Extreme Mass Ratio Inspirals in non-Kerr spacetimes

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EMRIs in Kerr: a short summary

- No matter, Petrov type V spacetime

➔ Geodesic equations separable

➔ Efficient way to compute metric perturbations (Teukolsky formalism)

- Hard to compute inspiral phase:

➔ Solve $\frac{Du^\mu}{d\tau} = -\frac{1}{2}(g^{\mu\nu} + u^\mu u^\nu)(2\nabla_\rho h_{\nu\lambda} - \nabla_\nu h_{\lambda\rho})u^\lambda u^\rho$: metric perturbations diverging at the particle $\Longrightarrow$ regularization needed $\Longrightarrow$ self-force approach: no inspirals computed so far

➔ Compute $\dot{E}, \dot{L}_z, \dot{Q}$: particle follows a geodesic with evolving parameters.
Less general than self-force approach, problems with $\dot{Q}$
First waveforms in 2005

Going beyond Kerr: why?

- Exotic alternatives to Kerr SMBHs: gravastars, boson stars, fermion balls
- Even if the SMBH is actually a BH, there's a lot of matter in galactic centers: accretion disks in AGNs, stellar disks, dark matter density profile is peaked
- Little is known about the strong field region close to the horizon: can GWs from EMRIs be a tool to detect deviations from Kerr?

Must be prepared for surprises!

Our approach

- Numerical solutions of BH+Torus spacetimes now available (Ansorg & Petroff, Phys. Rev. D 72 024019)

- Tori are heavy, compact and close to the horizon: unrealistic but useful as a strawman to maximize impact of “astrophysical bumpiness” on EMRIs

- Approach follows in spirit Glampedakis and Babak's (CQG 23, 4167): compared kludge waveforms produced in Kerr and Kerr+quadrupole spacetimes; found that waveforms containing the same frequencies are indistinguishable by LISA (confusion problem)

What we do: a summary

Numerical integration of geodesics
in BH+Torus spacetimes:

latus rectum — eccentricity — inclination

$$p = \frac{2\, r_{max}\, r_{min}}{r_{max} + r_{min}} \quad e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} \quad z_{-} = [\cos^2(\theta)]_{max}$$

Compute kludge waveforms using geodesic motion
(with cutoff in time to avoid radiation reaction effects)
in quasi-BL coordinates

Address the confusion problem: compare BH+Torus
waveforms to the “corresponding” waveforms
produced in a Kerr spacetime

What are kludge waveforms?

Folk etymology: klumsy, lame, ugly, dumb, but good enough

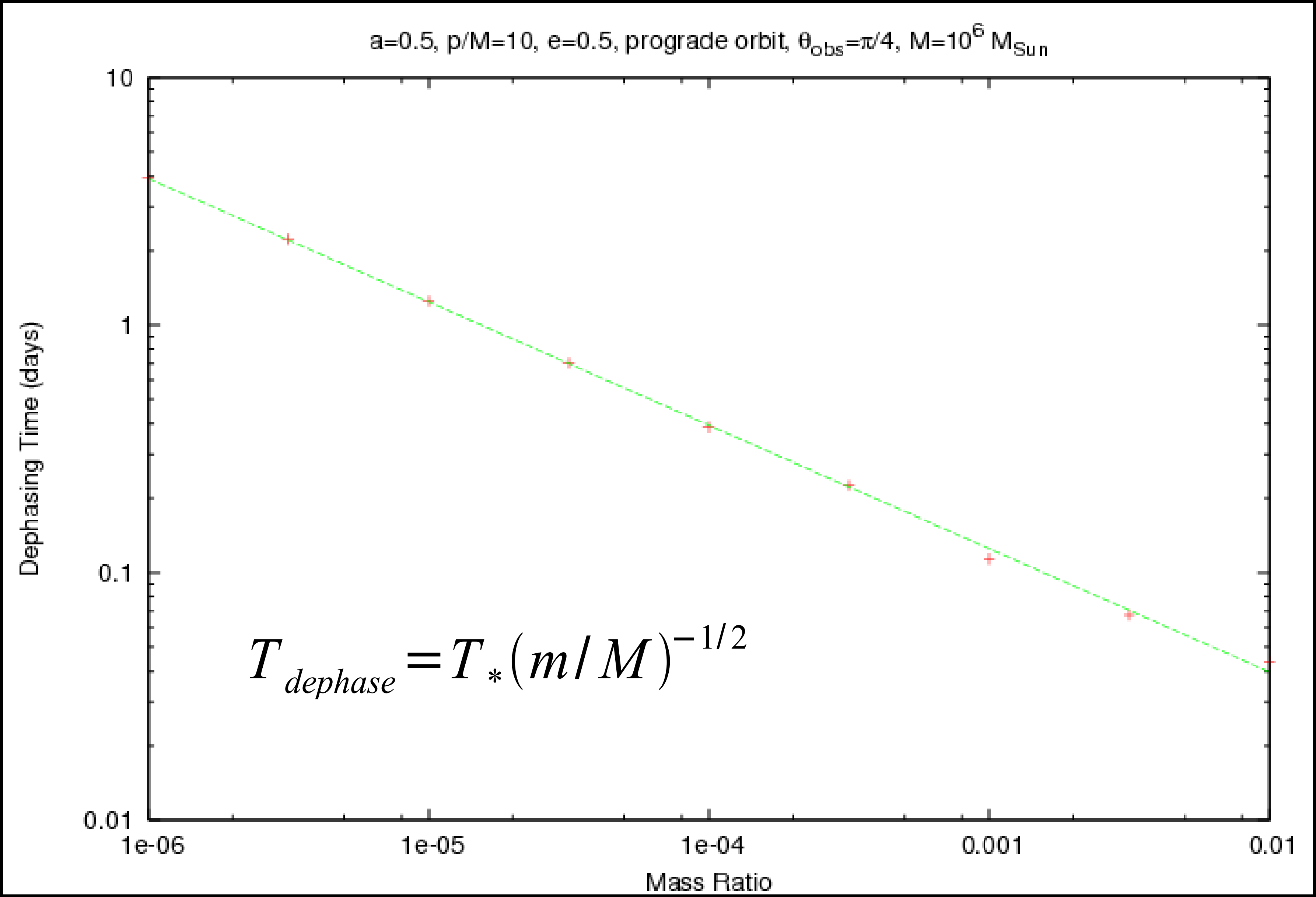
- Identify BL or quasi BL-coordinates with spherical coordinates
- Use $\partial_\nu T^{\mu\nu}_{particle} = 0$ (incorrect in curved spacetimes) + Green's function of $\Box$ in Minkowski spacetime (incorrect too)
- Surprisingly good agreement with Teukolsky-based waveforms in Kerr
- Natural preliminary approach in quasi-Kerr spacetimes, where Teukolsky formalism is not applicable

How long can we trust our waveforms?

Consider an orbit with initial parameters *p*, *e* and *z*_
Compute waveforms with (h_{RR})
and without ($h_{no\,RR}$) radiation reaction:
radiation reaction becomes important when
$overlap(h_{no\,RR}, h_{RR})$ drops below 0.95 (dephasing time)

- In pure Kerr we can use "kludge" fluxes for E, L_z and Q (based on post-newtonian expansions and fits to Teukolsky data)
- In BH+Torus we use the dephasing time of the orbit in Kerr having the same *p*, *e* and *z*_ in quasi-Boyer-Lindquist coordinates

How long is the dephasing time?



Recalling the signal to noise ratio...

$$s(t) = h(t) + n(t)$$

signal → $s(t)$; waveform → $h(t)$; gaussian noise → $n(t)$

$$\frac{S}{N}[\hat{h}] = \frac{\int \hat{h}(t)\, w(t-\tau)\, s(\tau)\, d\tau dt}{\text{rms} \int \hat{h}(t)\, w(t-\tau)\, n(\tau)\, d\tau dt} = \frac{(\hat{h}, s)}{(\hat{h}, \hat{h})^{1/2}}$$

template → $\hat{h}$

$$\tilde{w}(f) = 1/S_n(f)$$

detector's sensitivity → $S_n(f)$

$$(h_1, h_2) \equiv 2 \int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$$

The overlap function

Template matching the waveform ($\hat{h} = h$):

$$S/N = (h, h)^{1/2} \equiv A$$

Template not matching the waveform:

$$S/N = A \cdot overlap(h, \hat{h})$$

$$overlap(h_1, h_2) \equiv (h_1, h_2)/[(h_1, h_2)^{1/2}(h_1, h_2)^{1/2}]$$

Overlap between BH+Torus
and the “closest” Kerr waveform
expresses how much the S/N ratio is degraded
if we match-filter a BH+Torus signal with Kerr templates

Comparing Kerr and BH+Torus

- Compare waveforms having the same frequencies:

➔ Get the frequencies for the BH+Torus orbit ("approximate" frequencies if non-equatorial)

➔ Get the Kerr orbit having the same frequencies by solving the system

$$\Omega_r^{Kerr}(p, e, z_-, a, M) = \Omega_r^{BH+Torus}$$
$$\Omega_\varphi^{Kerr}(p, e, z_-, a, M) = \Omega_\varphi^{BH+Torus}$$
$$\Omega_\theta^{Kerr}(p, e, z_-, a, M) = \Omega_\theta^{BH+Torus}$$

in 3 variables chosen among *p, e, z_, a, M*
(variables not solved for are fixed to their BH+Torus values, e. g. $p = p_{BH+Torus}$, $e = e_{BH+Torus}$, etc.)

An example: equatorial orbits

- 2 equations (equality of Ω_r and Ω_φ) and 4 variables (*p, e, a, M*)
- Can solve for *p* and *e* and compute overlap between BH+Torus wfms and Kerr wfms with the same *a*, *M*, Ω_r and Ω_φ but different *p* and *e*:
  if overlap > 0.95 wfms are indistinguishable
  ("confusion problem in *p* and *e*")
- Can solve for *a* and *M* and compute overlap between BH+Torus wfms and Kerr wfms with the same *p*, *e*, Ω_r and Ω_φ but different *a* and *M*:
  if overlap > 0.95 wfms are indistinguishable
  ("confusion problem in *a* and *M*")

Spacetime A $M_{BH} = 10^6 M_\odot$ $M_{Torus} = 7 \cdot 10^4 M_\odot$

$a_{BH} = -1.74 \cdot 10^{-3}$ $a_{BH+Torus} = 0.224$

$r_{QI\ inner}/M_{BH} = 9.16$ $r_{QI\ outer}/M_{BH} = 10$

$\epsilon = 2.63$

Spacetime B $M_{BH} = 4.13 \cdot 10^6 M_\odot$ $M_{Torus} = 1.21 \cdot 10^6 M_\odot$

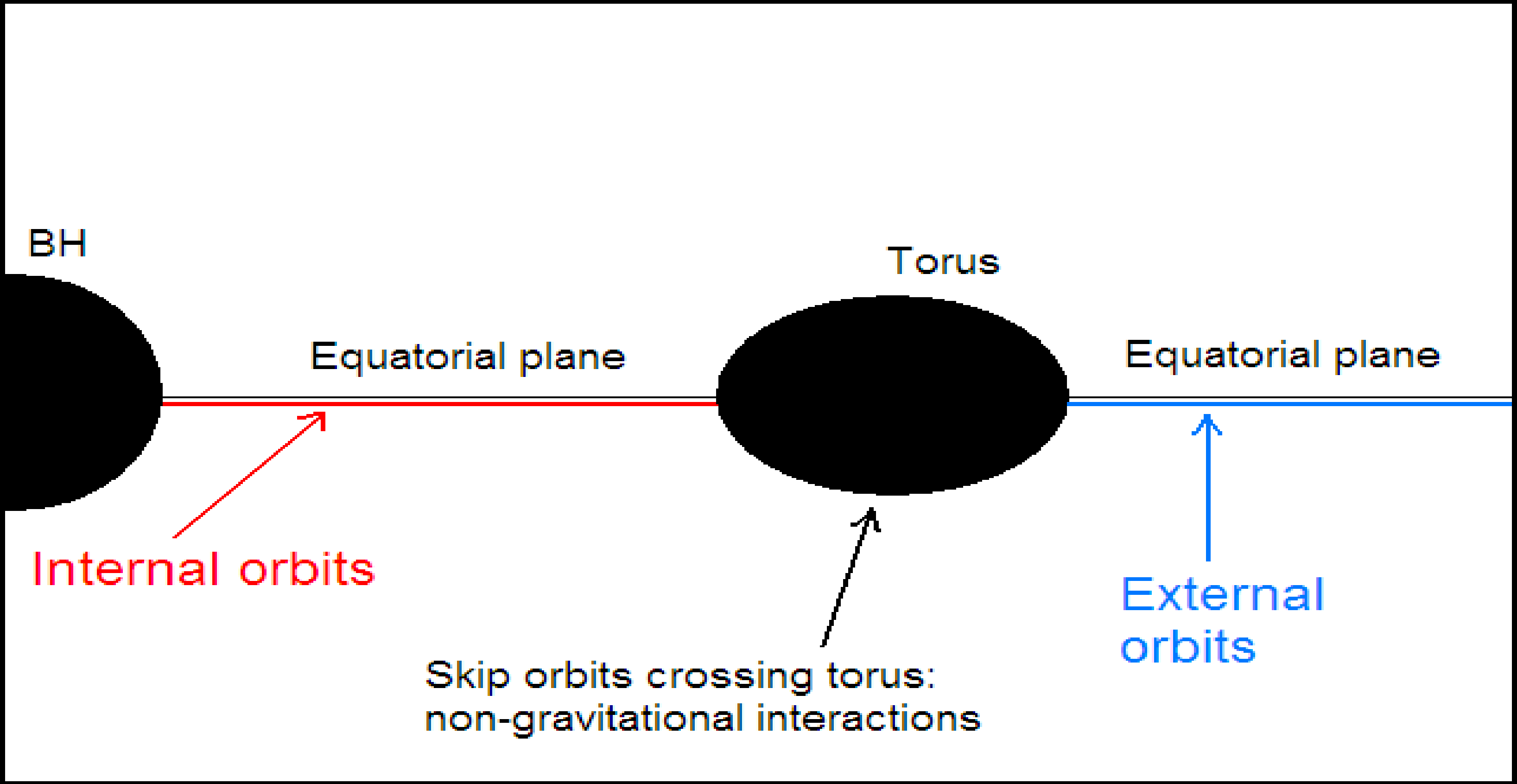
$a_{BH} = 0.528$ $a_{BH+Torus} = 0.728$

$r_{QI\ inner}/M_{BH} = 1.468$ $r_{QI\ outer}/M_{BH} = 1.527$

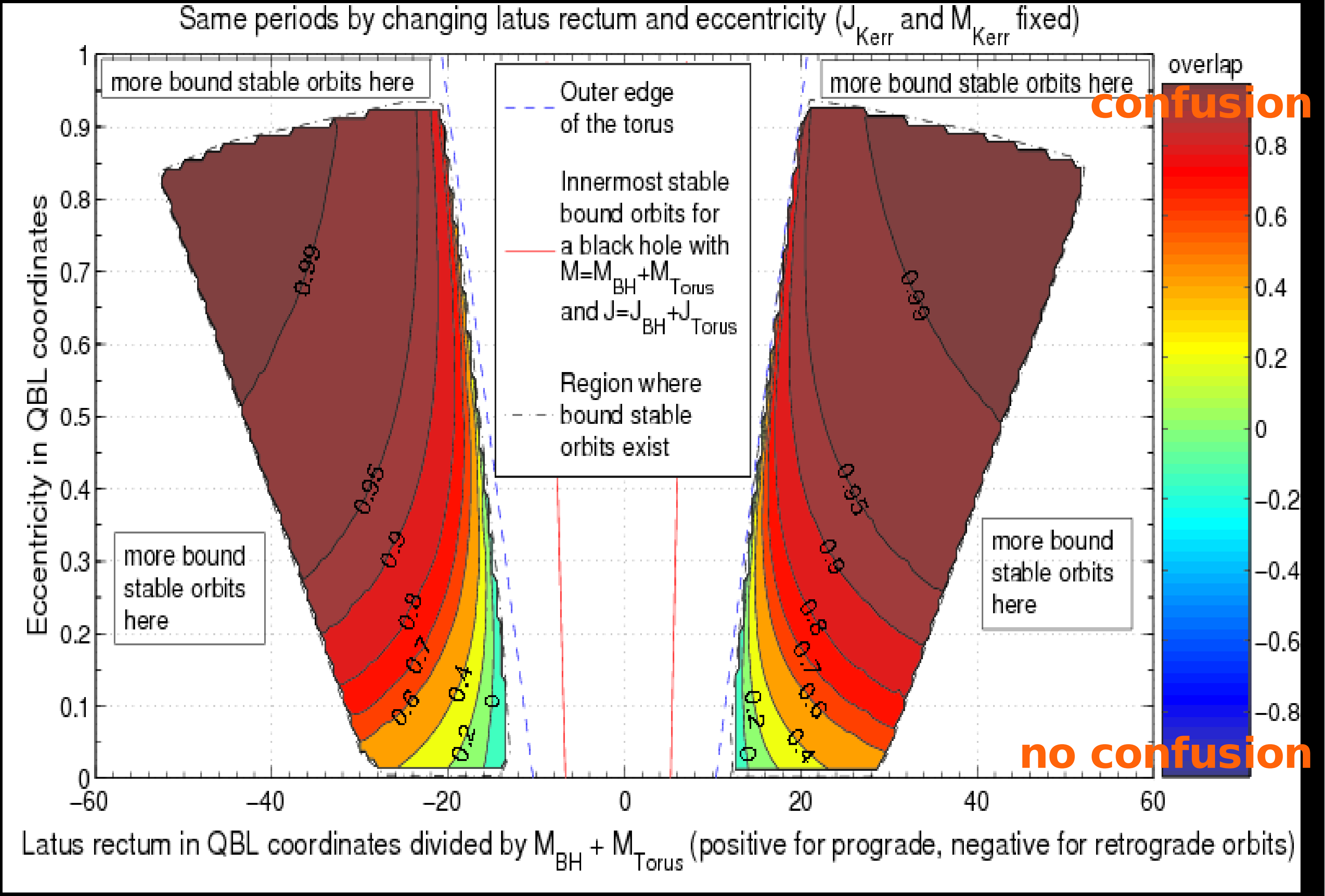
$\epsilon = 0.1103$

Small BH $m = 1 M_\odot$

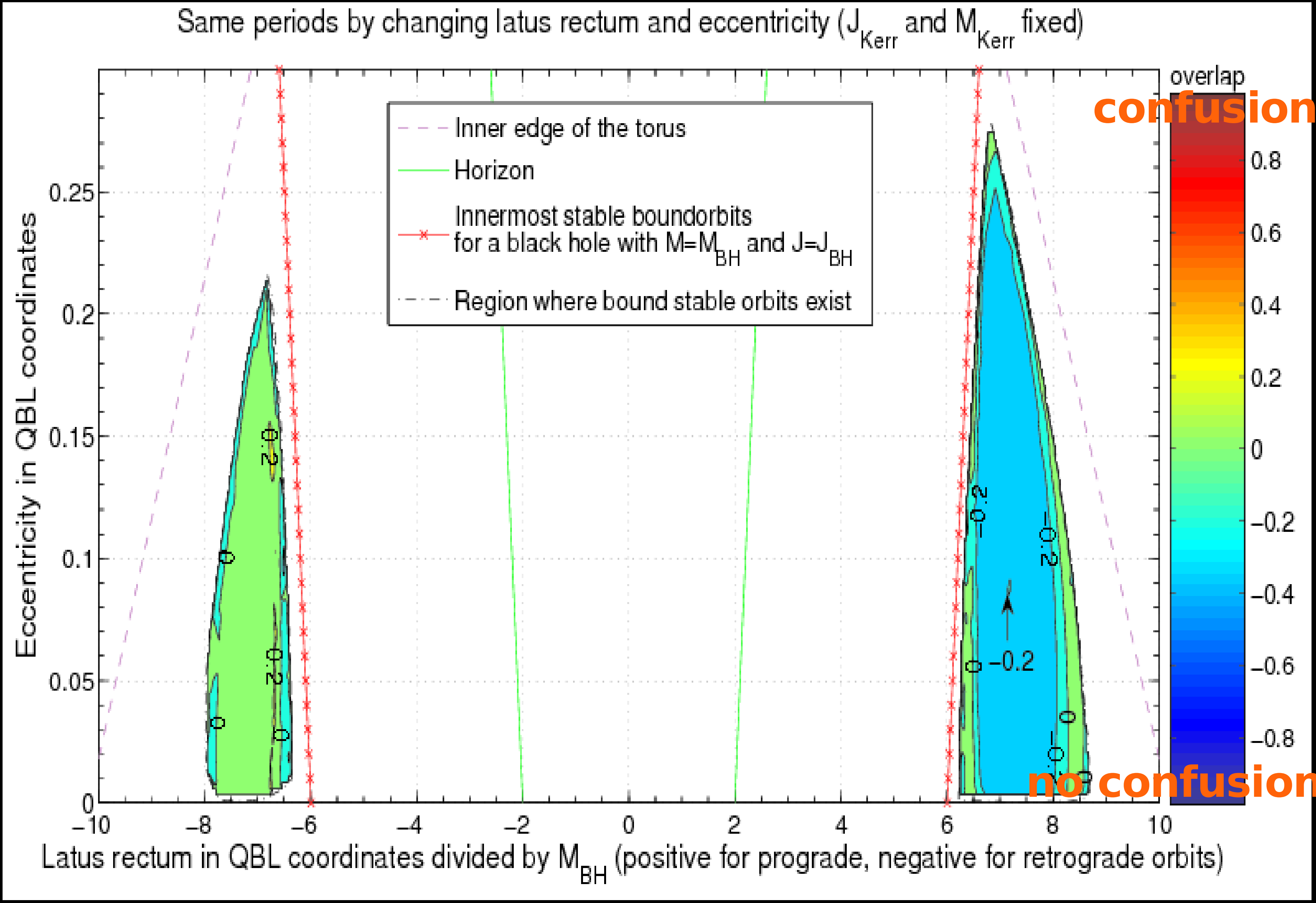
Equatorial orbits: summary of results



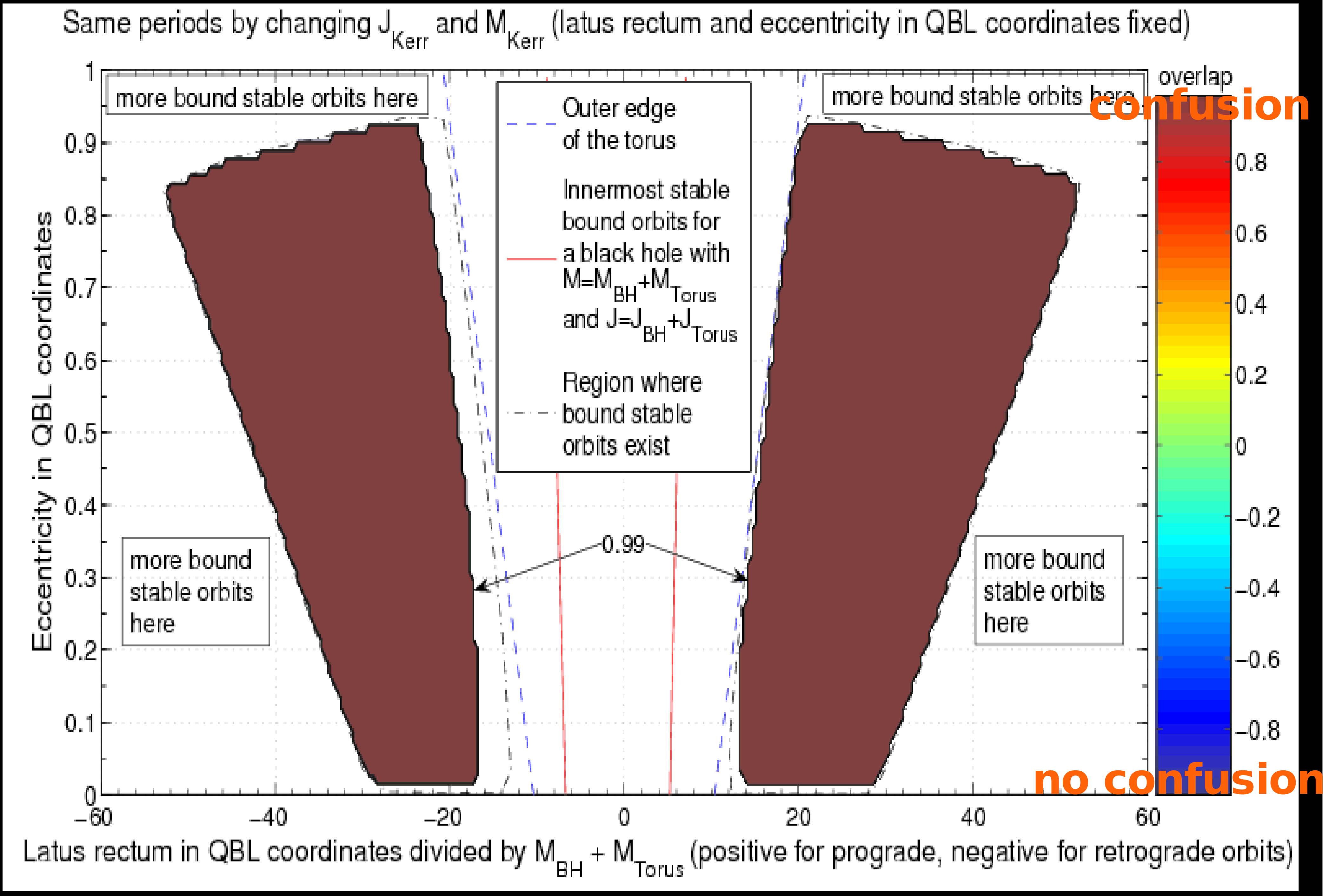
		Confusion in p and e	Confusion in a and M
Spacetime A	External Orbits	Only far from the system	Yes
	Internal Orbits	No	Yes
Spacetime B	External Orbits	Only far from the system	Yes



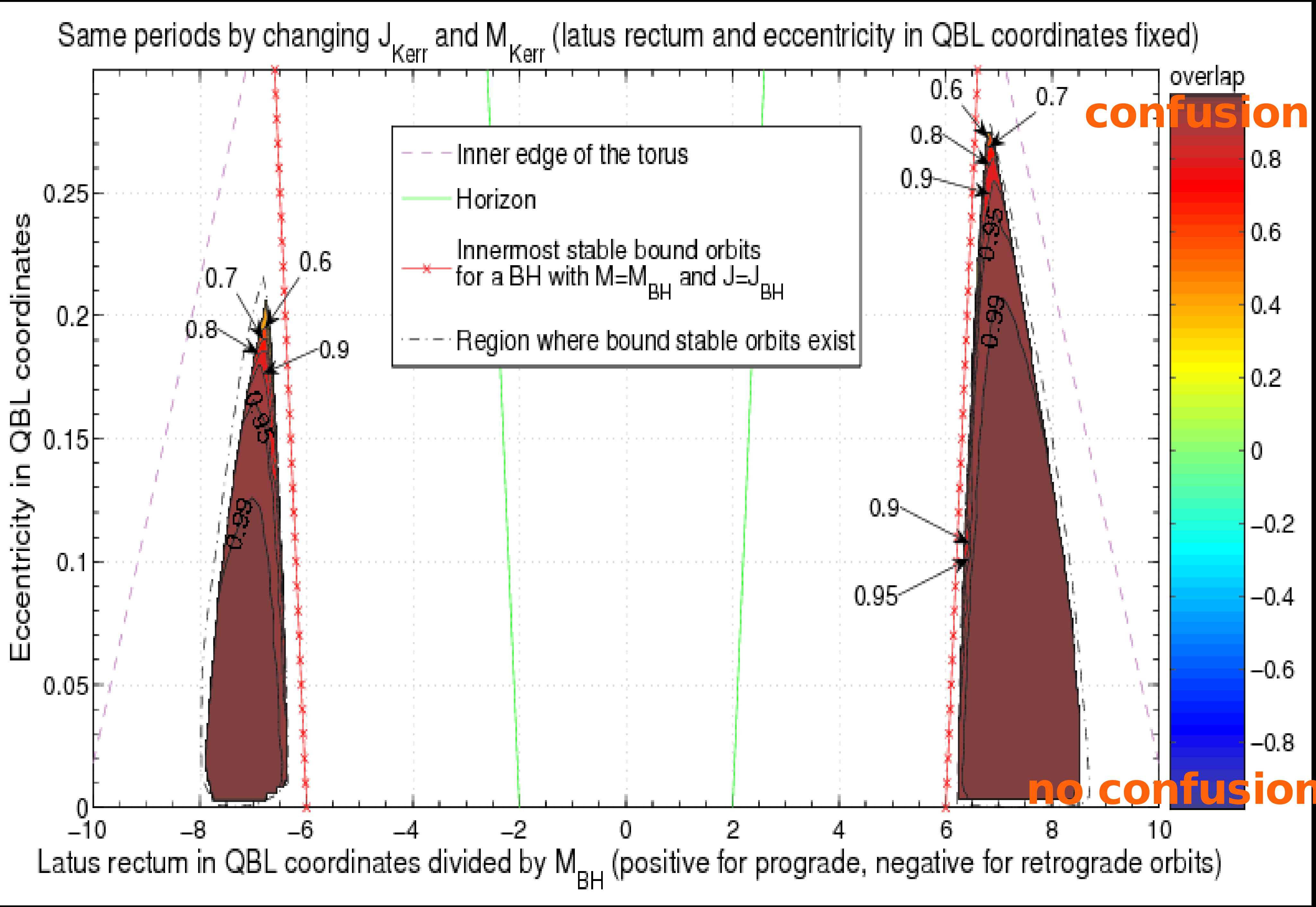
Same periods by changing latus rectum and eccentricity (J_{Kerr} and M_{Kerr} fixed)
more bound stable orbits here
more bound stable orbits here
Outer edge of the torus
Innermost stable bound orbits for a black hole with $M=M_{BH}+M_{Torus}$ and $J=J_{BH}+J_{Torus}$
Region where bound stable orbits exist
more bound stable orbits here
more bound stable orbits here
overlap
confusion
no confusion
Eccentricity in QBL coordinates
Latus rectum in QBL coordinates divided by M_{BH} + M_{Torus} (positive for prograde, negative for retrograde orbits)



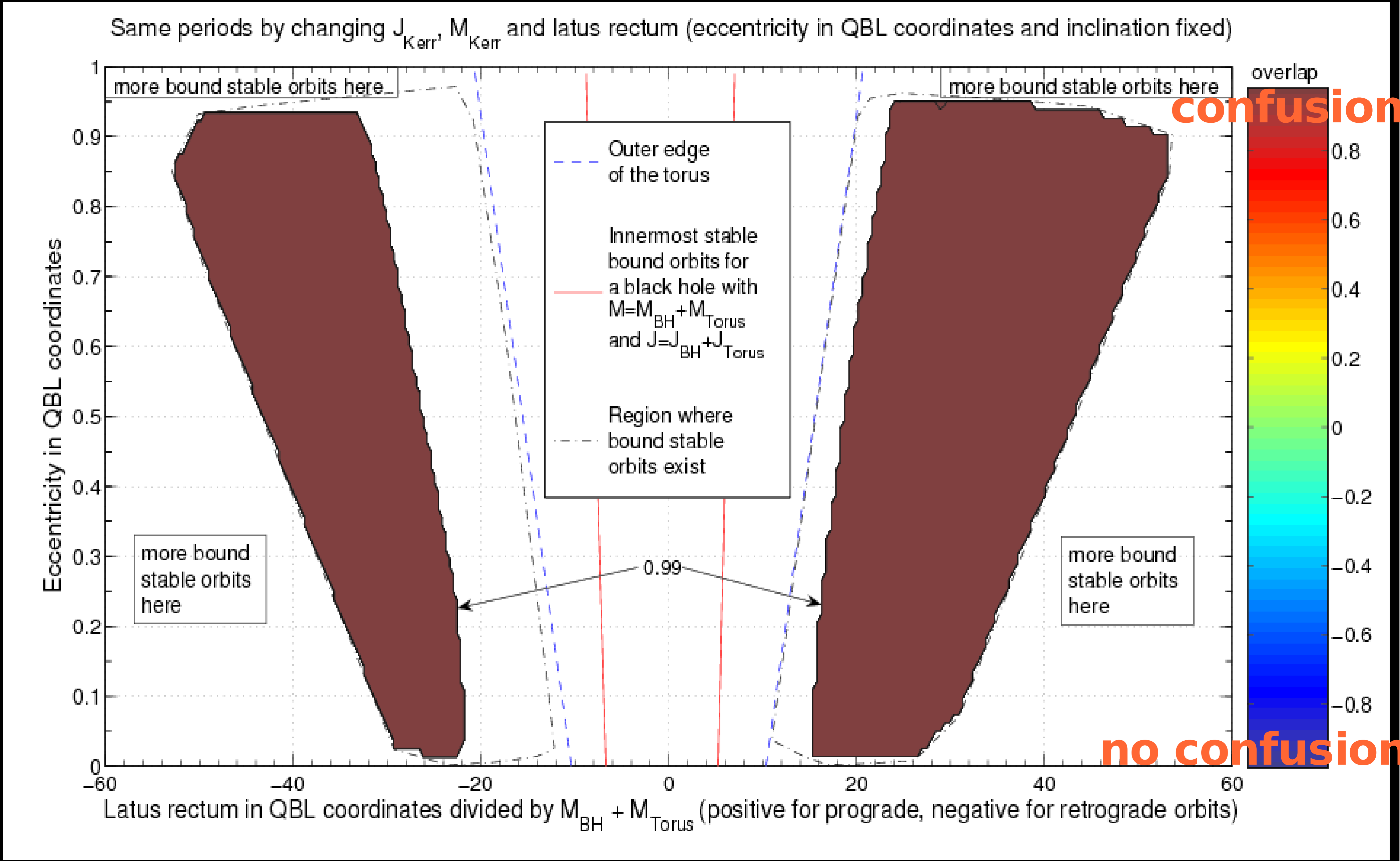
Same periods by changing latus rectum and eccentricity (J_{Kerr} and M_{Kerr} fixed)
Inner edge of the torus
Horizon
Innermost stable boundorbits
for a black hole with $M=M_{BH}$ and $J=J_{BH}$
Region where bound stable orbits exist
Eccentricity in QBL coordinates
Latus rectum in QBL coordinates divided by M_{BH} (positive for prograde, negative for retrograde orbits)
overlap
confusion
no confusion
−0.2



Same periods by changing J_{Kerr} and M_{Kerr} (latus rectum and eccentricity in QBL coordinates fixed)
more bound stable orbits here
more bound stable orbits here
Outer edge of the torus
Innermost stable bound orbits for a black hole with $M=M_{BH}+M_{Torus}$ and $J=J_{BH}+J_{Torus}$
Region where bound stable orbits exist
0.99
more bound stable orbits here
more bound stable orbits here
Eccentricity in QBL coordinates
Latus rectum in QBL coordinates divided by $M_{BH} + M_{Torus}$ (positive for prograde, negative for retrograde orbits)
overlap
confusion
no confusion


Same periods by changing J_Kerr and M_Kerr (latus rectum and eccentricity in QBL coordinates fixed)
Inner edge of the torus
Horizon
Innermost stable bound orbits for a BH with M=M_BH and J=J_BH
Region where bound stable orbits exist
overlap
confusion
no confusion
Eccentricity in QBL coordinates
Latus rectum in QBL coordinates divided by M_BH (positive for prograde, negative for retrograde orbits)

Non equatorial orbits: an example (i =10°)



Conclusions and future work

- A confusion problem in the orbital parameters p and e arises only far from the BH+Torus system

- A confusion problem in the BH parameters a and M is present in most of the spacetime

Can this affect LISA's measurements?

Probably no. To confirm, we need

- More realistic tori (larger or lighter)
- Inclusion of (approximate) radiation reaction