A template bank for gravitational waves from coalescing binary black holes

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in collaboration with

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LIGO Seminar CALTECH **27 Nov 2007**

Motivation of the work

- Great progress in analytical and numerical relativity in solving the binary BH problem.

- **Post-Newtonian calculations (non-spinning)**
  - GW phase accurate up to 3.5PN order. Blanchet *et al* (2004)
  - GW Amplitude up to 2.5PN order. Arun *et al* (2004)
  - Dominant (l=m=2) mode amplitude up to 3PN order. Kidder (2007)

- **Numerical simulations (non-spinning)**
  - "Breakthrough" simulations. Pretorius (2005), Campanelli (2006), Baker (2006)
  - Unequal-mass binaries, kicks. Jena, AEI, Goddard, UTB-RIT, Penn State...
  - Highly accurate waveforms. Caltech-Cornell, Jena
  - Many more exciting things!

Motivation of the work

- Gravitational waveforms from all the three (inspiral, merger and ring down) stages can be accurately computed.

- Finally allows us to *coherently* search for all three stages of the binary BH coalescence *using a single template family*.

Advantage Increased SNR due to coherent search over all stages → improved sensitivity → improved event rate.

Advantage More "structure" in the template waveform → harder for noise artifacts to mimic the template → potential reduction in the false alarm rate (speculation!).

Advantage Improved SNR and complex structure in the waveform → improvement in the parameter estimation (work in progress).

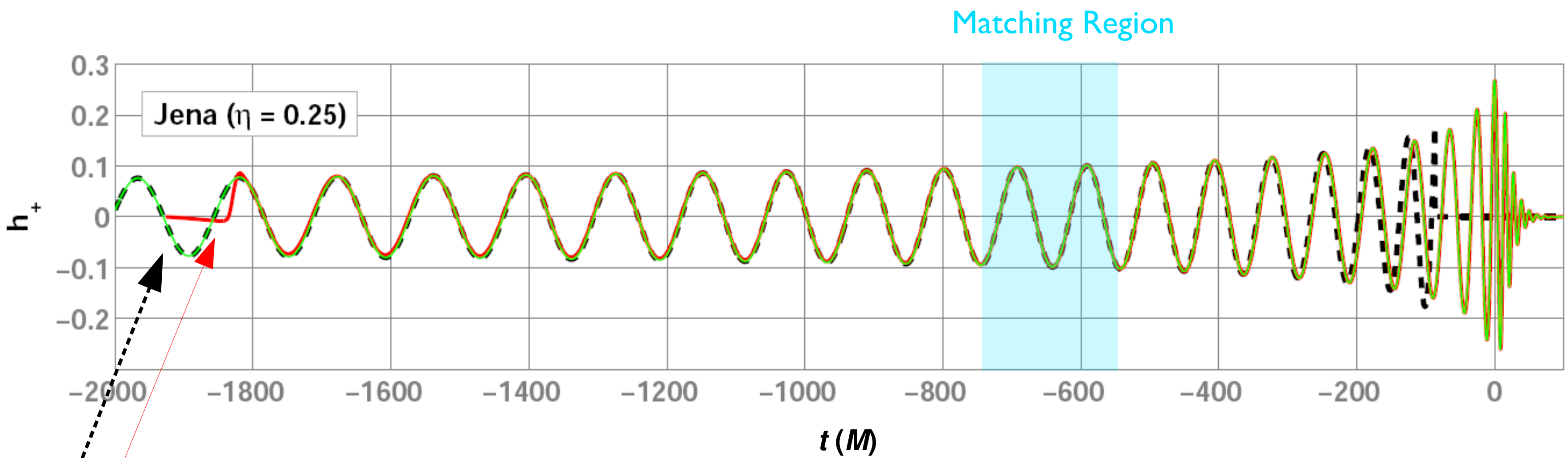
Recent progress in constructing waveform templates

- Buonanno *et al* Match effective-one-body (EOB) waveforms with NR waveforms by adding a 4PN correction to the EOB waveforms. arXiv:0706.3732[gr-qc]
- Damour *et al* Modify EOB framework to produce waveforms close to the NR waveforms. arXiv:0705.2519 [gr-qc]
- Pan *et al* Add a "pseudo 4 PN" term in the PN SPA waveforms; free upper cutoff frequency. arXiv:0704.1964[gr-qc]
- Ajith *et al* Frequency-domain parametrization of the "hybrid" waveforms; mapping to the physical parameter space; interpolated template bank. arXiv:0704.3764 [gr-qc], arXiv:0710.2335 [gr-qc].

A single template bank for BH coalescence (non-spinning)

- **Issue** How to construct a bank of templates?
  - Too expensive to compute a bank of numerical-relativity (NR) waveforms containing all the three stages, densely covering the parameter space.
  - An interpolated template bank (with parametrized analytical waveforms) which has very good overlap with the "target signals".
- **Issue** How to construct the "target" signals?
  - Need waveforms containing all three stages of the BH coalescence - too expensive to (numerically) evolve the binary from very large separations.
  - Match PN inspiral waveforms with NR (merger + ring down) waveforms in a region where both calculations are valid; thus construct "hybrid waveforms".

Matching PN and NR waveforms



PN *Restricted* 3.5PN TaylorT1 waveform

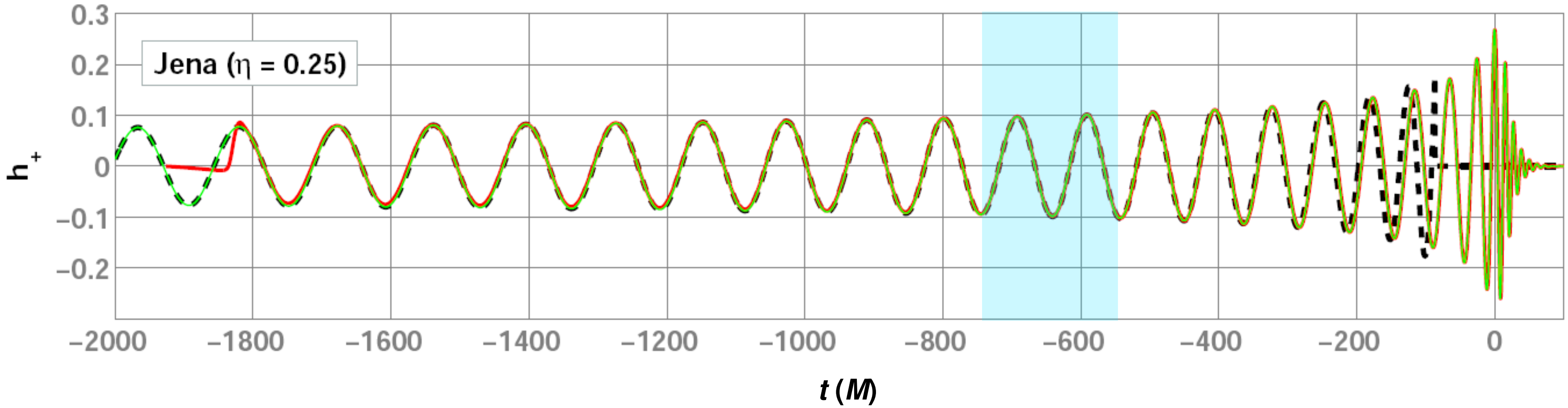
NR Jena equal-mass simulation

- Minimise the "distance" between PN and NR waveforms over a matching region $t_1 < t < t_2$

$$\delta \equiv \left[\int_{t_1}^{t_2} \left| \mathrm{h}^{\mathrm{PN}}(t, \boldsymbol{\mu}) - a\, \mathrm{h}^{\mathrm{NR}}(t, \boldsymbol{\mu}) \right|^2 \mathrm{d}t \right]$$

- Minimisation is carried out over the "extrinsic" parameters $\boldsymbol{\mu} = \{t_0, \phi_0\}$ and an amplitude scaling factor a, keeping the "intrinsic" parameters (M, η) of both the waveforms the same.

Constructing hybrid waveforms



- Combine NR waveforms with the best-matched PN waveforms

$$\mathsf{h}^{\rm hyb}(t,\boldsymbol{\mu}) \equiv a_0\,\tau(t)\,\mathsf{h}^{\rm NR}(t,\boldsymbol{\mu}) + (1-\tau(t))\,\mathsf{h}^{\rm PN}(t,\boldsymbol{\mu}_0)$$

Best-matched amplitude scale

Best-matched "extrinsic" parameters

Weighting function

$$\tau(t) \equiv \begin{cases} 0 & \text{if } t < t_1 \\ \frac{t-t_1}{t_2-t_1} & \text{if } t_1 \le t < t_2 \\ 1 & \text{if } t_2 \le t. \end{cases}$$

How robust is the matching?

- NR (PN) wave can be erroneous in the early (late) inspiral. Constructed hybrid waveform can be highly dependent on the matching region chosen.
  - But, if the hybrid waveforms constructed using different matching regions are "close" to each other, our analysis is not heavily dependent on this choice. The closeness can be measured by *overlap* (noise-weighted inner product).

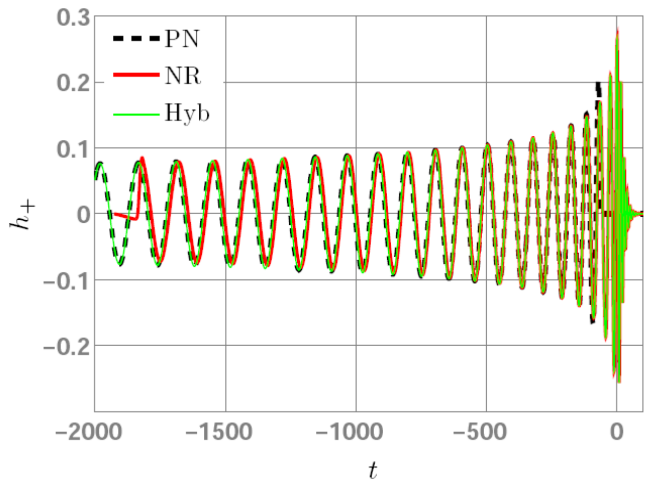
- A plausible test of the robustness of the matching:
  - Compute the overlap between hybrid waveforms h_n and $h_{n\text{-}1}$

$$\mathcal{M}(h_n, h_{n-1}) = \max_{t_0} \left[4\,\mathrm{Re} \int_0^\infty \frac{h_n(f)\, h_{n-1}^*(f)\, e^{\mathrm{i}2\pi f t_0}\, \mathrm{d}f}{S_h(f)} \right]$$

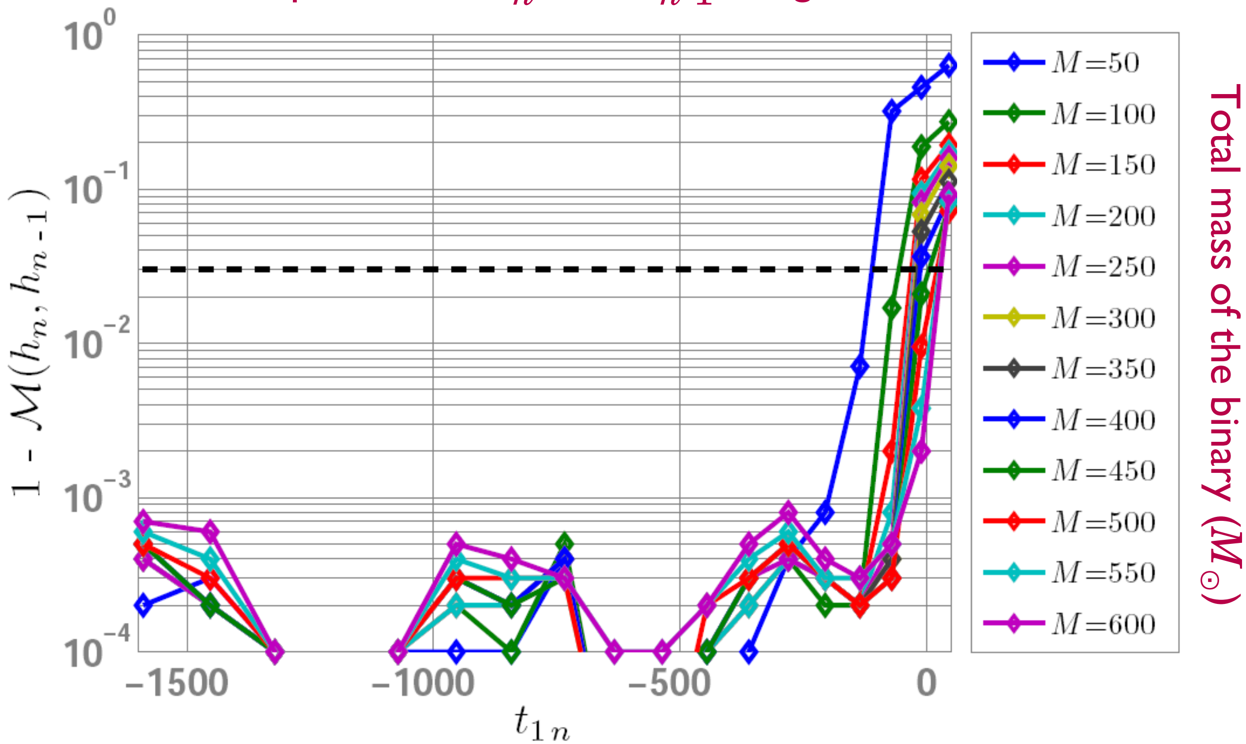
Overlap > 0.97 (say) is an indication of the robustness of h_n.

Hybrid wave constructed by matching PN and NR waves at the *n*th cycle of the NR wave.

Testing the robustness of the matching

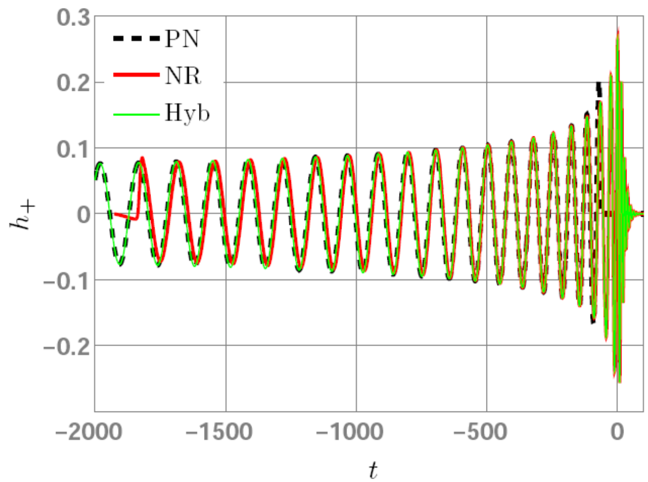


Overlap between h_n and $h_{n\text{-}1}$ using LIGO noise curve



Start time of the matching region

Testing the robustness of the matching



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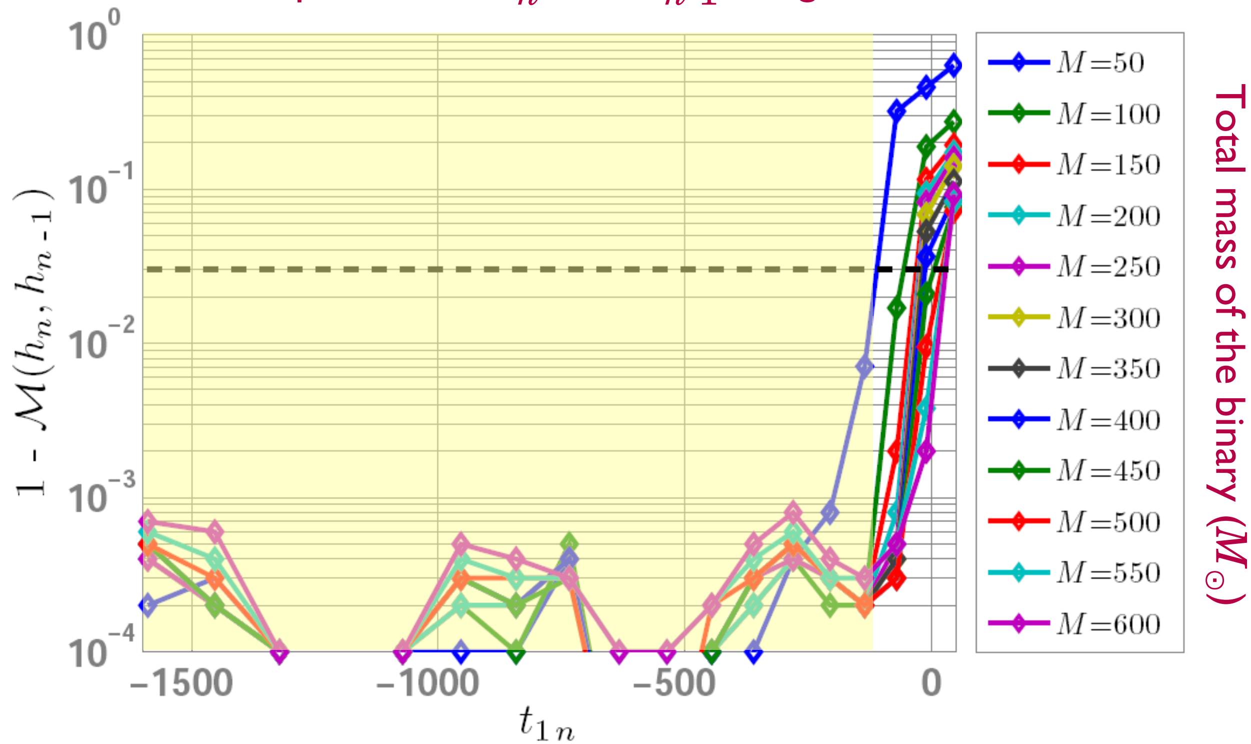
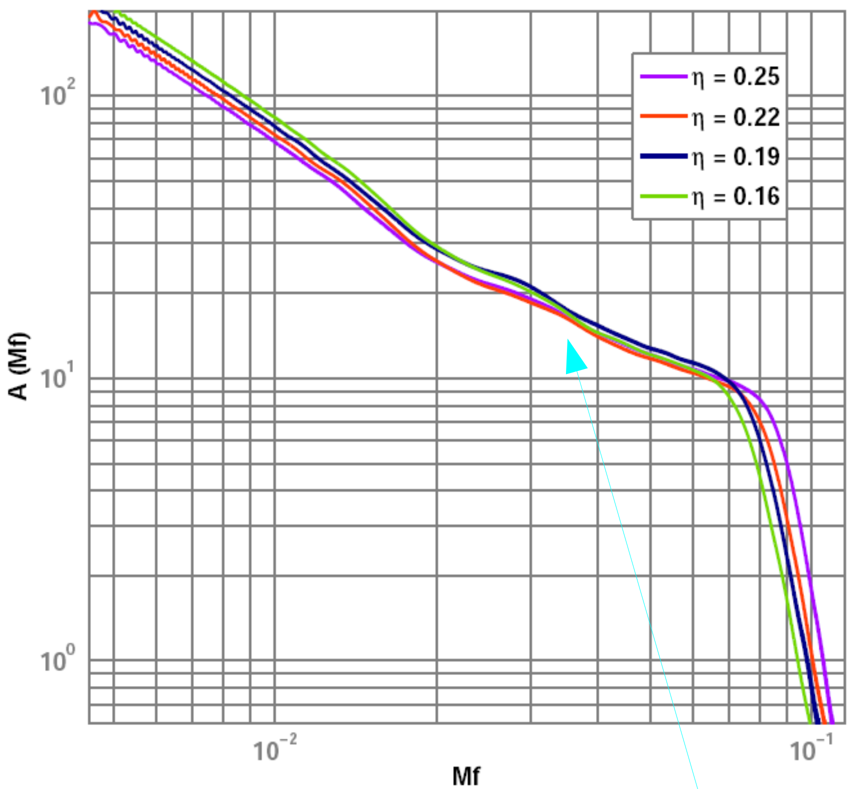
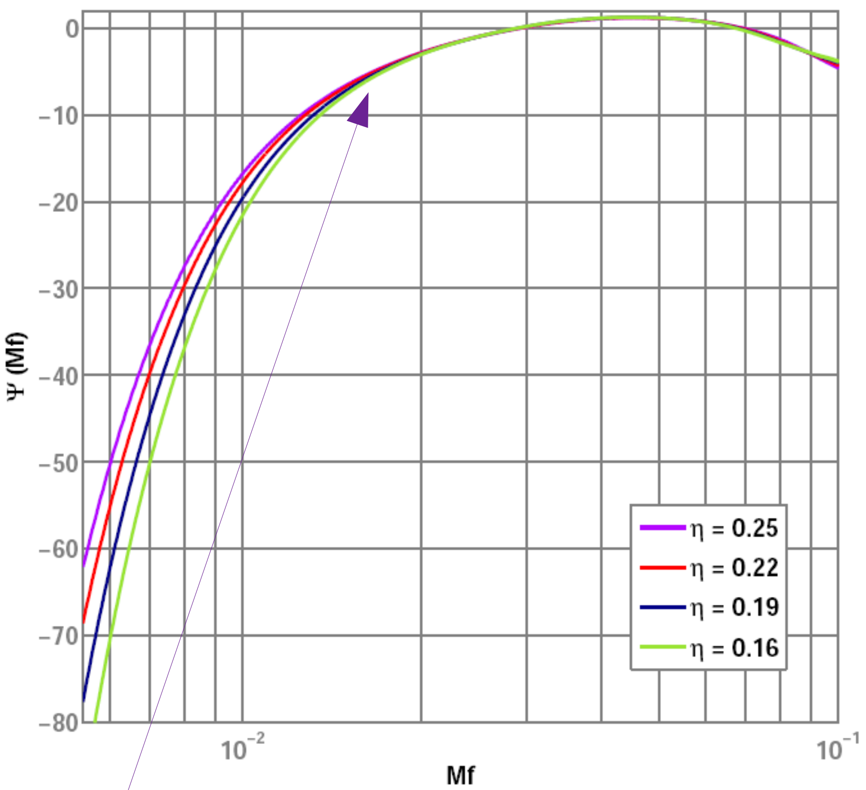


Start time of the matching region

- PN-NR matching is robust in matching regions with $t_1 < -150\ M$.

Hybrid waveforms in the Fourier domain

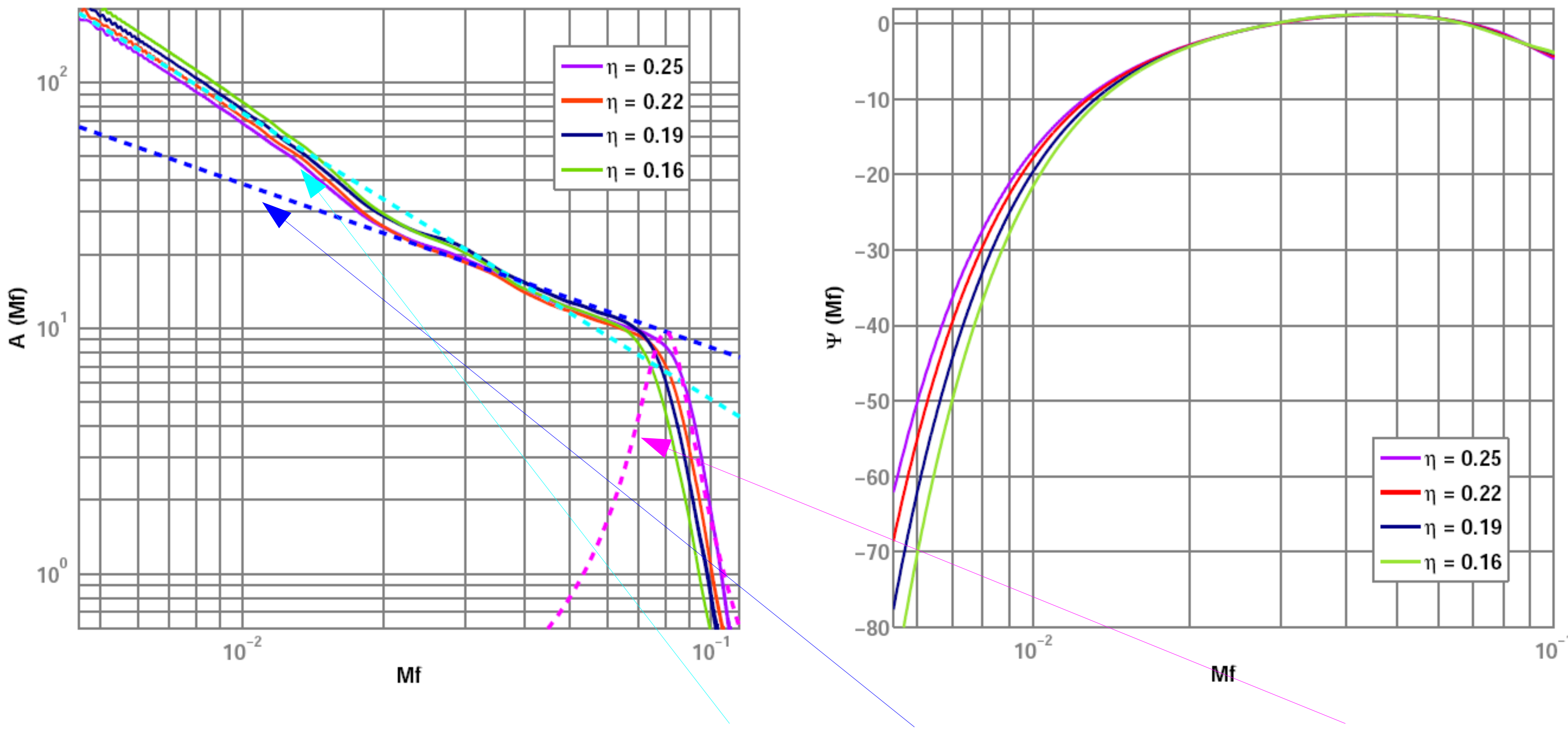




- Fourier domain magnitude and phase of hybrid waveforms.

PN *Restricted* 3.5PN TaylorT1 waveform
NR Jena unequal-mass simulations

A phenomenological parametrization



- **Magnitude** by two power-laws ($f^{-7/6}$ and $f^{-2/3}$) and a Lorentzian $L(f_{\rm ring}, \sigma)$.
- **Phase** Expansion in powers of f (motivation from the SPA).

Phenomenological waveforms

- The phenomenological waveform:

$$u(f) = \mathcal{A}_{\rm eff}(f)\, e^{i\Psi_{\rm eff}(f)}$$

where

$$\mathcal{A}_{\rm eff}(f) = \begin{cases} (f/f_{\rm merg})^{-7/6} & \text{if } f < f_{\rm merg} \\ (f/f_{\rm merg})^{-2/3} & \text{if } f_{\rm merg} \le f < f_{\rm ring} \\ w\,\mathcal{L}(f, f_{\rm ring}, \sigma) & \text{if } f_{\rm ring} \le f < f_{\rm cut} \end{cases}$$

$$\Psi_{\rm eff}(f) = 2\pi f t_0 + \varphi_0 + \sum_{k=0}^{7} \psi_k\, f^{(k-5)/3}$$

Phenomenological waveforms

- The phenomenological waveform:

Parametrized by
4 amplitude params + 6 phase params

$$u(f) = \mathcal{A}_{\rm eff}(f)\, e^{i\Psi_{\rm eff}(f)}$$

where

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Transition frequencies

'Spread' of the Lorentzian

Cutoff frequency

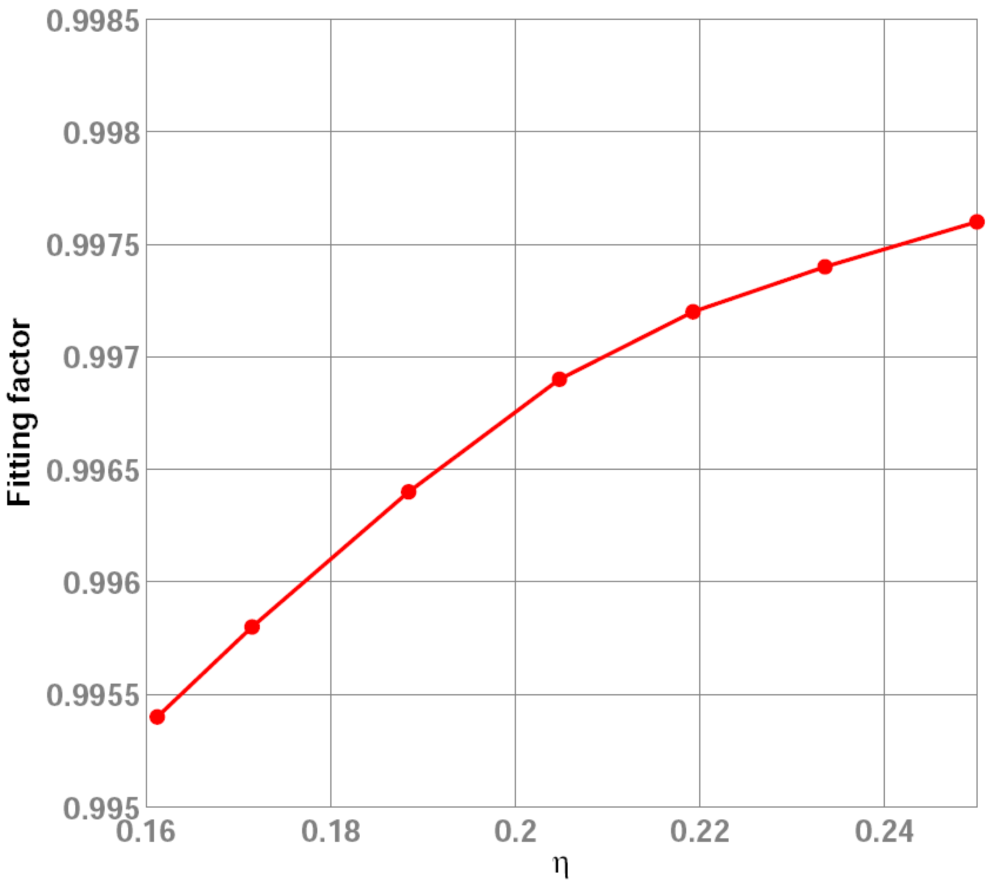
$$\Psi_{\rm eff}(f) = 2\pi f t_0 + \varphi_0 + \sum_{k=0}^{7} \psi_k\, f^{(k-5)/3}$$

Phasing coefficients (motivated from SPA)

Fitting factors with the hybrid waveforms

- **Hybrid waveforms**
  Jena UM simulations
  $(0.16 \leqslant \eta \leqslant 0.25)$ +
  3.5PN inspiral.

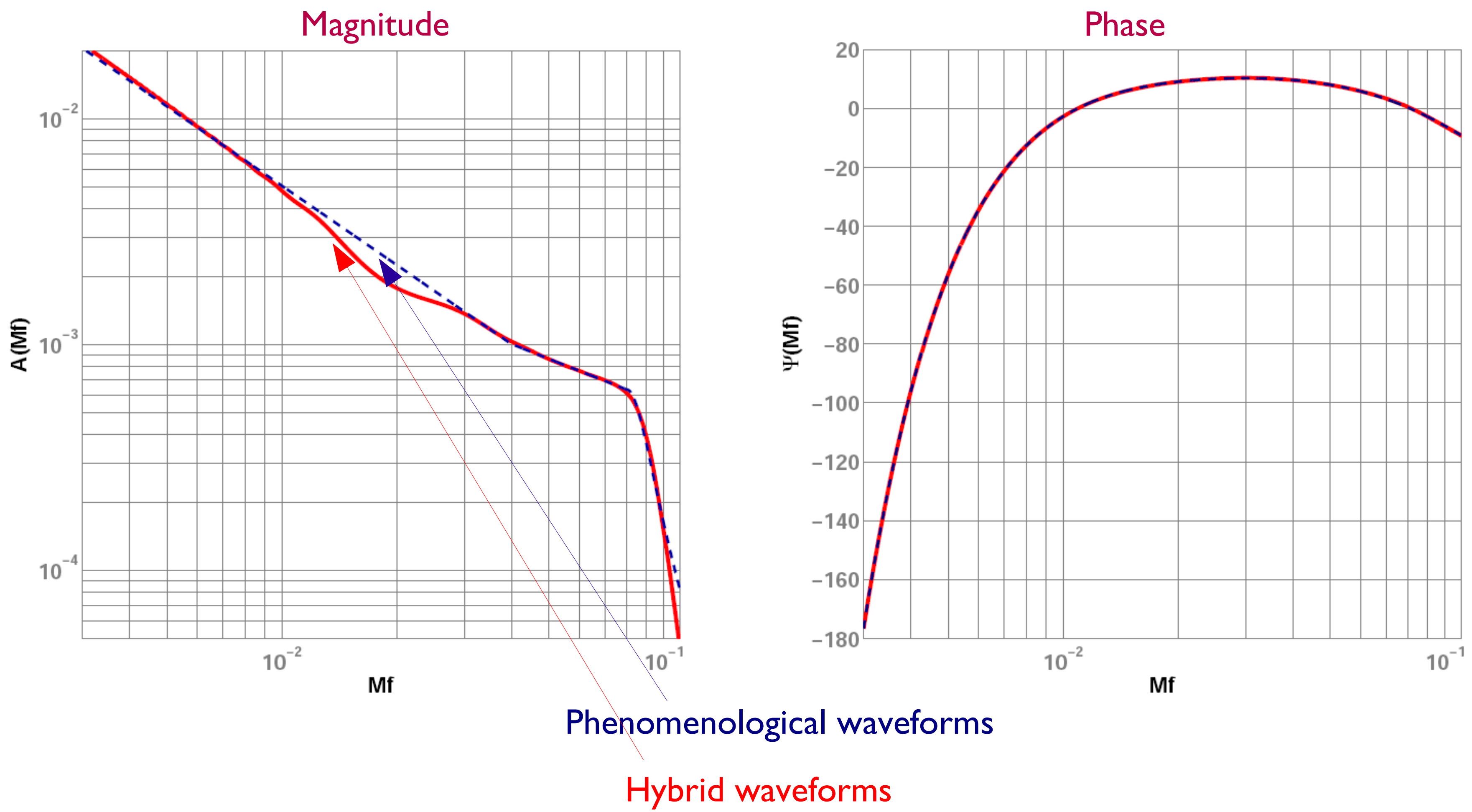
- **Noise spectrum**
  White noise.



Fitting factor Overlap between the (normalized) template and signal, maximized over the parameters of the template.

Hybrid waveforms and the "best-matched" templates

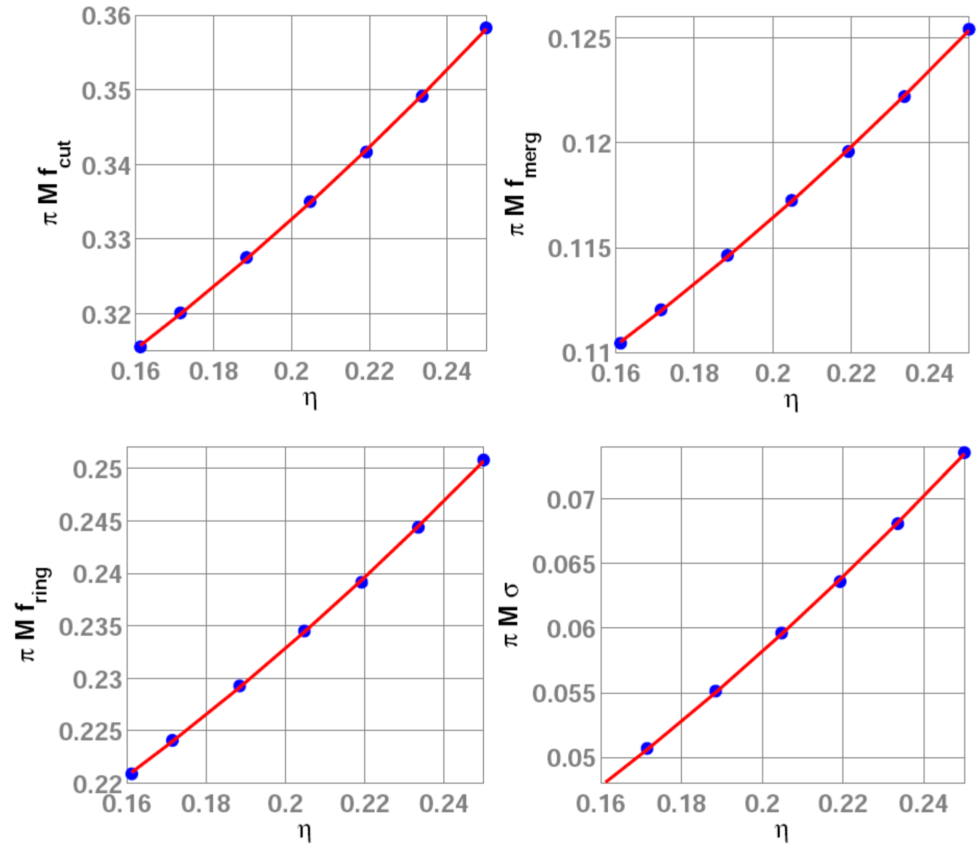
- Equal-mass system, White noise spectrum.



From phenomenological parameters to physical parameters

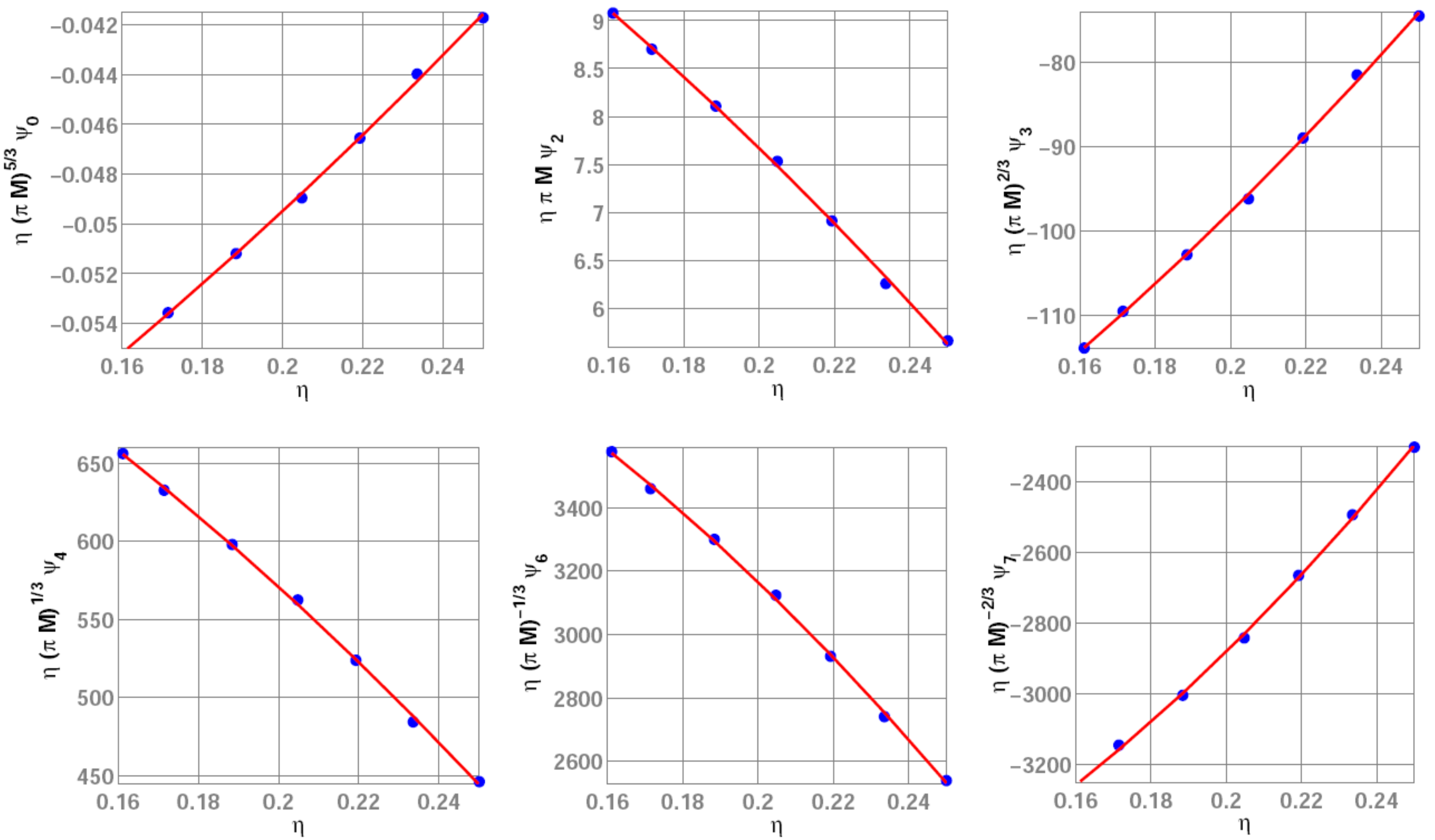
- Possible to reparametrize the “best-matched” phenomenological waveforms in terms of the physical parameters of the binary.

Best-matched amplitude parameters



Symmetric mass-ratio of the binary

From phenomenological parameters to physical parameters



Best-matched phase parameters

Symmetric mass-ratio of the binary

Reparametrization

- "Best-matched" phenomenological parameters can be written in terms of the physical parameters of the binary as

Polynomial fits to the amplitude parameters

$$\alpha_{j\ \mathrm{int}} = \frac{a_j\,\eta^2 + b_j\,\eta + c_j}{\pi M},$$

Polynomial fits to the phase parameters

$$\psi_{k\ \mathrm{int}} = \frac{x_k\,\eta^2 + y_k\,\eta + z_k}{\eta\,(\pi M)^{(5-k)/3}},$$

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Polynomial coefficients (see table)

Parameter	a_k	b_k	c_k
f_{merg}	2.9740×10^{-1}	4.4810×10^{-2}	9.5560×10^{-2}
f_{ring}	5.9411×10^{-1}	8.9794×10^{-2}	1.9111×10^{-1}
σ	5.0801×10^{-1}	7.7515×10^{-2}	2.2369×10^{-2}
f_{cut}	8.4845×10^{-1}	1.2848×10^{-1}	2.7299×10^{-1}

Parameter	x_k	y_k	z_k
ψ_0	1.7516×10^{-1}	7.9483×10^{-2}	-7.2390×10^{-2}
ψ_2	-5.1571×10^{1}	-1.7595×10^{1}	1.3253×10^{1}
ψ_3	6.5866×10^{2}	1.7803×10^{2}	-1.5972×10^{2}
ψ_4	-3.9031×10^{3}	-7.7493×10^{2}	8.8195×10^{2}
ψ_6	-2.4874×10^{4}	-1.4892×10^{3}	4.4588×10^{3}
ψ_7	2.5196×10^{4}	3.3970×10^{2}	-3.9573×10^{3}

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Note These polynomial coefficients have a unique meaning only related to the family of hybrid waveforms considered here. But the formulation should hold for the whole family of non-spinning waveforms.

2D template family

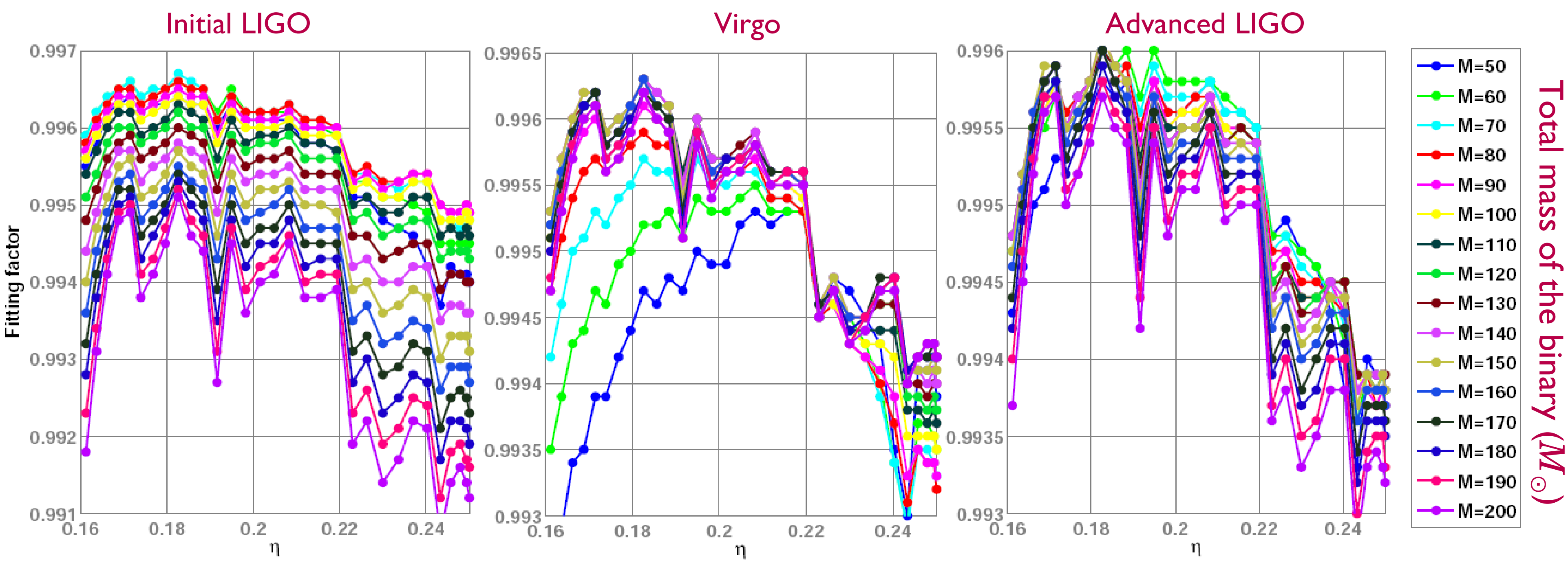
- 2D template family (explicitly parametrized by the physical parameters):

$$A_{\rm eff}(f) \equiv C \begin{cases} \left(\frac{\pi M f}{a_0\eta^2+b_0\eta+c_0}\right)^{-7/6} & \text{if } f < \frac{a_0\eta^2+b_0\eta+c_0}{\pi M} \\ \left(\frac{\pi M f}{a_0\eta^2+b_0\eta+c_0}\right)^{-2/3} & \text{if } \frac{a_0\eta^2+b_0\eta+c_0}{\pi M} \le f < \frac{a_1\eta^2+b_1\eta+c_1}{\pi M} \\ w\,\mathcal{L}\left(f, \frac{a_1\eta^2+b_1\eta+c_1}{\pi M}, \frac{a_2\eta^2+b_2\eta+c_2}{\pi M}\right) & \text{if } \frac{a_1\eta^2+b_1\eta+c_1}{\pi M} \le f < \frac{a_3\eta^2+b_3\eta+c_3}{\pi M}, \end{cases}$$

$$\Psi_{\rm eff}(f) = 2\pi f t_0 + \varphi_0 + \frac{1}{\eta}\sum_{k=0}^{7}(x_k\,\eta^2 + y_k\,\eta + z_k)\,(\pi M f)^{(k-5)/3},$$

$$C = \frac{M^{5/6} f_{\rm merg}^{-7/6}}{d\,\pi^{2/3}}\left(\frac{5\,\eta}{24}\right)^{1/2}.$$

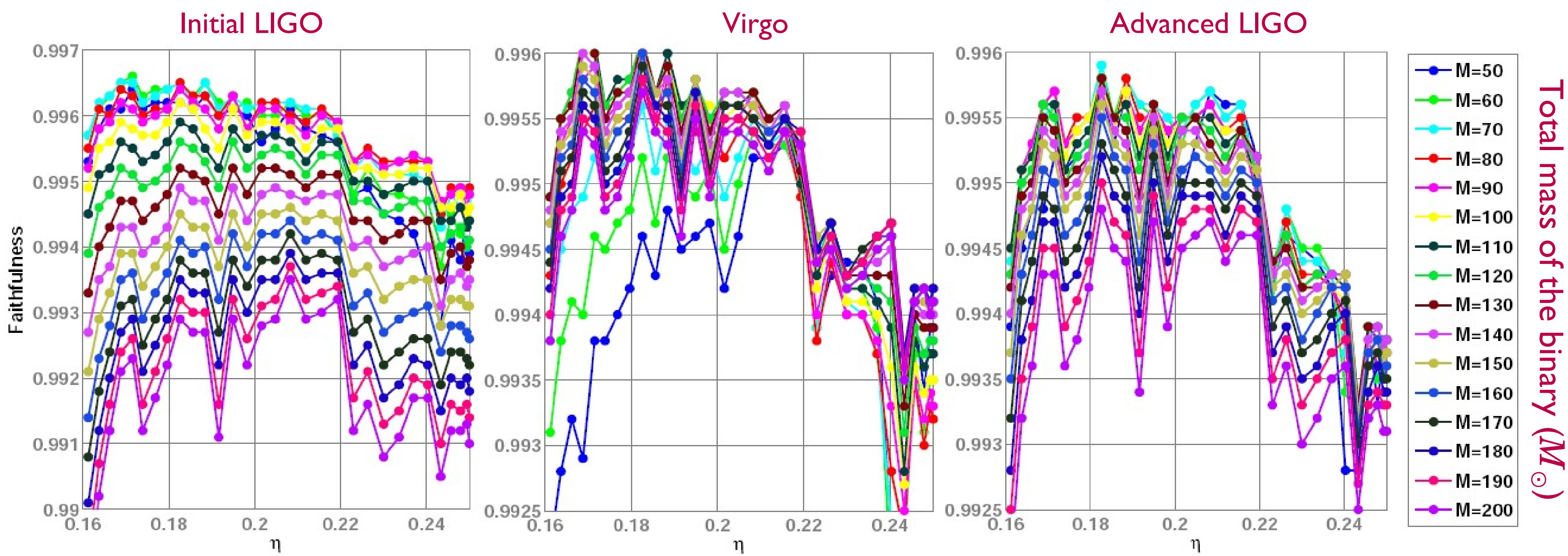
Fitting factor of the 2D template family



Symmetric mass ratio of the binary

- Two-parameter templates with > 0.99 overlaps with the hybrid waveforms!

“Faithfulness” of the 2D template family



Symmetric mass ratio of the binary

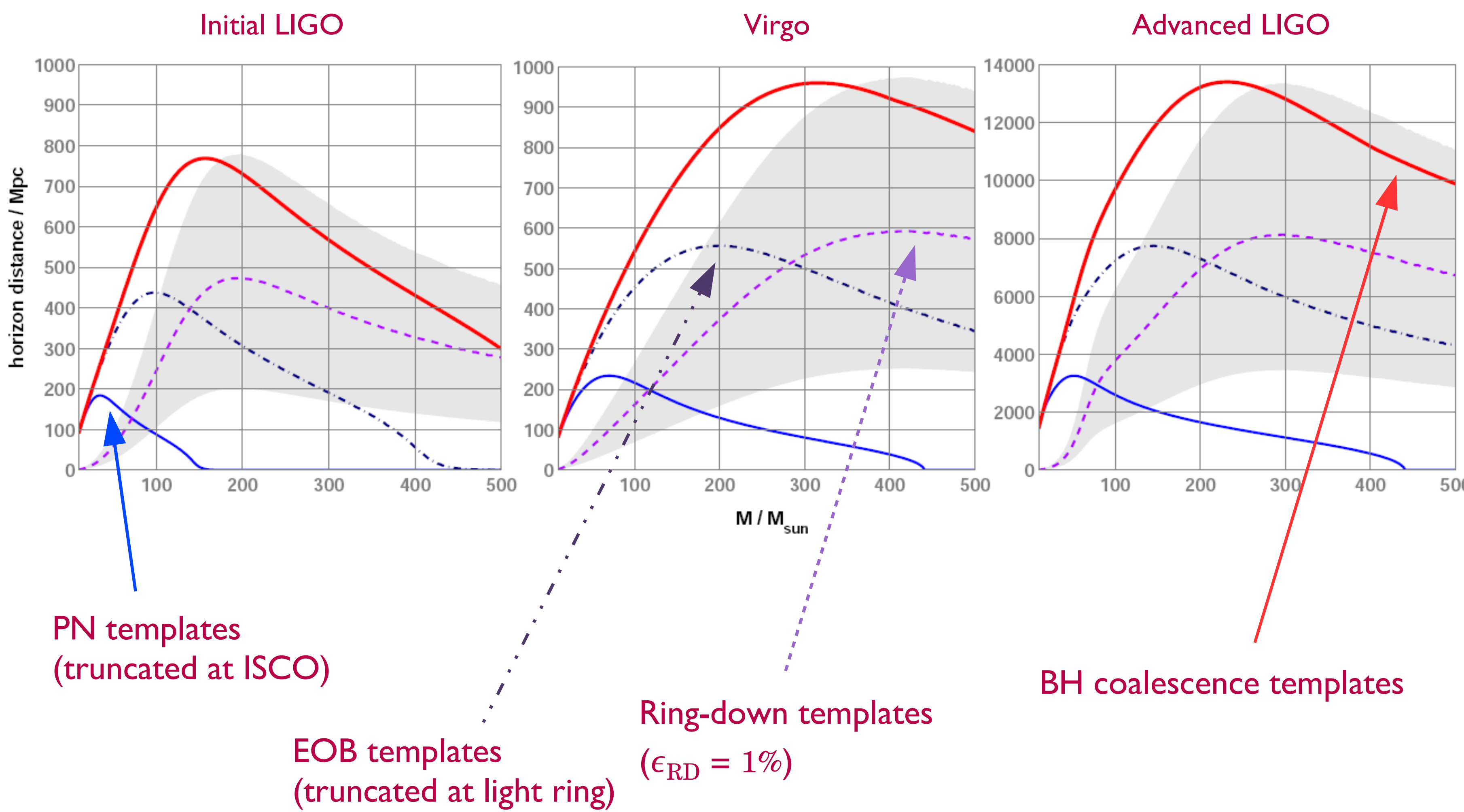
- Template family is “faithful” in estimating the parameters of the binary!

Faithfulness Overlap between the (normalized) template and signal, maximized over only the *extrinsic* parameters (t_0 , ϕ_0) of the template. A measure of the quality of the parameter estimation.

Damour, Iyer, Sathyaprakash 1998

Sensitivity of the search

- Effective distance to optimally-oriented systems producing optimal SNR of 8.



Summary

- Great progress in analytical and numerical relativity in modelling the binary BH coalescence problem. All three stages can be *coherently* searched over.

- "Complete" BBH waveforms (*hybrid* waveforms) are constructed by matching PN and NR waveforms.

- Proposed analytical template family which is very close to the hybrid waveforms. Can be explicitly parametrized in terms of the physical parameters of the system – 2D template bank for non-spinning binaries.

- **Advantages** Improved sensitivity and event rate, Potential reduction in the false alarm rate, Improved parameter estimation.

Future work

- **Work in progress** Template placement using the metric formalism (Owen 1996), Parameter estimation using the "complete" BBH waveforms, Injecting the hybrid waveforms into LIGO data.

- **Future LIGO-related work**

  - Tune the template bank using more accurate NR simulations and PN waveforms. Will be useful to have inputs from Caltech-Cornell NR group.
  - Take into account the higher harmonics → important for higher mass ratios.
  - Search for *intermediate-mass* BH binaries using S5 and S6 data. The most sensitive (in terms of distance reach) search to date. Potential detection, interesting (e.g., stellar dynamics in globular clusters) upper limit.
  - Develop a spinning template bank. Making use of the Caltech effort in constructing the spinning PN bank.

- **Future work related to advanced detectors**

  - Improved parameter estimation → interesting impacts in Cosmological parameter estimation using joint GW-EM observations (Schutz 1986), such as, in constraining the EOS of dark energy using LISA (Holz & Hughes 2005).