Phenomenological template bank for black hole coalescence waveforms

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in collaboration with

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Outline

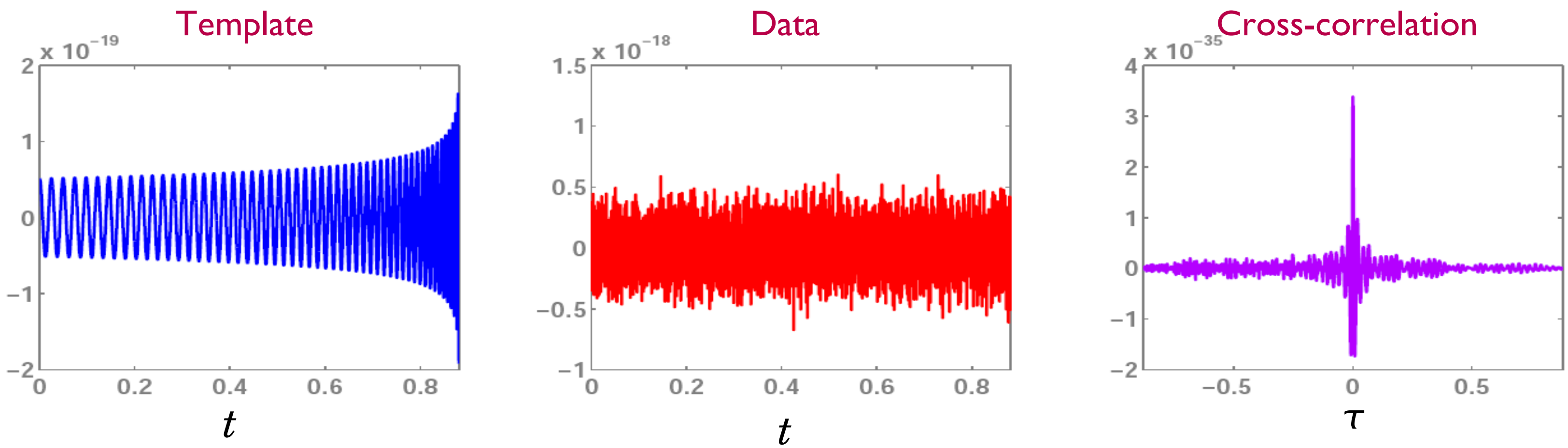
- Data analysis from coalescing compact binaries – state of the art.
- Motivation and plans – look for BH coalescences with a single template family.
- Constructing full coalescence waveforms – Matching PN and NR waveforms.
- A phenomenological waveform family.
- Laying down templates – the template bank.
- Sensitivity & astrophysical range of the search.

Disclaimer

This is a DA talk!

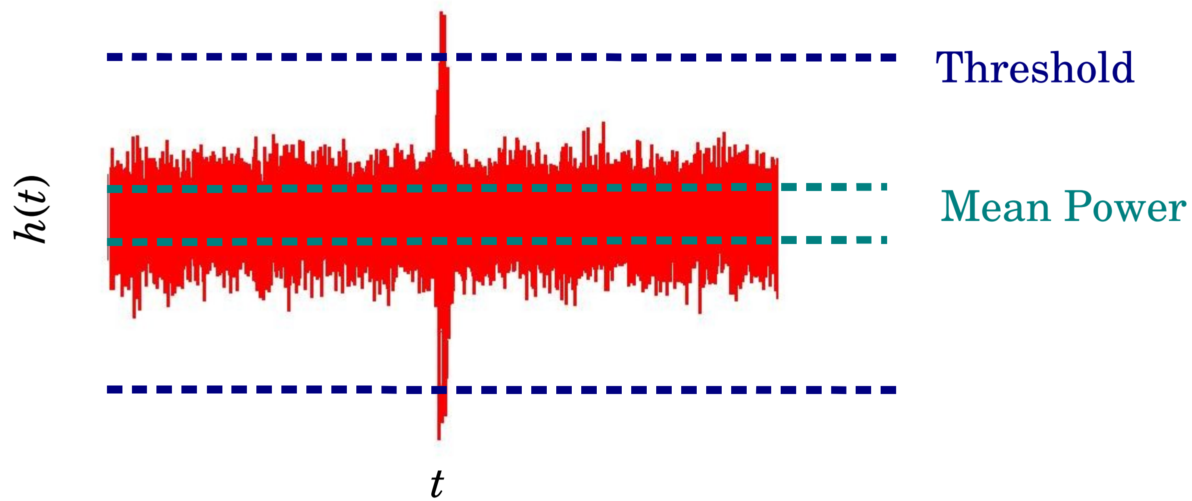
Data analysis for coalescing compact binaries

- **Inspiral phase**
  - Physical (analytical) waveforms from PN theory. Also, detection template families (*e.g.* BCV) constructed to match with a number of theoretical waveforms.
  - Since waveforms are *known,* the optimal filter to search these signals in the noise is *matched filter,* which essentially involves cross-correlating the data and the signal template.

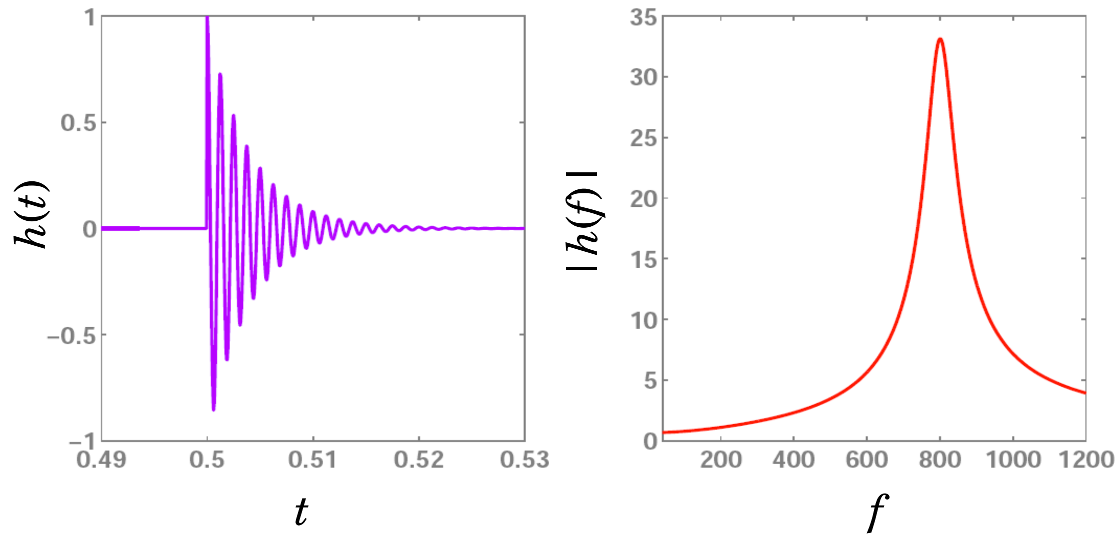


Data analysis for coalescing compact binaries

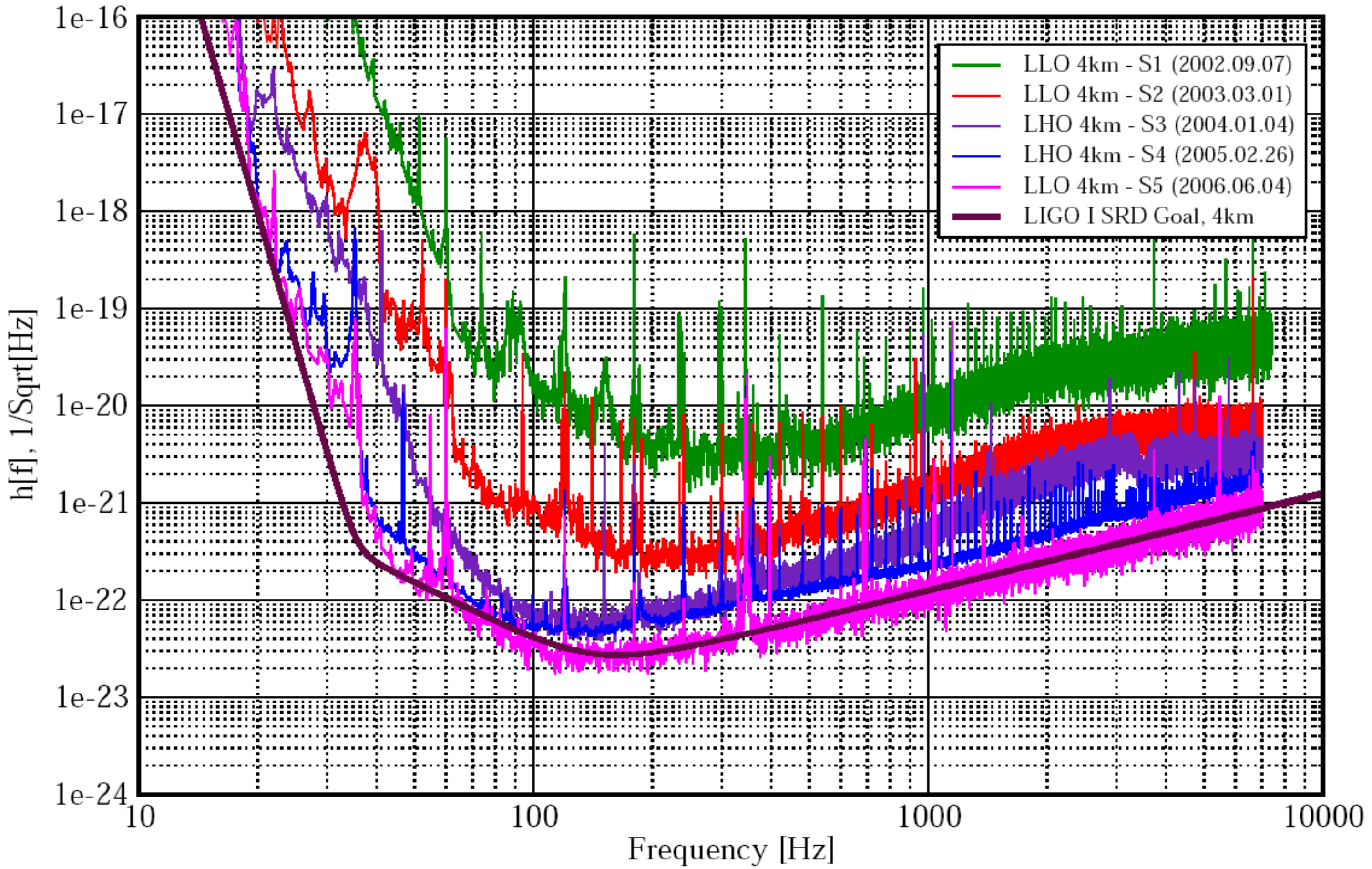
- **Merger phase**
  - Waveforms were not known until recently.
  - Searches use *excess-power* techniques, which are tuned to detect large morphologies of waveforms. Look for short-lived excitations of power in the data, which are less-likely to be associated with the background noise distribution – a non-optimal filter.



- **Ring-down phase**
  - Waveforms are exponentially-damped sinusoids, parametrized by the mass and spin of the BH.
  - Since waveforms are known, matched filtering can be used to search for the signals.



Motivation of the work



The ongoing science run of large-scale interferometers
at the design sensitivity

Motivation of the work

- Recent progress in Numerical Relativity in solving the binary BH problem.
- Gravitational waveforms from the non-perturbative merger phase can also be computed.
- Finally allows us to coherently search for all three stages (inspiral, merger and ring-down) of the binary BH coalescence.

A single template bank for BH coalescence

- Coherently search for all three stages of the BH coalescence signals (non-spinning binaries) using a single template bank.
- But the high computational cost makes it infeasible to generate enough numerical waveforms to densely cover the entire parameter space to be searched over using matched filtering technique.

A single template bank for BH coalescence

- **Issue** How to construct a bank of templates?
  - Too expensive to compute a bank of NR waveforms dense enough in the (M, η) parameter space.
  - A phenomenological template bank (with parametrized waveforms) which has very good overlap with the 'target signals'.
- **Issue** How to construct the 'target' waveforms?
  - Need waveforms containing all three stages of the BH coalescence - too expensive to (numerically) evolve the binary from very large separations.
  - Match PN inspiral waveforms with NR (merger+ring-down) waveforms in a region where both calculations are valid.

Matching PN and NR waveforms

- Minimize the 'distance' between PN and NR waveforms over a matching region (a few cycles long), thus construct hybrid waveforms.

$$\delta = \min_{\boldsymbol{\mu},a}\left[\sum_{i=+,\times}\int_{t_1}^{t_2}\left[h_i^{\mathrm{PN}}(t,\boldsymbol{\mu}) - a\, h_i^{\mathrm{NR}}(t,\boldsymbol{\nu})\right]^2 \mathrm{d}t\right]$$

- Minimisation is carried out over the parameters $\boldsymbol{\mu} = \{t_0, \phi_0, M, \eta\}$ and an amplitude scaling factor a.

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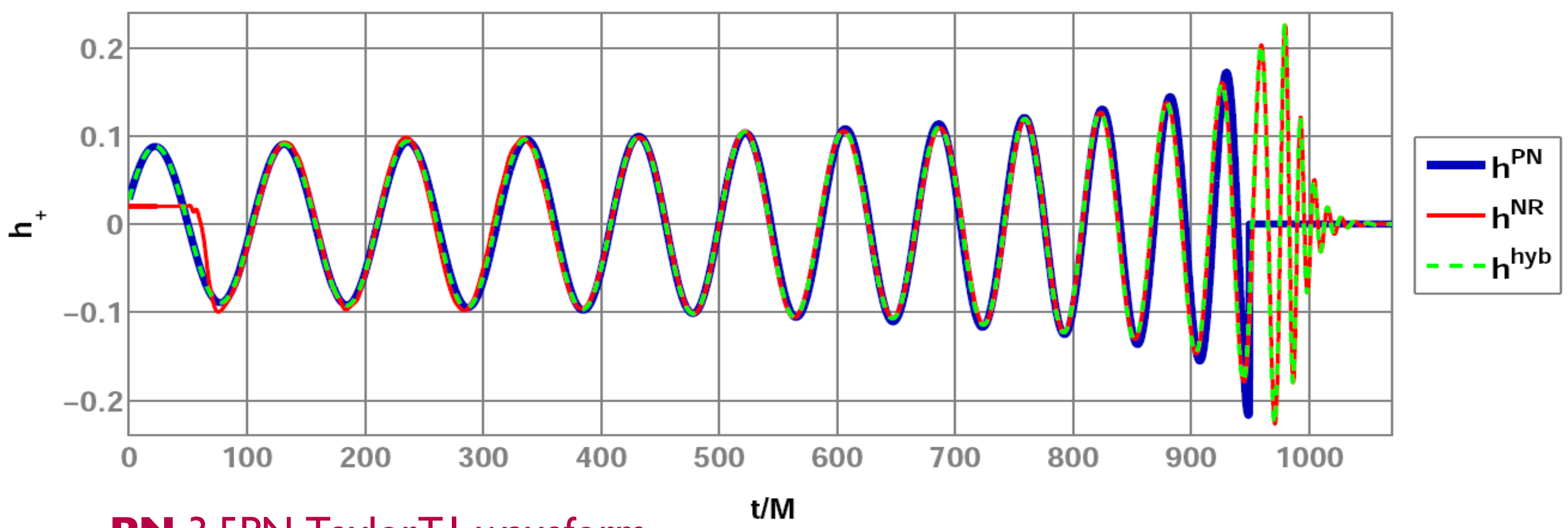
Constructing hybrid waveforms

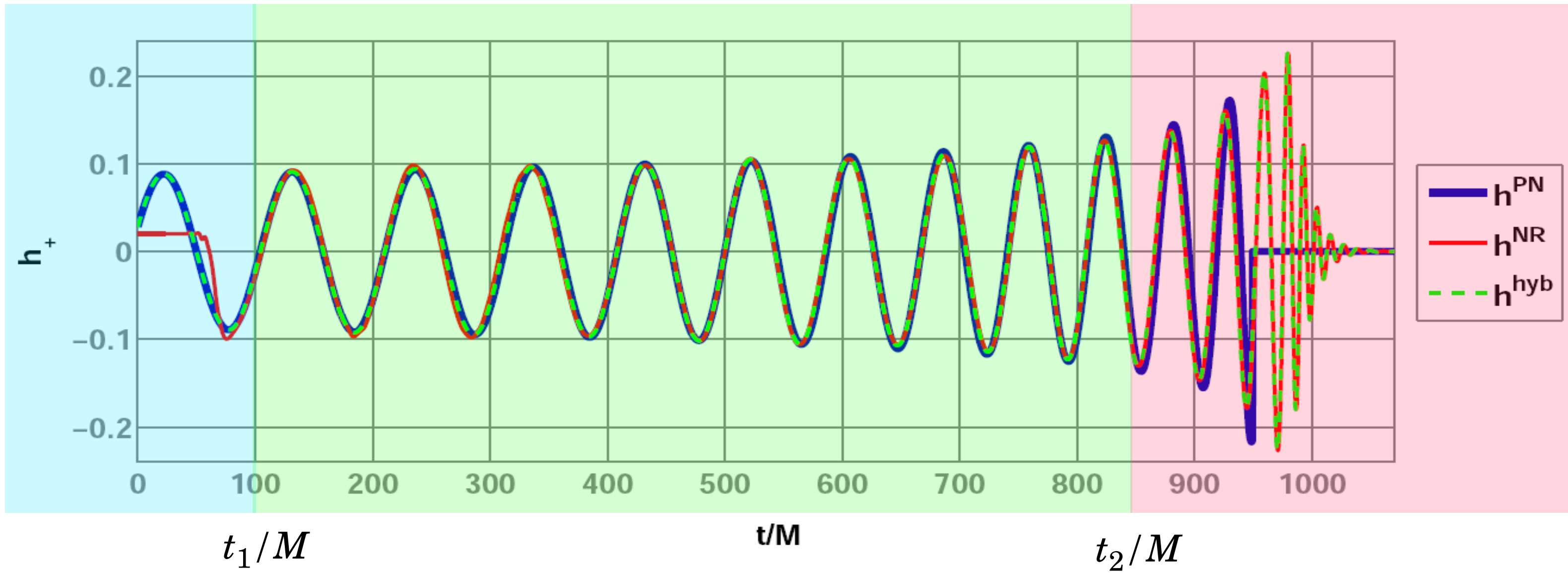


PN 3.5PN TaylorT1 waveform
NR AEI equal-mass simulation

- Combine NR waveforms with the best-matched PN waveforms

$$h^{\rm hyb}_{+,\times}(t,\boldsymbol{\nu}) = \begin{cases} h^{\rm PN}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t < t_1 \\ a_0\,\tau\,h^{\rm NR}_{+,\times}(t,\boldsymbol{\nu}) + (1-\tau)\,h^{\rm PN}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t_1 \le t < t_2 \\ a_0\,h^{\rm NR}_{+,\times}(t,\boldsymbol{\nu}) & \text{if } t_2 \le t \end{cases}$$

Constructing hybrid waveforms



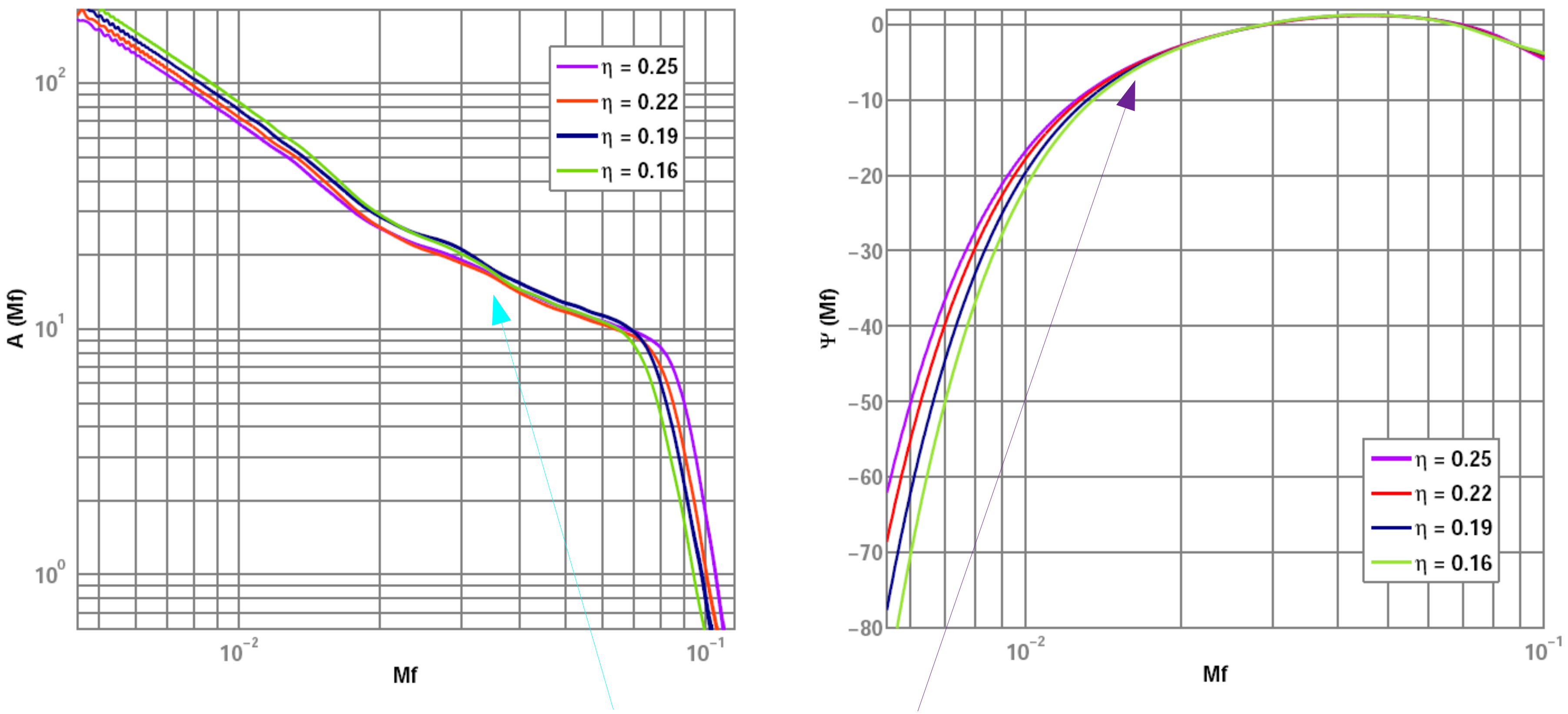
'Best-matched' parameters

$$h^{\rm hyb}_{+,\times}(t,\boldsymbol{\nu}) = \begin{cases} h^{\rm PN}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t < t_1 \\ a_0\,\tau\, h^{\rm NR}_{+,\times}(t,\boldsymbol{\nu}) + (1-\tau)\, h^{\rm PN}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t_1 \le t < t_2 \\ a_0\, h^{\rm NR}_{+,\times}(t,\boldsymbol{\nu}) & \text{if } t_2 \le t \end{cases}$$

Linearly increasing weighting function such that

$\tau(t_1) = 0$ & $\tau(t_2) = 1$

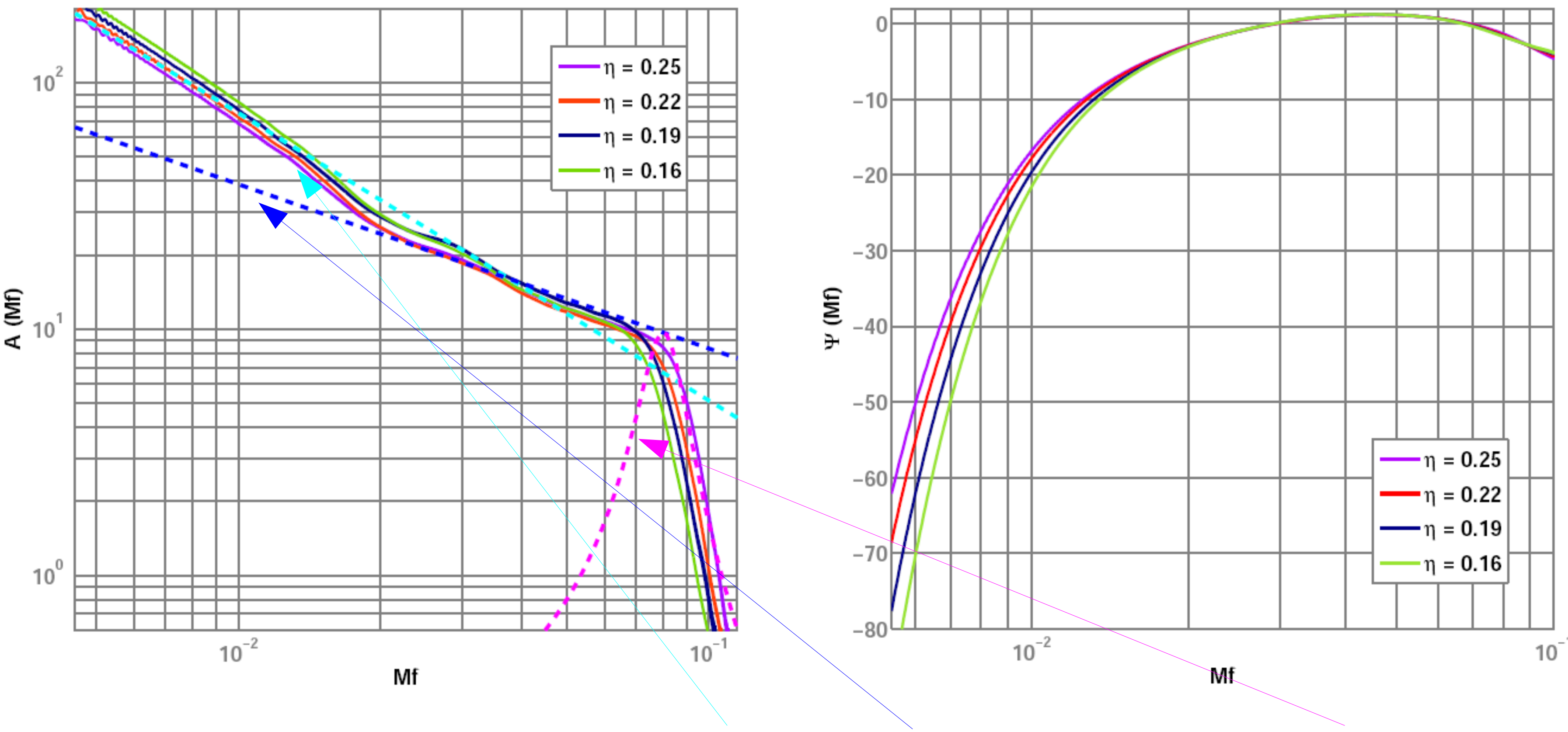
Hybrid waveforms in the Fourier domain



- Fourier domain magnitude and phase of hybrid waveforms.

PN 3.5PN TaylorT1 waveform
NR Jena unequal-mass simulations

A phenomenological parametrization



- **Magnitude** by two power-laws ($f^{-7/6}$ and $f^{-2/3}$) and a Lorentzian $L(f_{ring}, \sigma)$.
- **Phase** Expansion in powers of f (motivation from the SPA).

Phenomenological waveforms

- The parametrised phenomenological waveform is written as

$$u(f) = \mathcal{A}_{\text{eff}}(f)\, e^{i\Psi_{\text{eff}}(f)}$$

where

$$\mathcal{A}_{\text{eff}}(f) = \begin{cases} (f/f_{\text{merg}})^{-7/6} & \text{if } f < f_{\text{merg}} \\ (f/f_{\text{merg}})^{-2/3} & \text{if } f_{\text{merg}} \le f < f_{\text{ring}} \\ w\,\mathcal{L}(f, f_{\text{ring}}, \sigma) & \text{if } f_{\text{ring}} \le f < f_{\text{cut}} \end{cases}$$

$$\Psi_{\text{eff}}(f) = 2\pi f t_0 + \phi_0 + \psi_0\, f^{-5/3} + \psi_2\, f^{-1} + \psi_3\, f^{-2/3} + \psi_4\, f^{-1/3} + \psi_6\, f^{1/3}.$$

Phenomenological waveforms

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$$u(f) = \mathcal{A}_{\rm eff}(f)\, e^{i\Psi_{\rm eff}(f)}$$

where

$$\mathcal{A}_{\rm eff}(f) = \begin{cases} (f/f_{\rm merg})^{-7/6} & \text{if } f < f_{\rm merg} \\ (f/f_{\rm merg})^{-2/3} & \text{if } f_{\rm merg} \le f < f_{\rm ring} \\ w\, \mathcal{L}(f, f_{\rm ring}, \sigma) & \text{if } f_{\rm ring} \le f < f_{\rm cut} \end{cases}$$

Transition frequencies

'Spread' of the Lorentzian

Cutoff frequency

$$\Psi_{\rm eff}(f) = 2\pi f t_0 + \phi_0 + \psi_0\, f^{-5/3} + \psi_2\, f^{-1} + \psi_3\, f^{-2/3} + \psi_4\, f^{-1/3} + \psi_6\, f^{1/3}.$$

Phasing coefficients

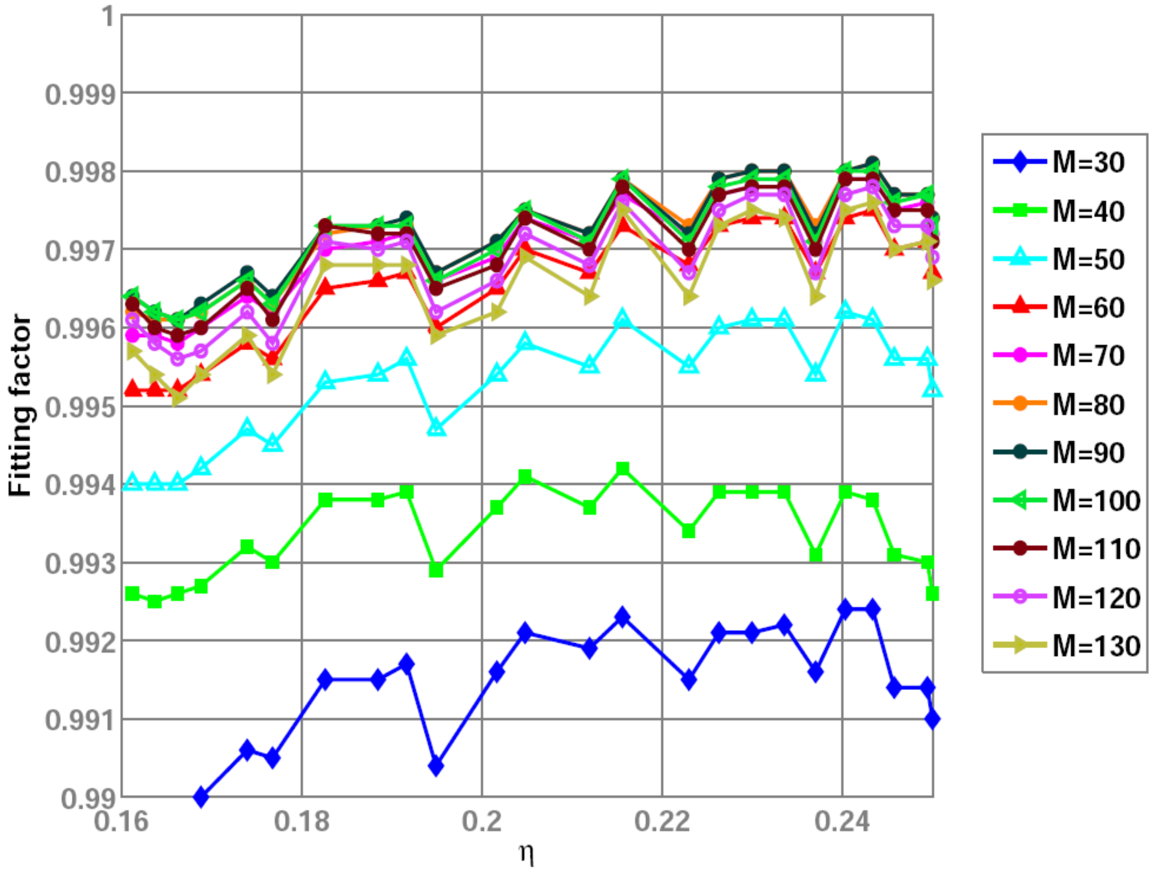
Characterising the quality of the phenomenological waveforms

- **Fitting factor** Ratio of the SNR that can be achieved with suboptimal template to the SNR obtained with an optimal template -- overlap between the (normalized) template and signal, maximized over the parameters of the template.

$$\mathrm{FF}\left(h(f,\boldsymbol{\mu}),u(f,\boldsymbol{\alpha})\right)=\frac{\max_{\boldsymbol{\alpha}}\langle h(f,\boldsymbol{\mu}),u(f,\boldsymbol{\alpha})\rangle}{\sqrt{\langle h(f,\boldsymbol{\mu}),h(f,\boldsymbol{\mu})\rangle}\sqrt{\langle u(f,\boldsymbol{\alpha}),u(f,\boldsymbol{\alpha})\rangle}},$$

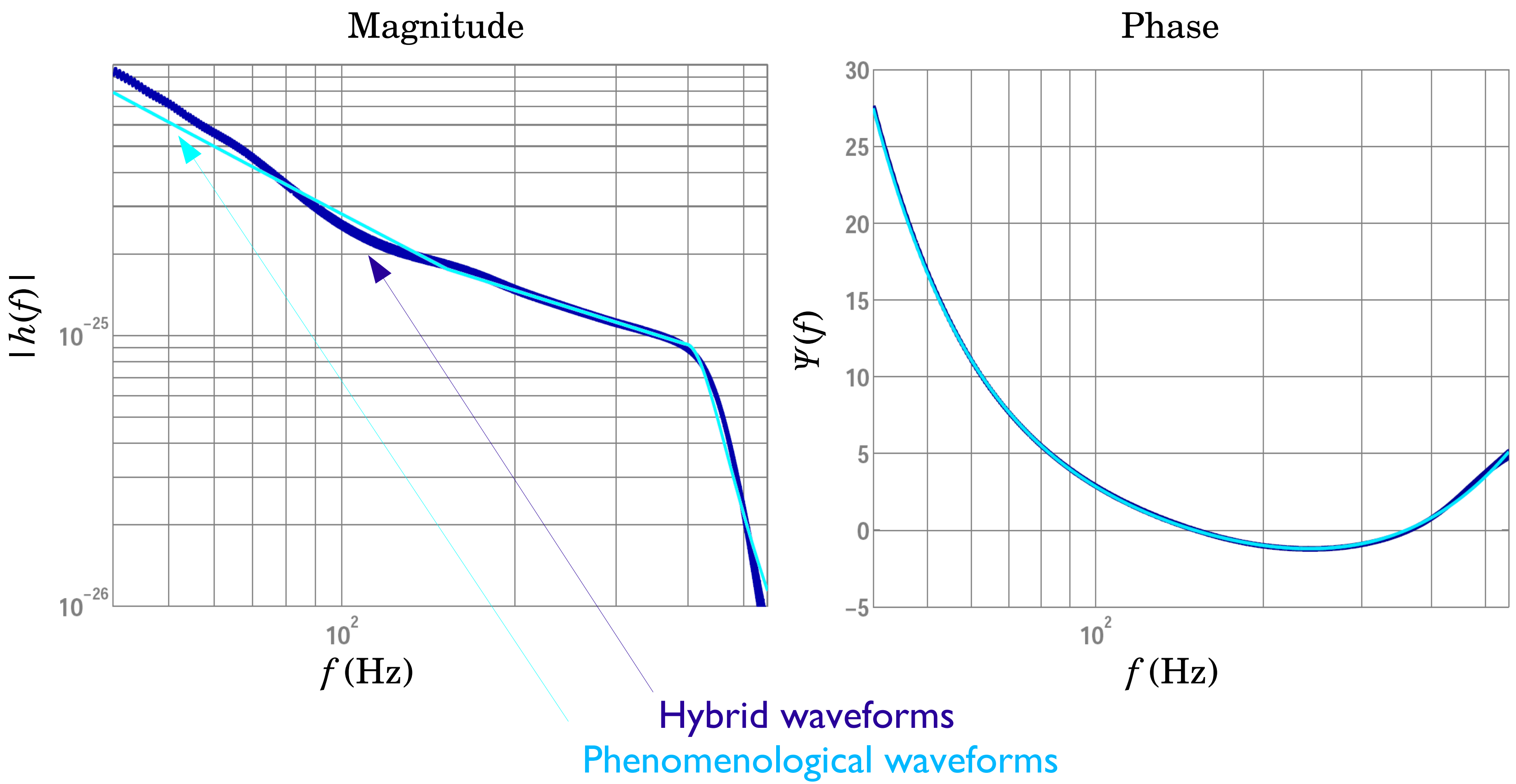
Fitting factors with the hybrid waveforms

- **Hybrid waveforms**
  Jena UM simulations $(0.16 \leqslant \eta \leqslant 0.25)$ + 3.5PN inspiral.
- **Noise spectrum**
  Initial LIGO design.



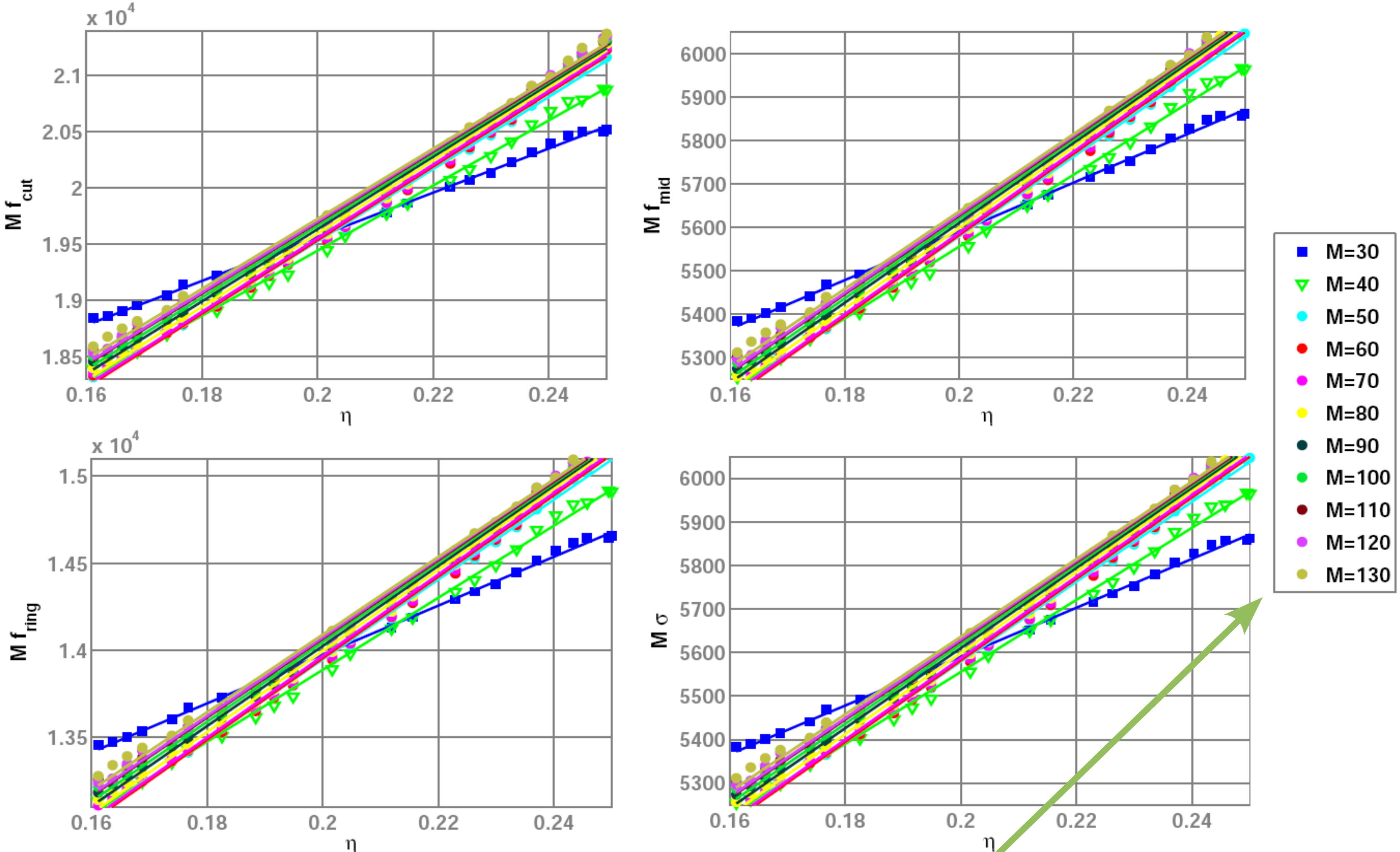
Hybrid waveforms and the 'best-matched' templates

- $M = 40\ M_\odot$, $\eta = 0.25$ system, Initial LIGO noise spectrum.



From phenomenological parameters to physical parameters

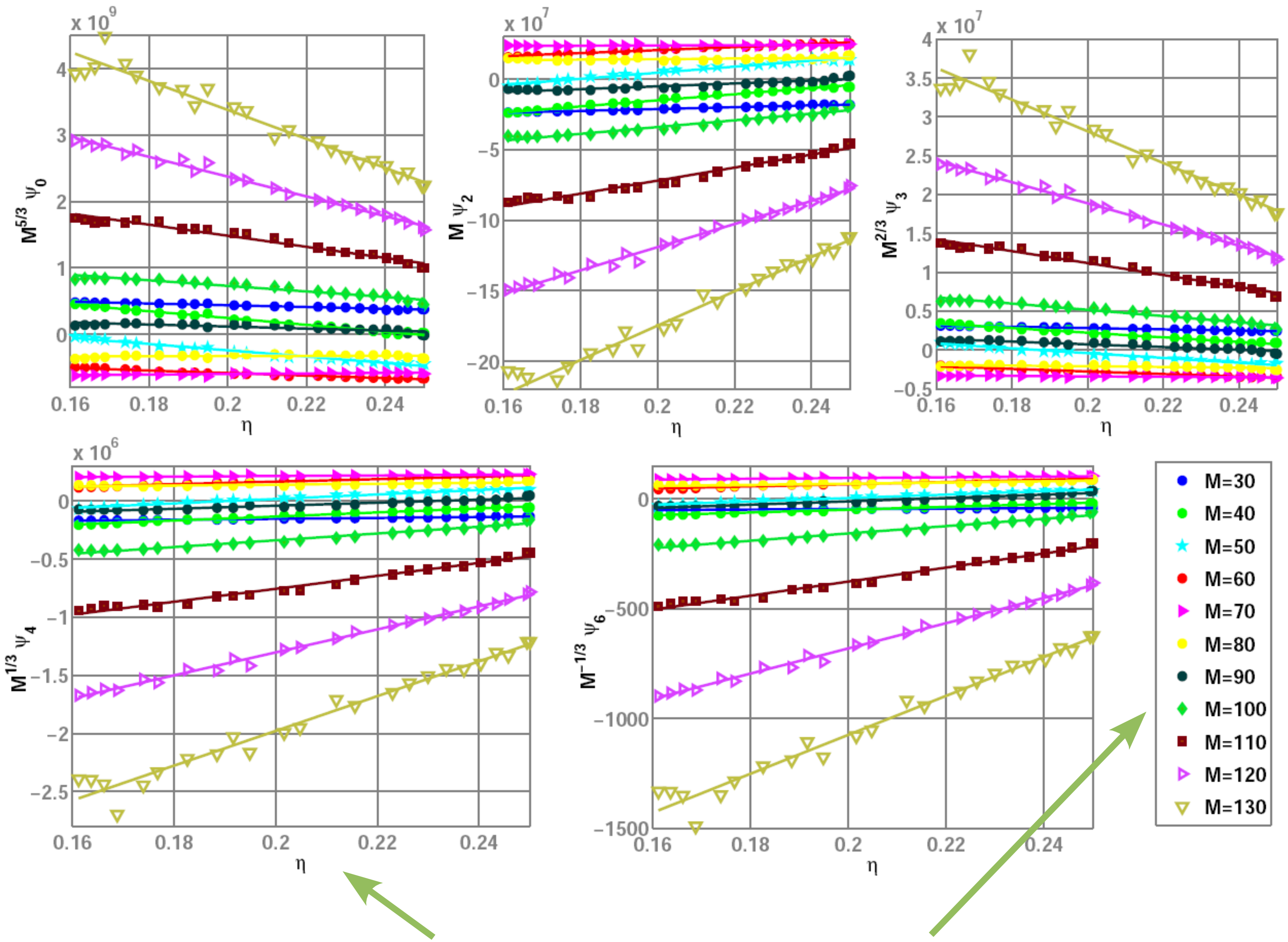
Best-matched phenomenological parameters



Physical parameters of the binary

From phenomenological parameters to physical parameters

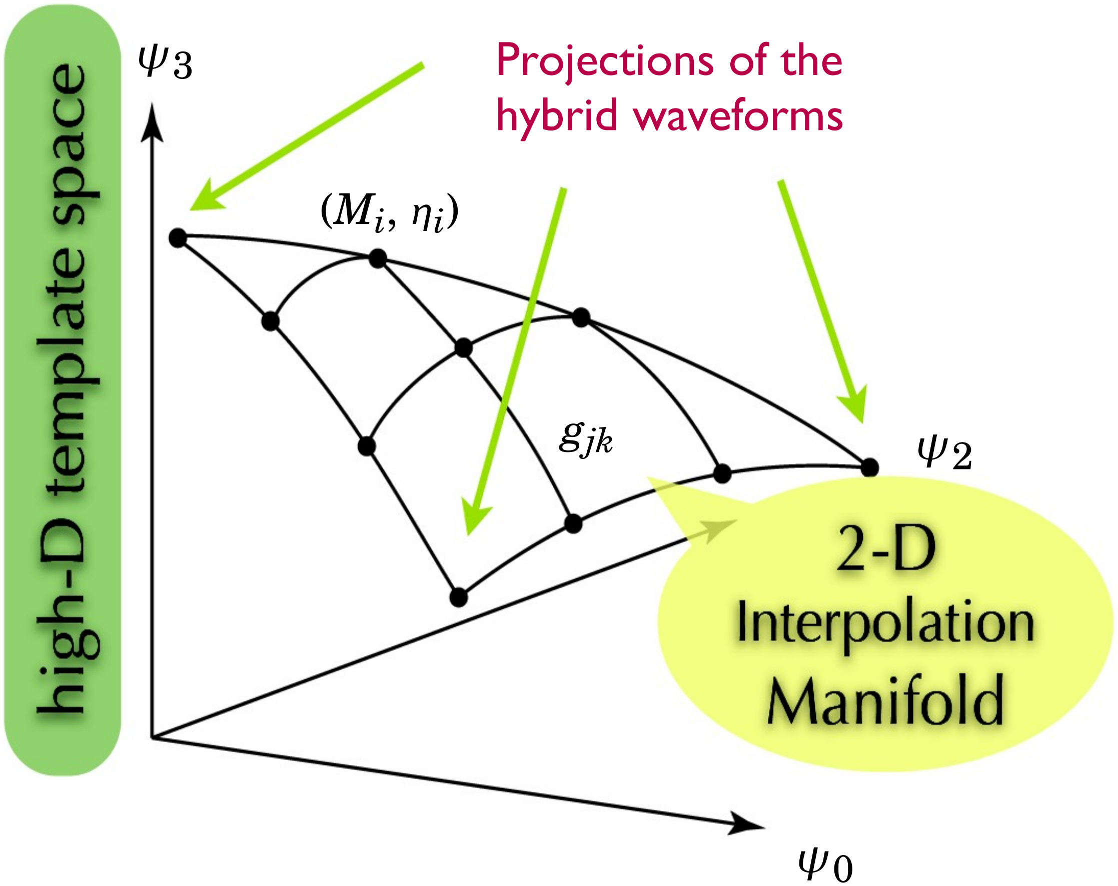
Best-matched phenomenological parameters



Physical parameters of the binary

Template bank

- Re-parametrize the templates in terms of M and η. The template family is a two dimensional manifold embedded in a higher dimensional space.
- A metric can be defined on this manifold, which gives the notion of distance. This can be used to lay down templates allowing a given missmatch between neighboring templates.

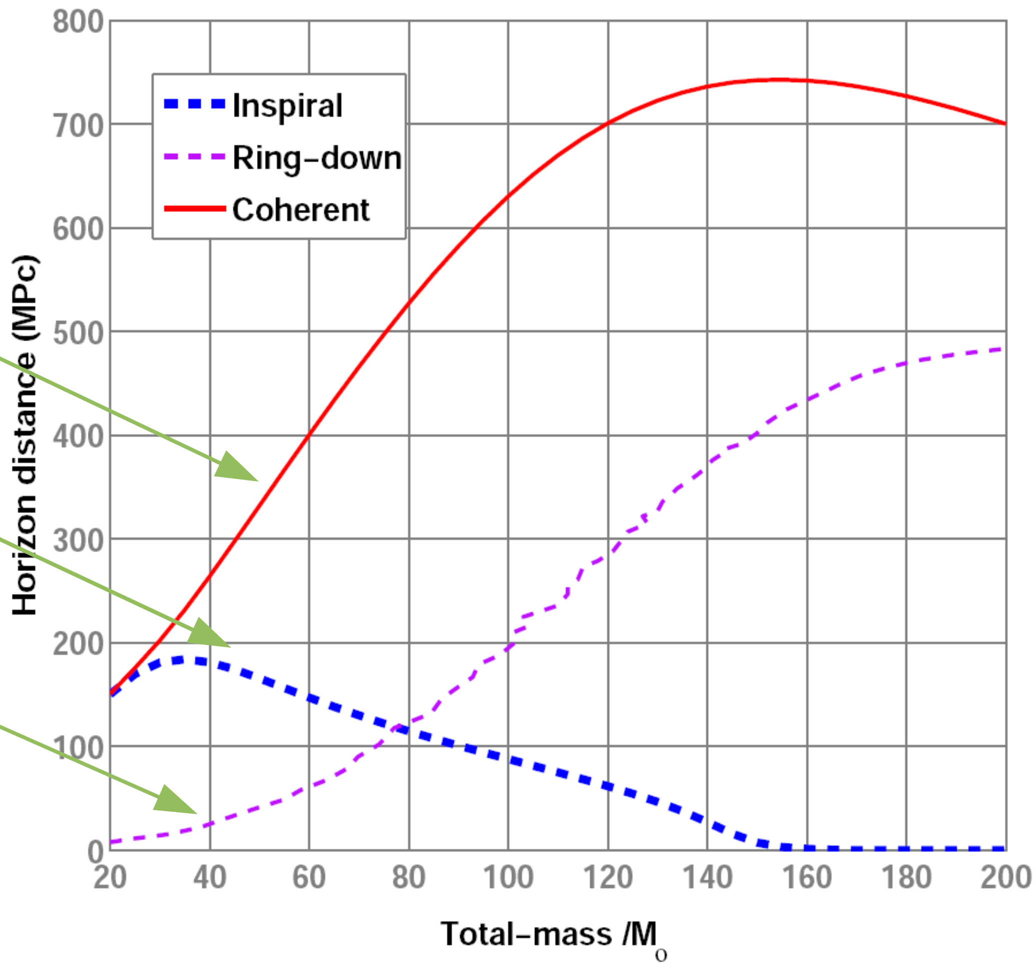


Sensitivity of the search

Using inspiral+merger+ring-down templates

Using standard PN templates (truncated at ISCO)

Using ring-down templates



- Effective distance to optimally-oriented systems which can produce an optimal SNR of 8 at Initial LIGO.

Summary

- Recent progress in Numerical Relativity in modelling the non-perturbative merger phase of the binary BH coalescence problem.
- Constructed a set of hybrid waveforms by matching PN and NR predictions – waveforms containing the inspiral, merger, and ring-down stages.
- Proposed a phenomenological waveform family which has very good overlaps with the hybrid waveforms. These waveforms can be parametrized in terms of the physical parameters of the system – a two-dimensional template bank (for non-spinning binaries).
- This template bank might enable us to extend the present inspiral searches to higher mass binary black hole systems – increased reach of the current generation of ground based detectors.