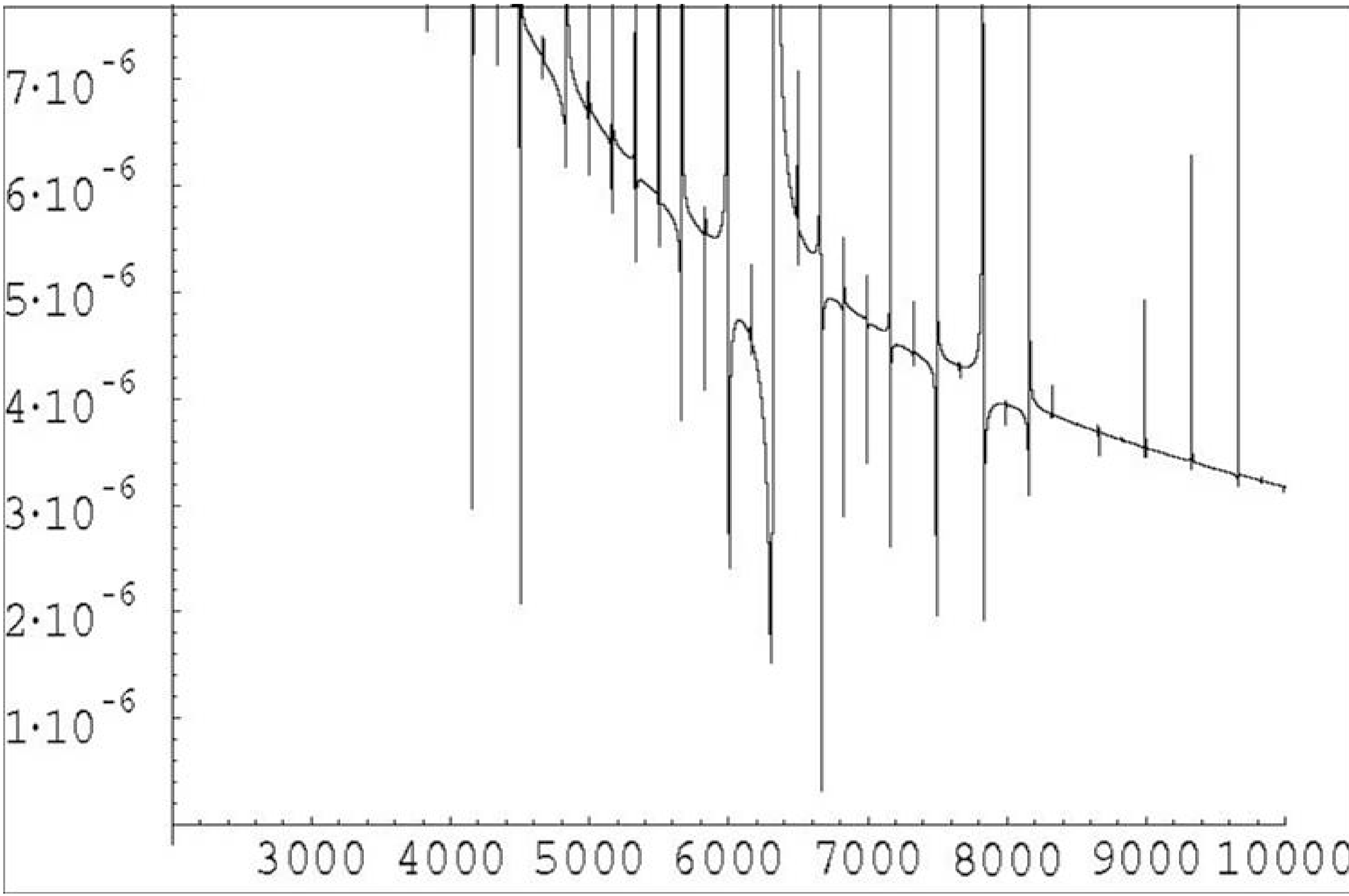
Orbital dynamics in perturbed spacetimes

- **It is remarkable that the equations separate for geodesic motion in the Kerr spacetime. Might not expect this to generalise.**
- **However, we find that most orbits in most “bumpy” spacetimes possess an apparent third integral and are tri-periodic to high accuracy. In principle, this allows application of Ryan’s theorem.**
- **In certain cases, the third integral can be lost. In general, this occurs in astrophysically uninteresting regions of parameter space, but if observed it is a ‘smoking gun’**
  - See loss of third integral in Poincaré map of orbits
  - Difference in Fourier transform of regular versus ergodic orbit is very striking.

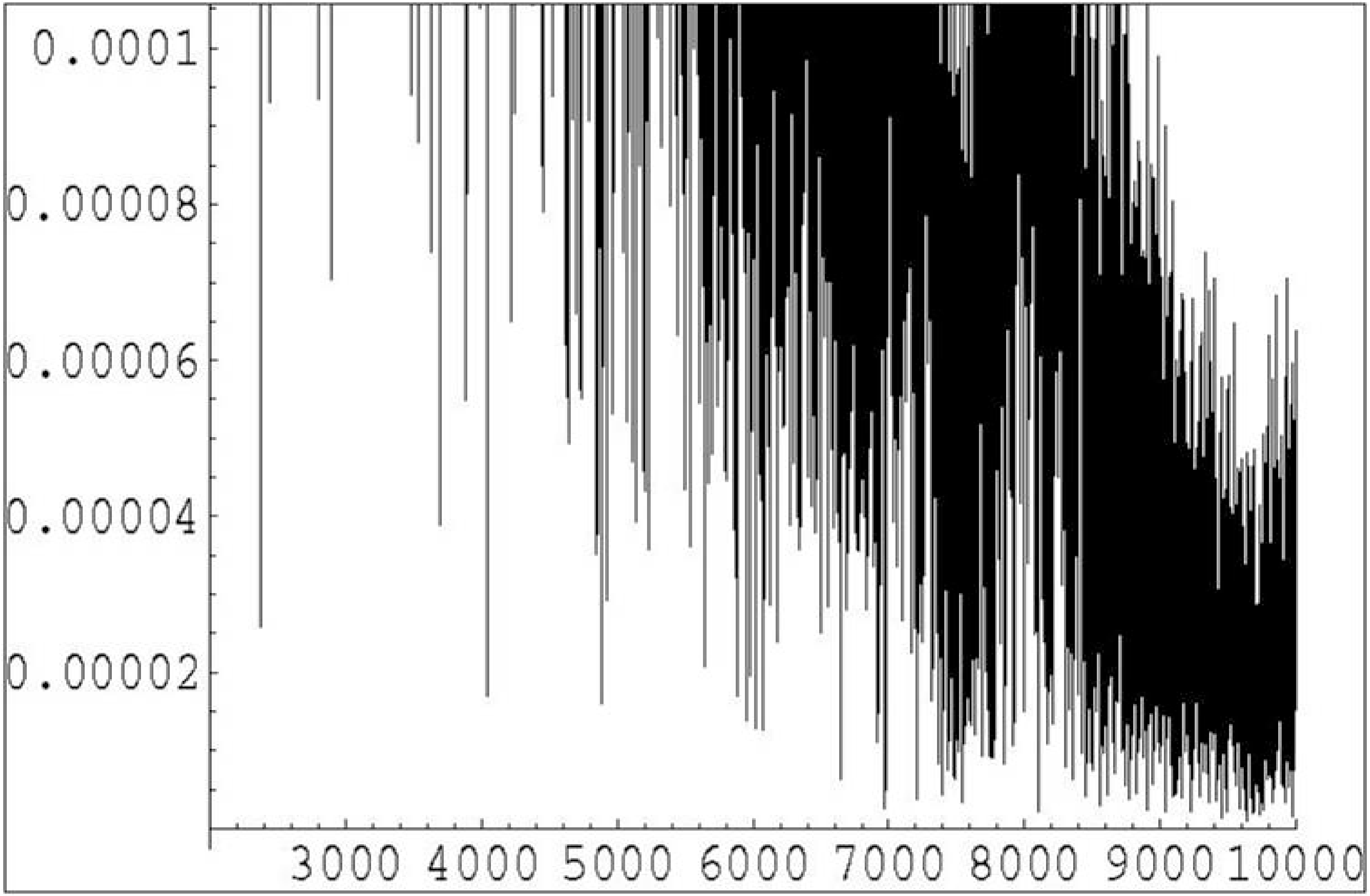
Orbital dynamics in perturbed spacetimes



Orbital dynamics in perturbed spacetimes



Orbital dynamics in perturbed spacetimes



Orbital dynamics in perturbed spacetimes

- **It is remarkable that the equations separate for geodesic motion in the Kerr spacetime. Might not expect this to generalise.**
- **However, we find that most orbits in most "bumpy" spacetimes possess an apparent third integral and are tri-periodic to high accuracy. In principle, this allows application of Ryan's theorem.**
- **In certain cases, the third integral can be lost. In general, this occurs in astrophysically uninteresting regions of parameter space, but if observed it is a 'smoking gun'**
    - See loss of third integral in Poincaré map of orbits
    - Difference in Fourier transform of regular versus ergodic orbit is very striking.
    - If timescale over which orbit wanders due to absence of a third integral is shorter than radiation reaction timescale, the effect will be observable in the GW emission.

Imprint of an excess quadrupole moment

- **For orbits that are tri-periodic, the quadrupole moment is revealed by the change in the three frequencies compared to their Kerr values.**

- **Compare precession frequencies for "same orbit". Naively, can detect quadrupole moment in time $t \sim 1/\delta\Omega(Q)$. But, have no way to identify the orbit *a priori*. Can't disentangle the moment-induced shifts from errors in the other orbital parameters.**

- **Inspiral breaks the degeneracies in time $t \sim 1/\sqrt{\delta\dot{\Omega}(Q)}$. Compute parameter errors using Fisher Matrix formalism to allow for correlations**

$$\Gamma_{ij} = \left(\frac{\partial \mathbf{h}}{\partial \lambda_i} \middle| \frac{\partial \mathbf{h}}{\partial \lambda_j} \right)$$

- **Compute estimation error for quadrupole moment δQ . Evaluate for Kerr orbit ($Q = 0$) to quantify accuracy of Kerr "null-hypothesis" test.**

Imprint of an excess quadrupole moment

- **Difficulty arises in computing accurate inspiral waveforms, since this requires modelling the wave emission.**

- **Make progress using post-Newtonian and "kludge" models**

  - Using pN models, Ryan estimated an estimation accuracy of $|\delta M_2| = 0.015$ compared to the Kerr value $|M_2| = 0.64$.

  - Ryan further estimated that as many as 7 spacetime multipoles could be extracted in an observation - distinguish a BH, boson star or something else.

  - Ryan's model (circular, equatorial, stationary phase) was extremely simple. Using more sophisticated pN models, find that the results are even better, e.g., Barack and Cutler find $\delta Q/Q \sim 10^{-4}$ in a one year observation.

  - Kludge models combine geodesic dynamics in perturbed spacetimes with flat space GW emission formulae. Have promising results for geodesic orbits. Inspiral is more challenging, but we can make progress using Kerr expressions. Evolution of the apparent third integral for generic orbits is very difficult.

- **See talks by Barausse, Glampedakis and Mandel.**

Probing the horizon

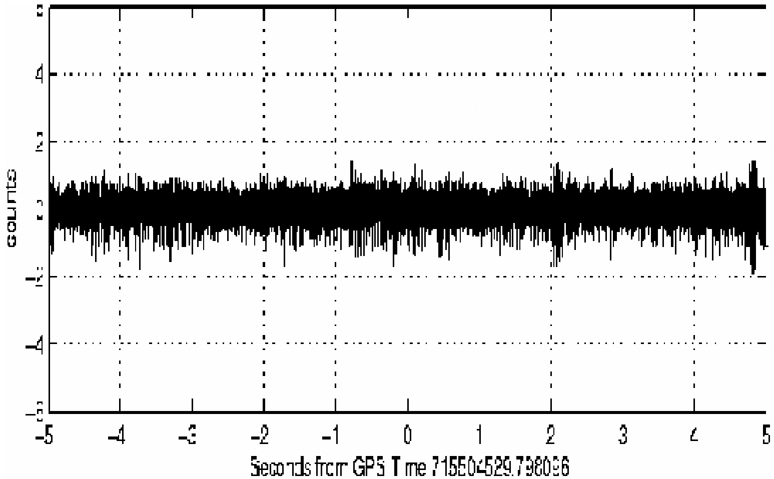
- Plunge frequency indicates the existence and location of a horizon in the spacetime.
- The horizon also interacts dynamically with the inspiralling body - the gravitational field of the small object raises a tide on the horizon that then influences the motion of the small body.
- Can model this tidal interaction as energy “going down the horizon”. Consider a phenomenological model of this process

$$\dot{E}^{(\epsilon)}_{GW} = \dot{E}_{GW}\left(1+\epsilon\frac{\dot{E}_H}{\dot{E}_{GW}}\right)$$

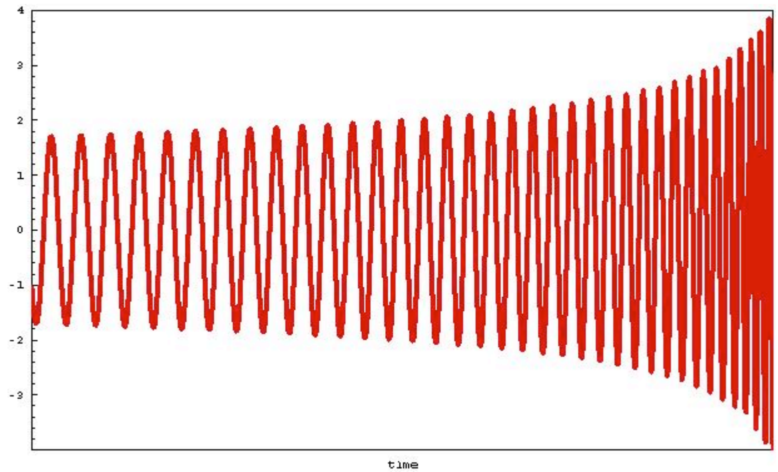
- The limit $\epsilon = 0$ corresponds to Kerr. The limit $\epsilon = 1$ represents a spacetime in which the rate of change of orbital energy is the same as Kerr, but all of that energy is radiated to infinity.
- In principle, a typical LISA observation can distinguish $\epsilon = 0$ from $\epsilon = 1$.

EMRI detection

- **EMRI events observed by LISA will typically be very faint.**
- **Basic underlying technique for detection is matched filtering using a bank of templates.**
- **Overlap of template with data pulls signal out of the noise.**
- **Difficult to detect GWs that differ from templates in our bank.**
- **Further complications in LISA from confusion with other EMRIs and other LISA sources.**
- **Need huge number of templates.**

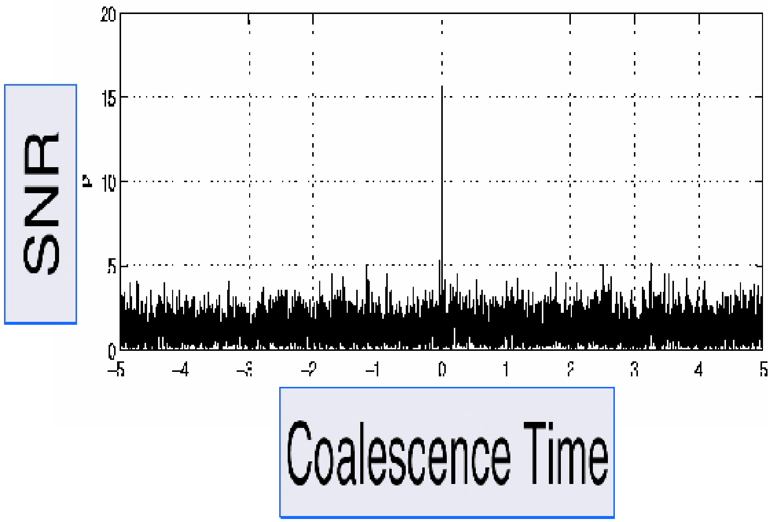


+



EMRI detection

- EMRI events observed by LISA will typically be very faint.
- Basic underlying technique for detection is matched filtering using a bank of templates.
- Overlap of template with data pulls signal out of the noise.
- Difficult to detect GWs that differ from templates in our bank.
- Further complications in LISA from confusion with other EMRIs and other LISA sources.
- Need huge number of templates.



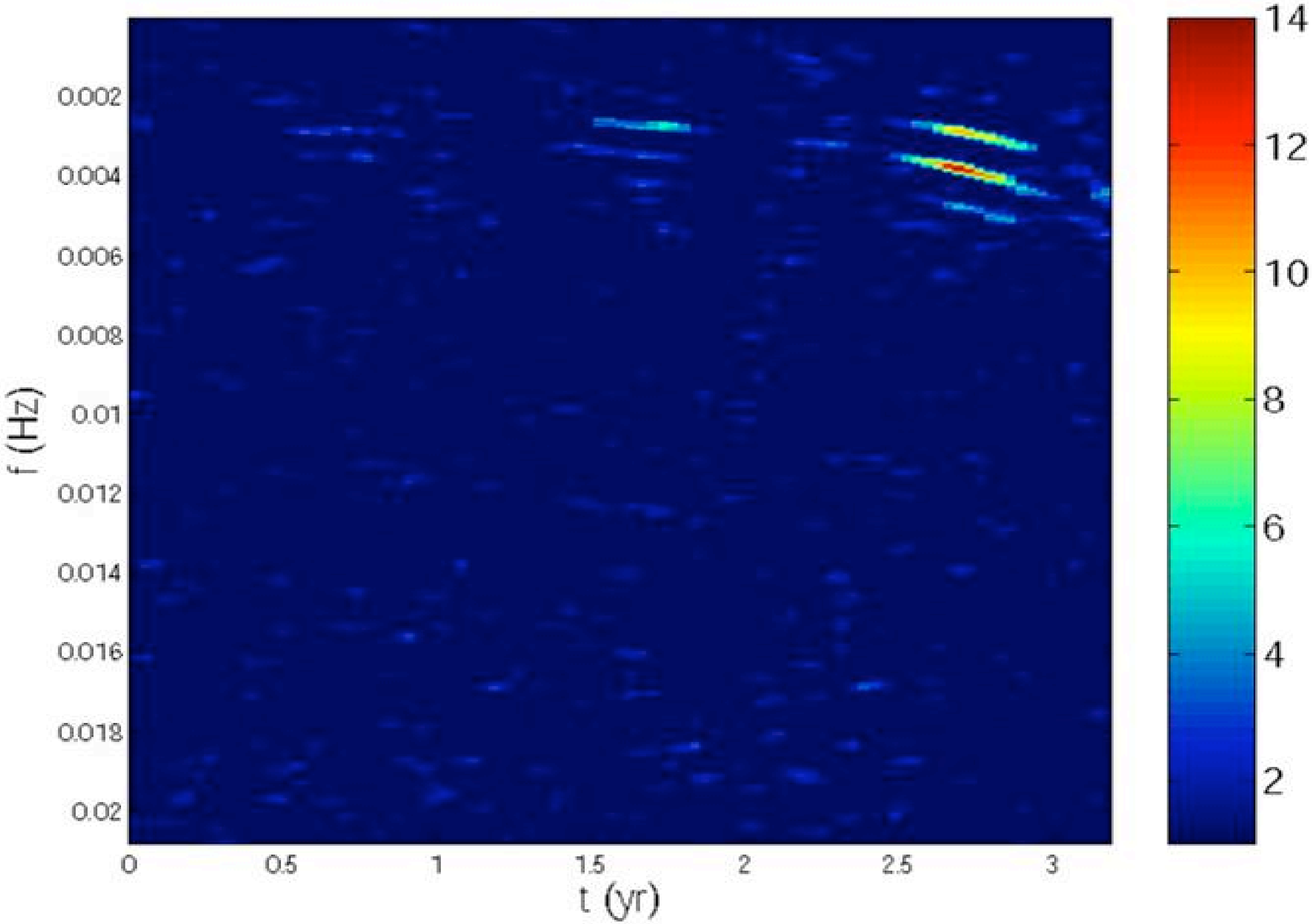
Detection of "quasi-Kerr" EMRIs

- **How do we identify GWs from non-Kerr systems? Difficult to detect deviations when using a template based search.**
    - Use only Kerr templates and regard observations as hypothesis tests. Need to be specific about what alternatives are being constrained. Difficult to detect something completely unexpected (and therefore very interesting!).
    - Use "kludge" or phenomenological templates designed to, e.g., extract spacetime multipole moments. Increases computational cost and makes interpretation complicated. Less efficient for detecting Kerr inspirals.
    - Hope to detect inspiral during a stage when it is close to a Kerr inspiral. If inspiral subsequently deviates, or the source vanishes or persists after it should have plunged (i.e., the horizon is in the wrong place), identify the system as non-Kerr.

Detection of "quasi-Kerr" EMRIs

- **Use non-template based methods. A time-frequency search could detect Kerr EMRIs out to 2-3Gpc under ideal assumptions.**
  - If a clear track is seen, can extract the frequency components, their rate of change and the power variation along the track. These track properties will evolve in certain ways for Kerr EMRIs.
  - By extracting the evolution of the frequency harmonics and comparing to Kerr models, we can use this for "spacetime mapping".
  - Could identify plunge time and frequency in this way, and see transition from regular to ergodic motion if this did occur.
  - Procedure is made much more difficult by source confusion. Do not have a clean set of tracks from each event, but they overlap in time and frequency. Difficult to identify tracks from the same event.

Detection of "quasi-Kerr" EMRIs



Interpreting the observations

- **If an excess multipole moment is detected, there could be several reasons**
  - Astrophysical "hair". The presence of other material, e.g., an accretion disc, could change the multipole structure of the system.
  - The central object could be an exotic object consistent with GR rather than a black hole, e.g., a supermassive boson star or a naked singularity (violating the Cosmic Censorship Hypothesis).
  - The central object could have hair consistent with GR, e.g., due to the existence of extra dimensions.
  - General Relativity could be the wrong theory of gravity.
- **Must distinguish violations of relativity theory from astrophysical effects, e.g., the difference between an "internal" and "external" quadrupole moment might be revealed by how the inspiral progresses if the object passes inside the perturbing material.**

Interpreting the observations

- **The ϵ model is a good straw man to probe tidal coupling, but we would prefer to extract more details of the interaction. Decoding this and other information requires an understanding of waveform generation in arbitrary spacetimes.**
- **Electromagnetic counterparts would be a useful additional diagnostic. Unlikely in practice, unless we are very lucky.**
- **Our ability to perform the "null hypothesis test" depends on how well we know EMRI gravitational waveforms in Kerr. Must be able to distinguish black hole hair from failures in the Kerr waveform model, e.g., second order effects.**
- **Can use the set of observed EMRI events to make statements**
  - A single EMRI event with an excess multipole moment might indicate an astrophysical effect, an observational error or a rare exotic object.
  - If many events are peculiar, it would suggest a significant failure in our understanding of massive compact objects.

Some outstanding questions

- **What is the optimal detection strategy? Do we only look for Kerr EMRIs and test for consistency or do we do more?**
- **Are there clear signatures for deviations from Kerr?**
- **At what level can LISA "test GR", i.e., test the null hypothesis that an EMRI is a Kerr EMRI? How generic a statement can be made, i.e., are we only ruling out certain models?**
- **Can LISA distinguish between deviations arising from the environment of the black hole, versus those that arise from a departure from relativity theory?**
- **How does adding potential excess multipole moments affect parameter estimation of other "standard" parameters?**
- **How much does our ability to make statements degrade when we make our model for LISA more realistic, i.e., including source confusion?**

Summary

- The gravitational waves from EMRI events encode an exquisite map of the spacetime in which they are generated, including the multipole structure and horizon properties.
- Decoding this map is difficult in theory, and decoding it from realistic LISA data is even more difficult in practice.
- Using template based methods or a time-frequency analysis, we might be able to extract information about the spacetime structure and see the signatures of a deviation from Kerr.
- We should certainly be able to make statements about the consistency of the observations with inspirals into Kerr.
- If we can decode the spacetime map, EMRI observations will provide us with a unique probe of the strong field region of black hole spacetimes.
- See talks by Barausse, Glampedakis, Li and Mandel for more details.