A template bank for gravitational waves from coalescing binary black holes

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with

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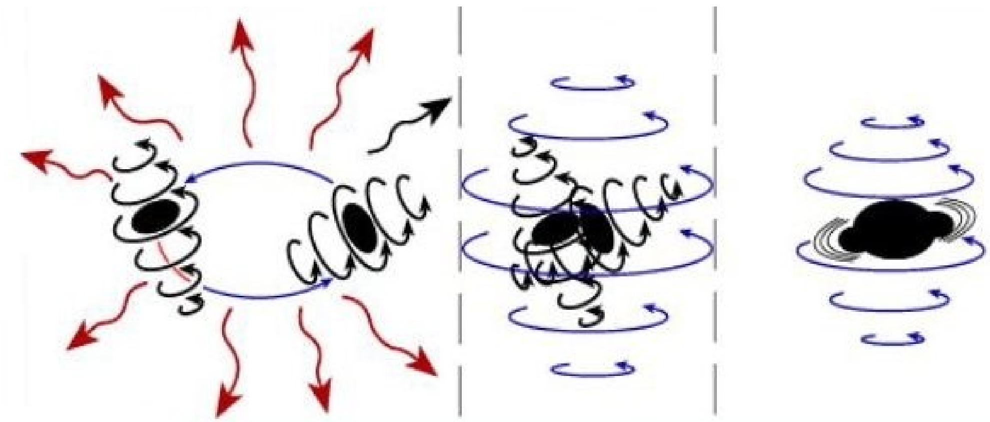
Binary black-hole coalescence

Inspiral | Merger | Ring down



Post-Newtonian theory | Numerical Relativity | BH perturbation theory

(Pic. K. Thorne)

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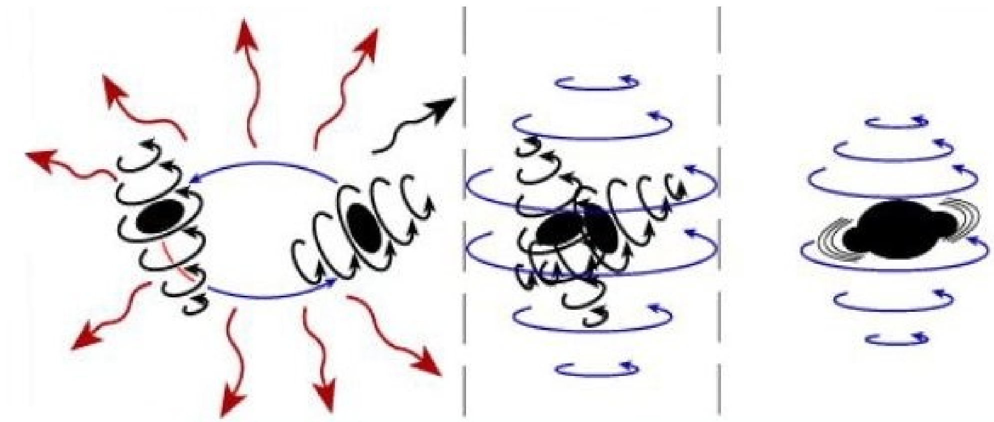

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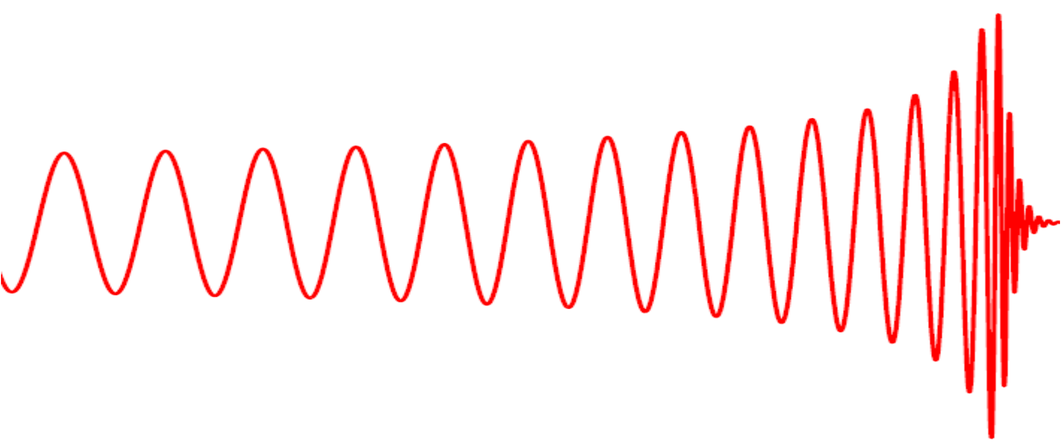


$h(t)$

- Great (recent) progress in analytical and numerical relativity

Gravitational waveforms from all the three stages can be computed.

A template bank for BBH coalescence

- Current searches for GWs look for the individual stages separately.
- Combining results from analytical and numerical relativity enables us to perform a *coherent* search for all stages *using a single template family*.

Advantages

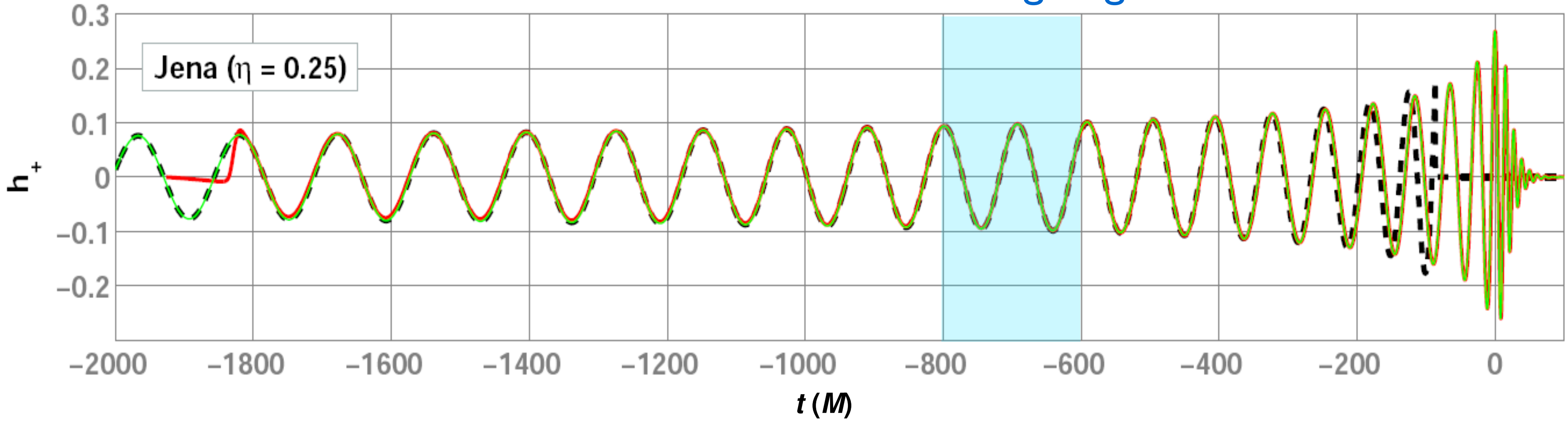
- Increased SNR due to coherent search → improved sensitivity & event rate.
- More "structure" in the template waveform → harder for noise artifacts to mimic the template → reduction in the false alarm rate.
- Improved SNR and complex "structure" → improved parameter estimation.

- How to construct a bank of templates?
  Too expensive to compute a bank of "complete" Numerical-Relativity waveforms.
  - An interpolated template bank (with parametrized analytical waveforms) which has very good overlap with the "target signals".

- How to construct the target signals?
  - Match PN inspiral waveforms with NR waveforms to construct "hybrid waveforms".

Matching PN and NR waveforms

Matching Region

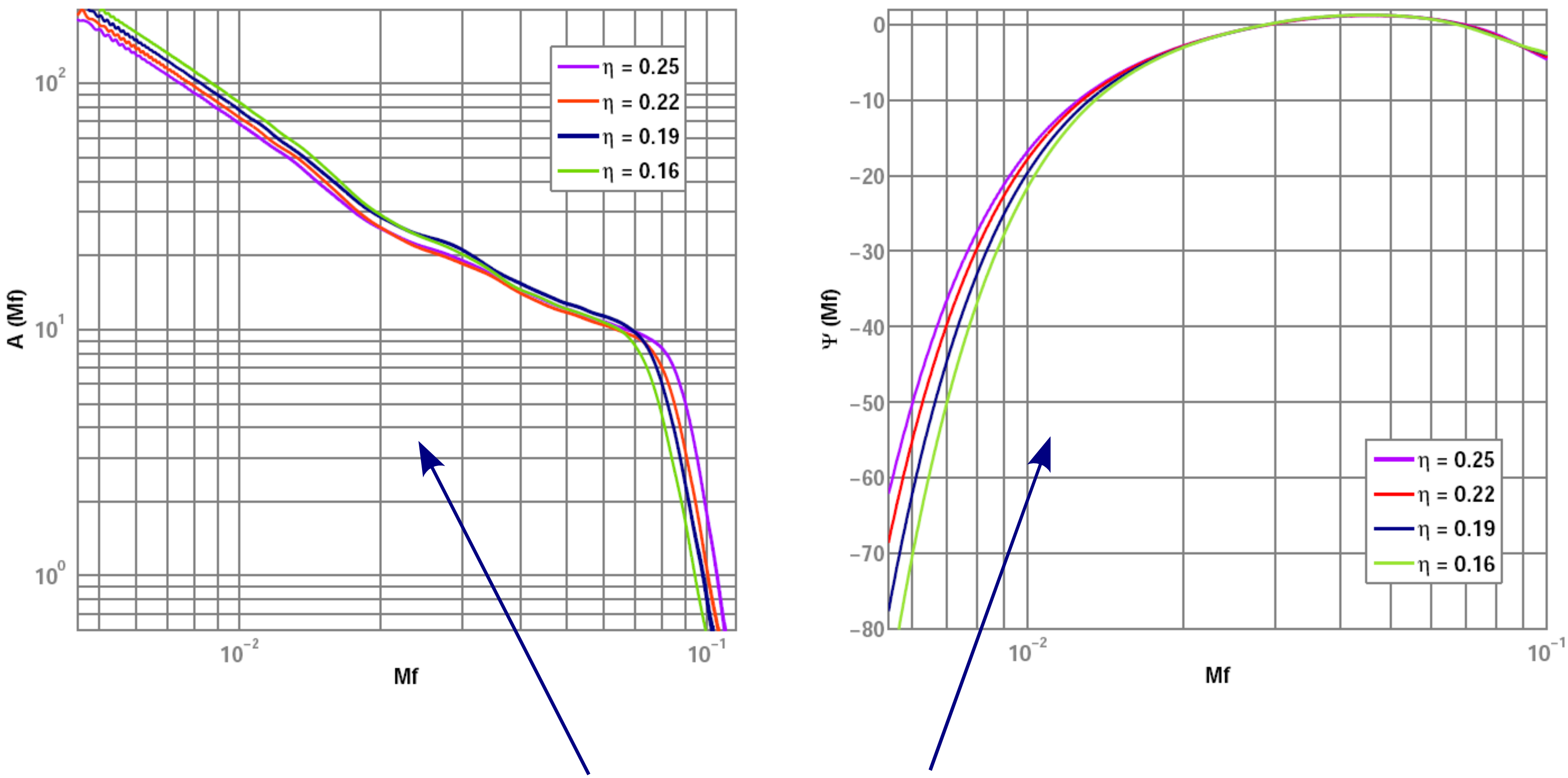


PN Restricted 3.5PN waveform
NR Jena equal-mass simulation
Hybrid

Matching parameters

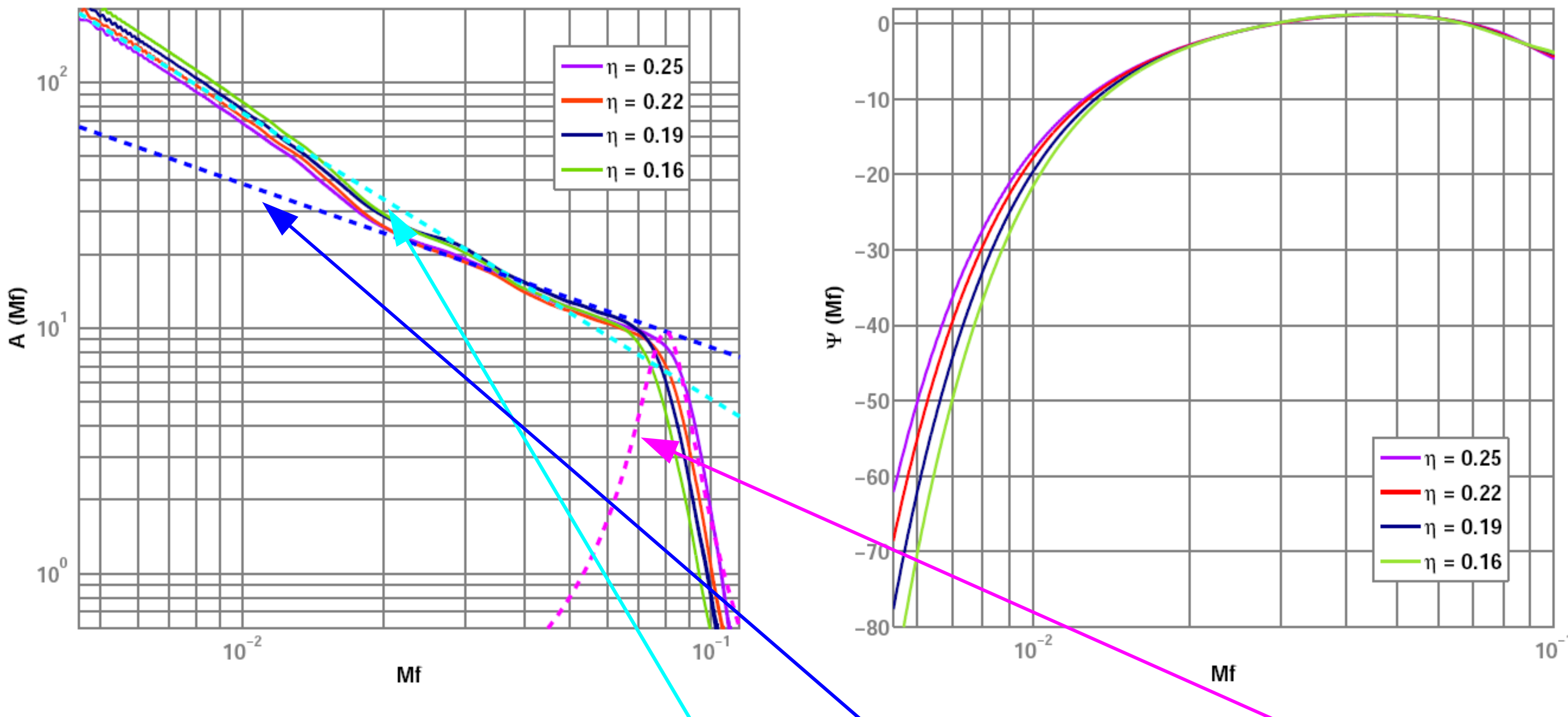
t_0, ϕ_0, a

- **Hybrid waveforms** combining best-matched NR & PN waveforms.



Fourier domain magnitude and phase of hybrid waveforms.

A phenomenological parametrization



- **Magnitude** Power-laws $f^{-7/6}$ and $f^{-2/3}$, and a Lorentzian $\mathcal{L}(f_{\rm ring}, \sigma)$.
- **Phase** Expansion in powers of f.

Phenomenological waveforms

- The phenomenological waveform:

$$u(f) = \mathcal{A}_{\text{eff}}(f)\, e^{i\Psi_{\text{eff}}(f)}$$

where

$$\mathcal{A}_{\text{eff}}(f) = \begin{cases} (f/f_{\text{merg}})^{-7/6} & \text{if } f < f_{\text{merg}} \\ (f/f_{\text{merg}})^{-2/3} & \text{if } f_{\text{merg}} \leq f < f_{\text{ring}} \\ w\,\mathcal{L}(f, f_{\text{ring}}, \sigma) & \text{if } f_{\text{ring}} \leq f < f_{\text{cut}} \end{cases}$$

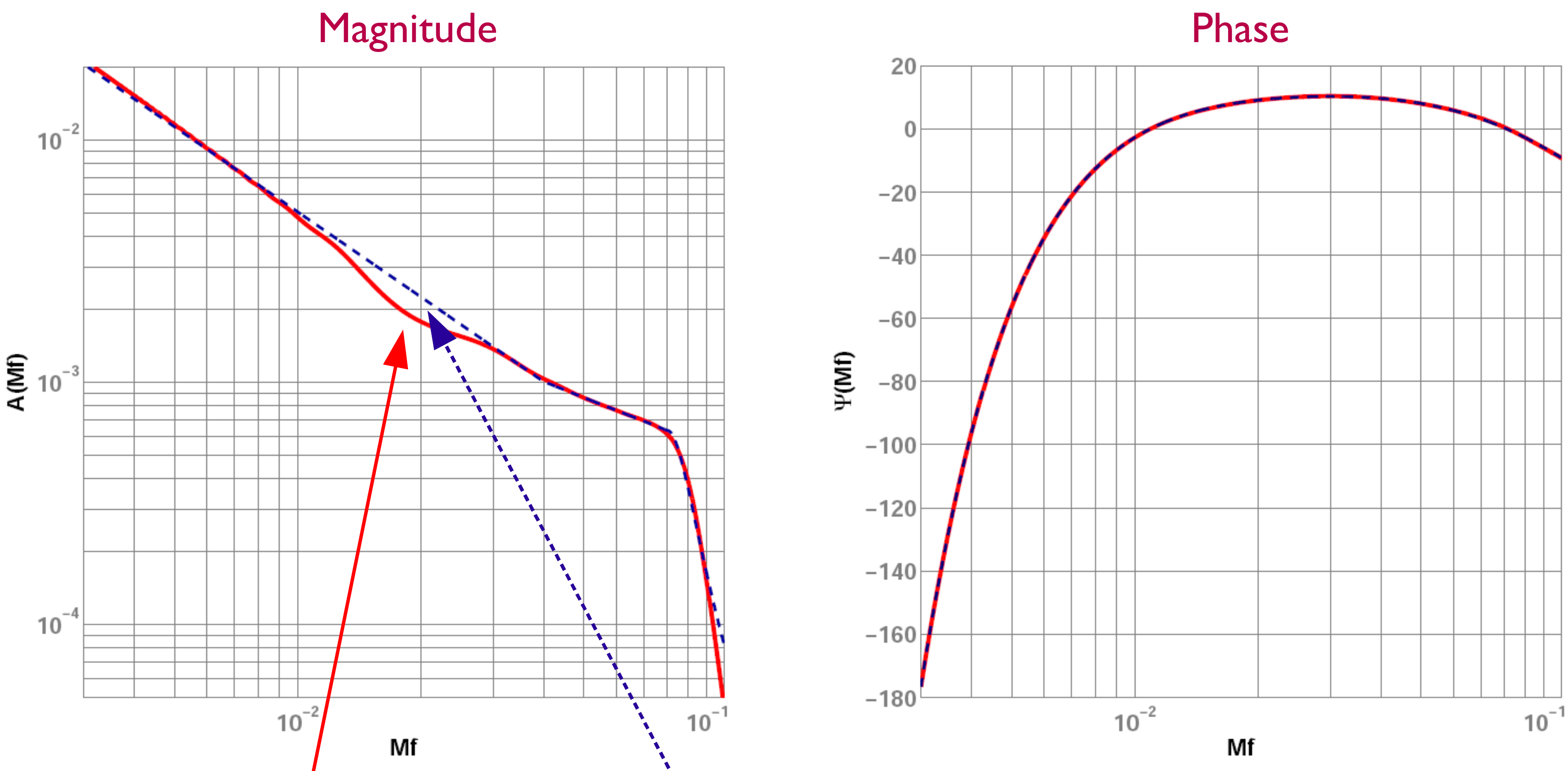
Amplitude parameters

$$\Psi_{\text{eff}}(f) = 2\pi f t_0 + \varphi_0 + \sum_{k=0}^{7} \psi_k\, f^{(k-5)/3}$$

Phase parameters

Hybrid waveforms and the “best-matched” templates

Equal-mass binary, White noise spectrum.

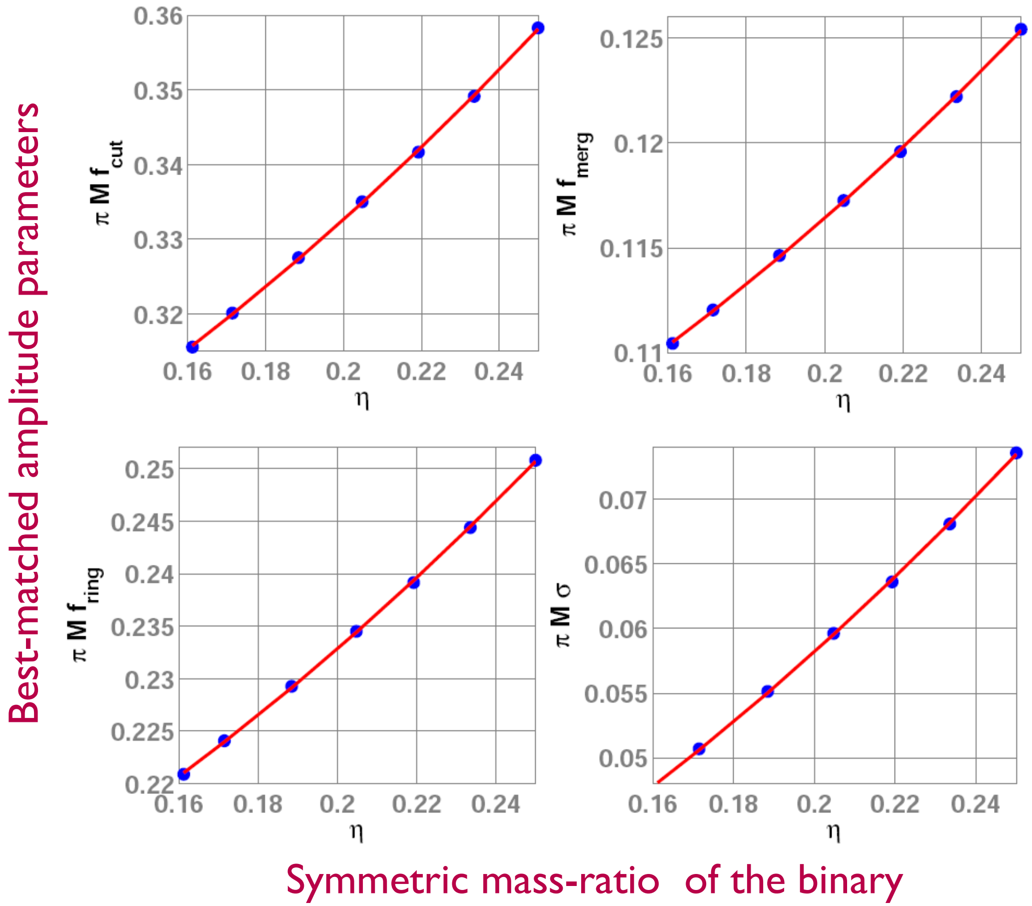


Hybrid waveforms

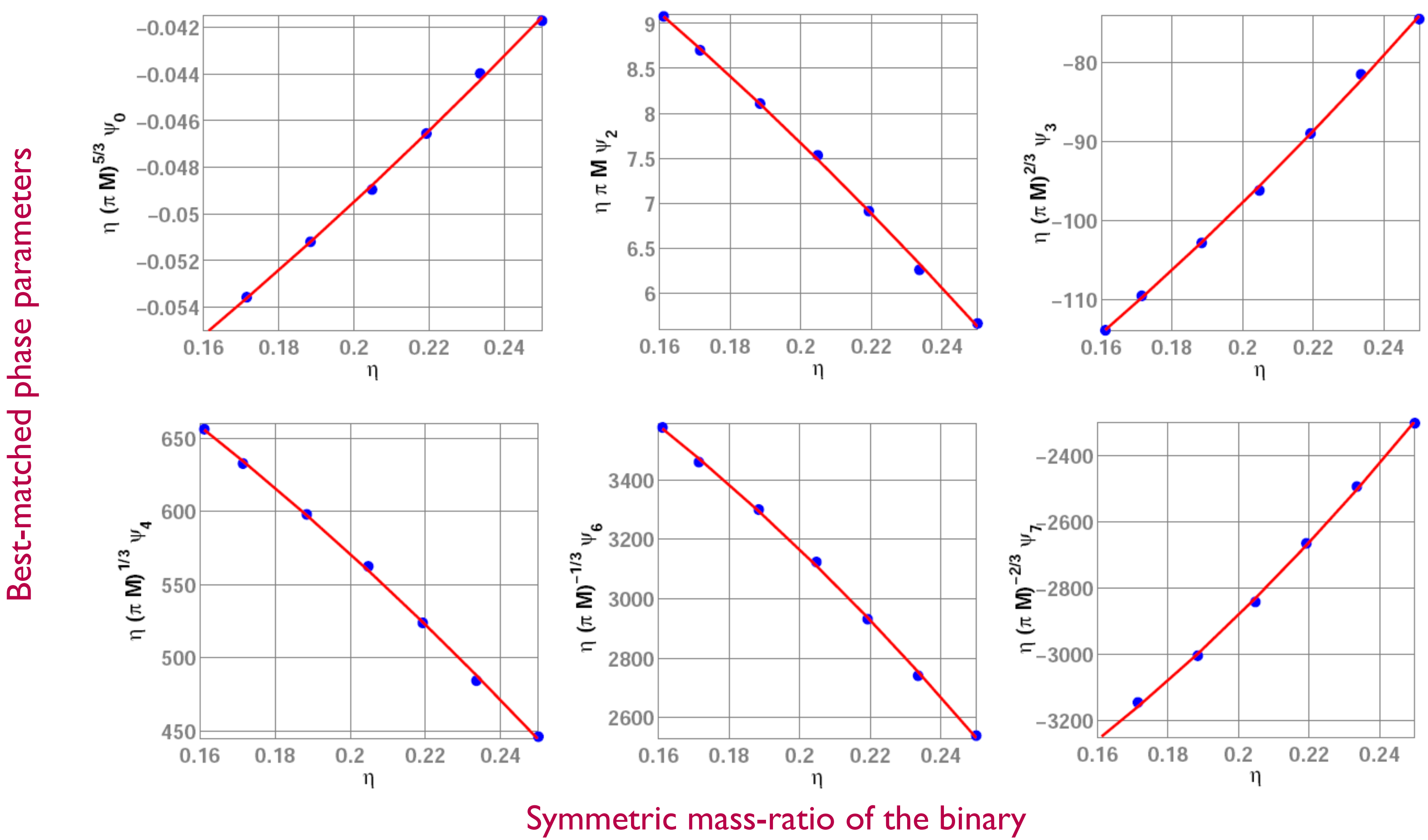
Phenomenological waveforms

From phenomenological parameters to physical parameters

- Reparametrize the "best-matched" phenomenological waveforms in terms of the physical parameters.



From phenomenological parameters to physical parameters



Reparametrization

- "Best-matched" phenomenological parameters can be written in terms of the physical parameters:

Polynomial fits to the amplitude parameters

$$\alpha_{j\ \mathrm{int}} = \frac{a_j\,\eta^2 + b_j\,\eta + c_j}{\pi M},$$

Polynomial fits to the phase parameters

$$\psi_{k\ \mathrm{int}} = \frac{x_k\,\eta^2 + y_k\,\eta + z_k}{\eta\,(\pi M)^{(5-k)/3}},$$

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Polynomial coefficients (see table)

Parameter	a_k	b_k	c_k
f_{merg}	2.9740×10^{-1}	4.4810×10^{-2}	9.5560×10^{-2}
f_{ring}	5.9411×10^{-1}	8.9794×10^{-2}	1.9111×10^{-1}
σ	5.0801×10^{-1}	7.7515×10^{-2}	2.2369×10^{-2}
f_{cut}	8.4845×10^{-1}	1.2848×10^{-1}	2.7299×10^{-1}

Parameter	x_k	y_k	z_k
ψ_0	1.7516×10^{-1}	7.9483×10^{-2}	-7.2390×10^{-2}
ψ_2	-5.1571×10^{1}	-1.7595×10^{1}	1.3253×10^{1}
ψ_3	6.5866×10^{2}	1.7803×10^{2}	-1.5972×10^{2}
ψ_4	-3.9031×10^{3}	-7.7493×10^{2}	8.8195×10^{2}
ψ_6	-2.4874×10^{4}	-1.4892×10^{3}	4.4588×10^{3}
ψ_7	2.5196×10^{4}	3.3970×10^{2}	-3.9573×10^{3}

“Closeness” of a template with the exact waveform

- **Fitting factor** measure of the “*effectualness*” of the template family $u(\boldsymbol{\mu},\boldsymbol{\nu})$ in detecting a signal h.

$$\mathrm{FF} = \max_{\boldsymbol{\nu},\boldsymbol{\mu}} \left\langle \hat{h}, \hat{u}(\boldsymbol{\mu}, \boldsymbol{\nu}) \right\rangle$$

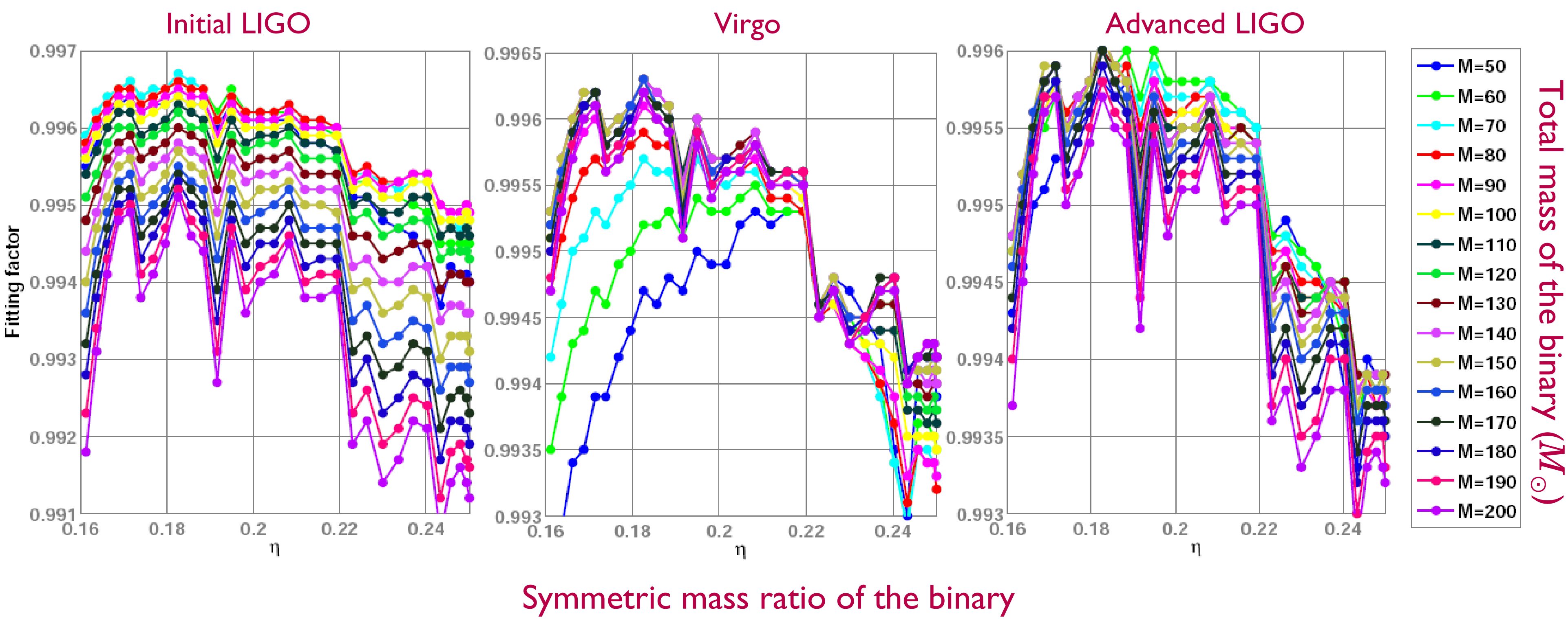
- **Faithfulness** measure of the quality of $u(\boldsymbol{\mu},\boldsymbol{\nu})$ for parameter estimation.

$$\mathbb{F} = \max_{\boldsymbol{\nu}} \left\langle \hat{h}, \hat{u}(\boldsymbol{\mu}, \boldsymbol{\nu}) \right\rangle$$

“intrinsic” parameters (m_1, m_2)

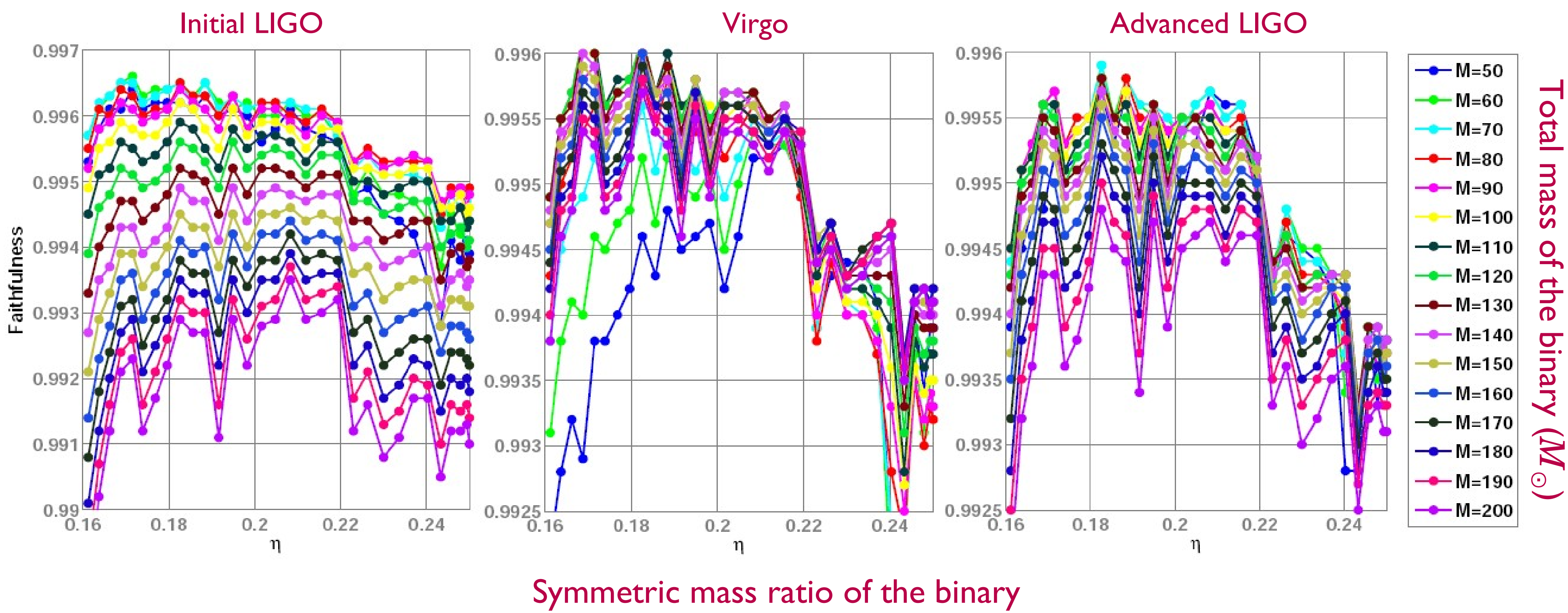
“extrinsic” parameters (φ_0, t_0)

Fitting factor of the 2D template family



2D templates with fitting factor > 0.99 with the hybrid waveforms!

Faithfulness of the 2D template family

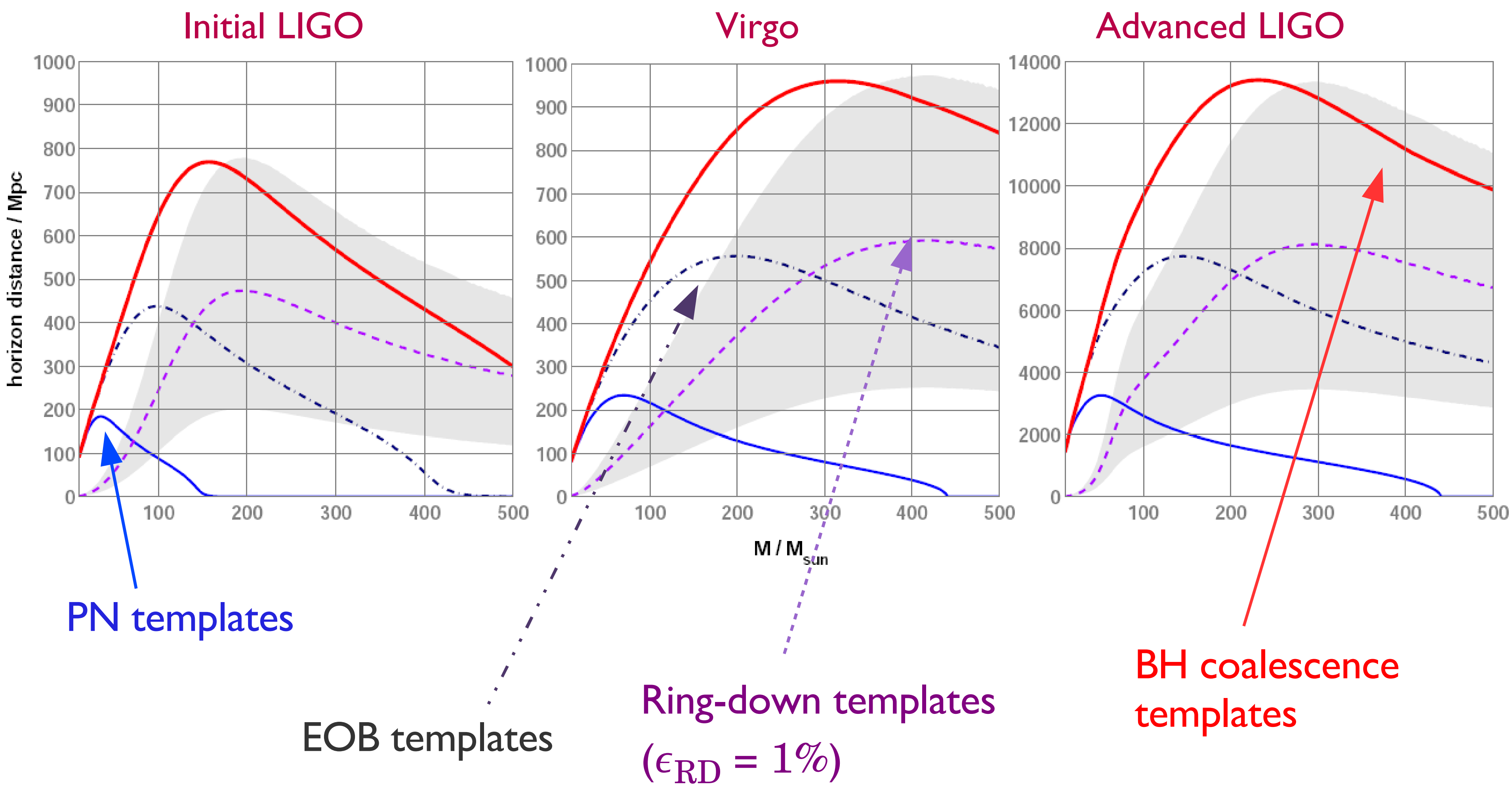


2D templates with fitting factor > 0.99 with the hybrid waveforms!

Also "faithful" in estimating the parameters of the binary!

Sensitivity of the search

- Effective distance to optimally-oriented binaries producing optimal SNR 8.



Summary

- Great progress in analytical and numerical relativity in solving BBH problem. All three stages can be *coherently* searched over.
- "Complete" BBH waveforms (*hybrid* waveforms) are constructed by matching PN and NR waveforms.
- Proposed analytical template family which is very close to the hybrid waveforms – 2D template bank for non-spinning binaries.
- Highly *effectual* and *faithful* templates.
- Significant improvement in the "distance reach".