



A Phenomenological template bank for black hole coalescence waveforms



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Gravitational waves from coalescing binary black holes

Coalescence of binary black holes (BHs) involves three stages of evolution. Gravitational waveforms from the “*inspiral*”, and “*ring down*” stages can be accurately modelled by approximation techniques in General Relativity. Recent progress in Numerical Relativity (NR) in solving the binary BH problem has enabled us to model the non-perturbative “*merger*” phase also. This enables us to “coherently” search for gravitational waves (GWs) from the three stages of the binary BH coalescence using a single template family.

It is computationally impossible, as to now, to compute enough

NR waveforms so that they densely cover the parameter space of the coalescing binary that will be searched over by the matched filtering technique. As a possible way out, we propose a family of phenomenological waveforms which has very good overlaps with the “target signals”.

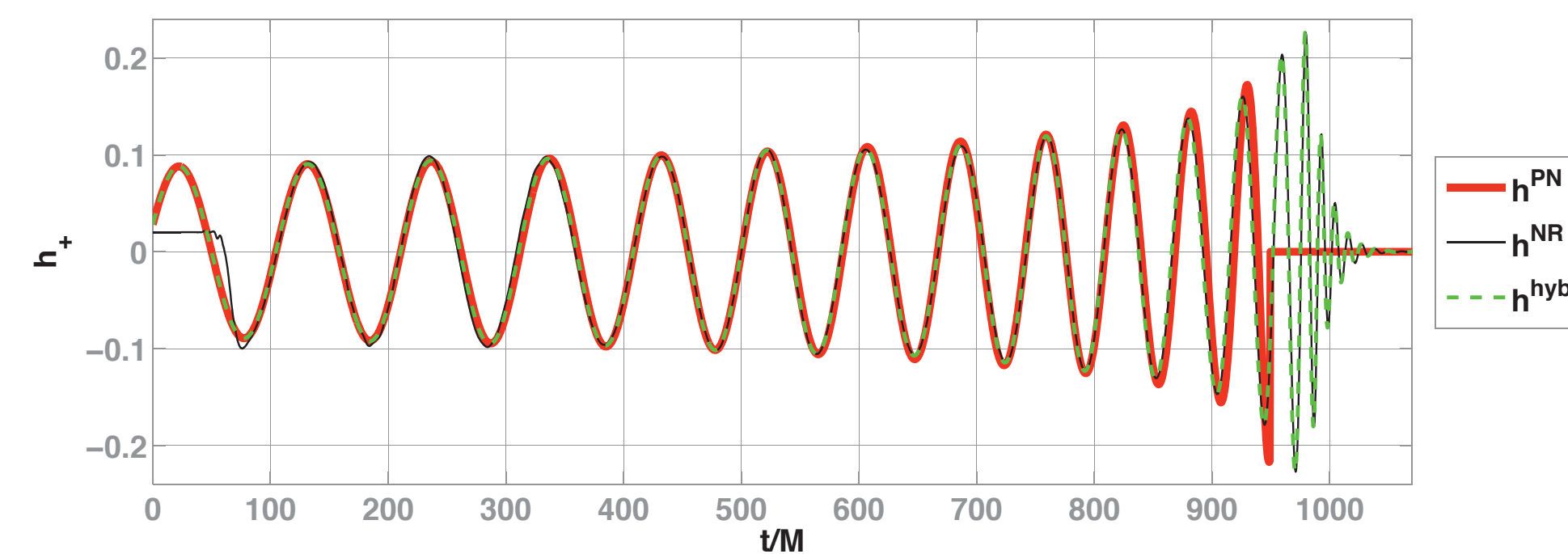


Fig 1 NR waveform from an equal-mass simulation (AEI) and the best-matched 3.5PN waveform. Constructed hybrid waveform is also shown.

Constructing the target signals: Matching PN & NR waveforms

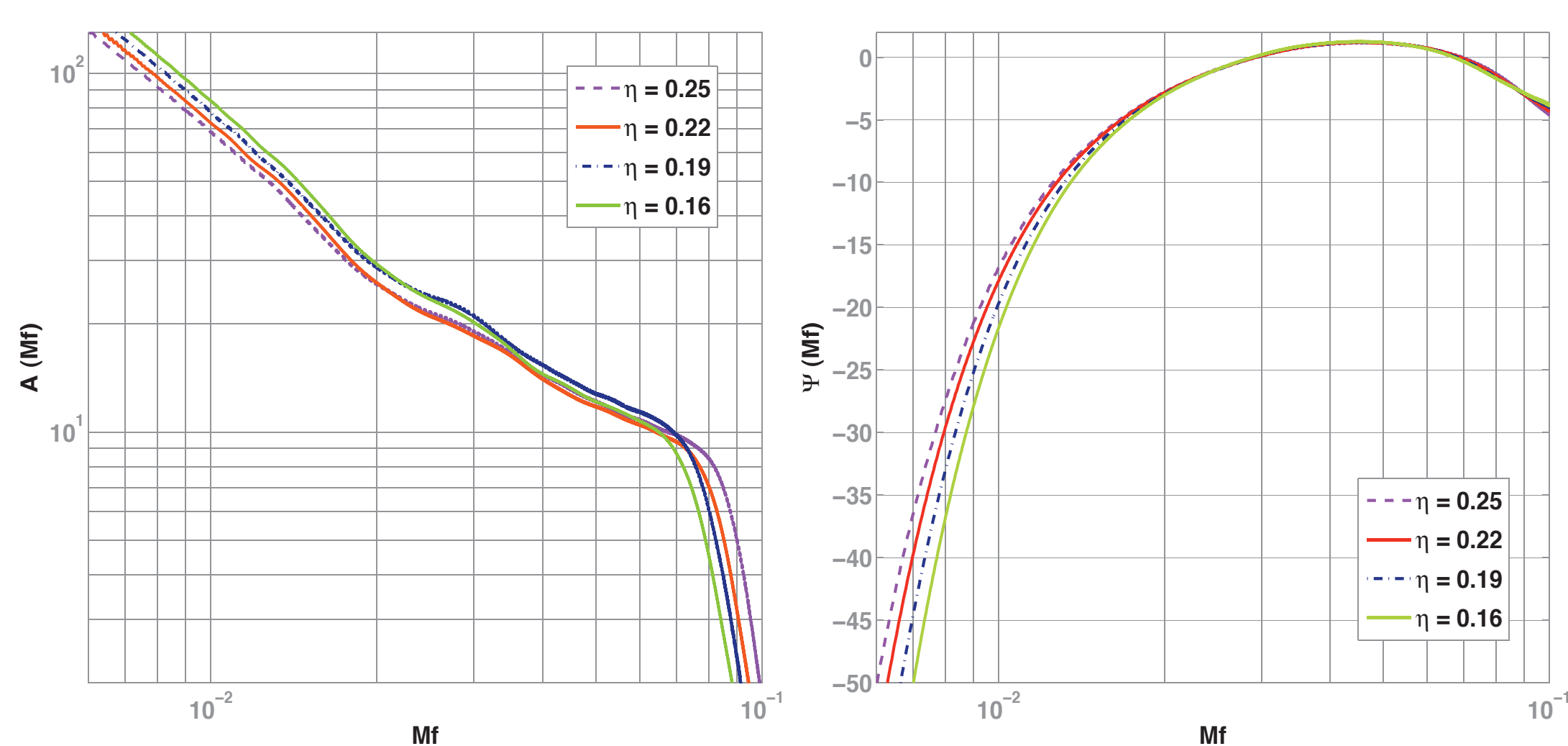


Fig 2 Hybrid waveforms in the frequency domain. These are produced by matching NR waveforms from an unequal mass simulation by Jena with 3.5PN waveforms.

We construct our target signals by matching *restricted* post-Newtonian (PN) waveforms with NR waveforms in an appropriate matching region $t_1 < t < t_2$. The integrated squared difference between the two waveforms is minimized over the extrinsic parameters (t_0, ϕ_0) of the PN waveform and an amplitude scaling factor a . A set of “hybrid waveforms” (Fig 1 & Fig 2.) are constructed by linearly combining the best-matched PN waveforms with NR waveforms.

$$h_{+, \times}^{\text{hyb}} = (1 - \tau) h_{+, \times}^{\text{PN}} + a \tau h_{+, \times}^{\text{NR}}$$

where,

$$\tau \equiv \begin{cases} 0 & \text{if } t < t_1 \\ \frac{t-t_1}{t_2-t_1} & \text{if } t_1 \leq t < t_2 \\ 1 & \text{if } t_2 \leq t \end{cases}$$

A phenomenological waveform family

We propose the following (frequency domain) waveform family, parametrized by the set of phenomenological parameters $\mathbf{A} = \{f_{\text{merg}}, f_{\text{cut}}, f_{\text{ring}}, \sigma\}$ and $\mathbf{P} = \{\psi_0, \psi_2, \psi_3, \psi_4, \psi_6, \psi_7, \psi_9\}$:

$$u(f) \equiv \mathcal{A}_{\text{eff}}(f) e^{i\Psi_{\text{eff}}(f)}$$

$$\mathcal{A}_{\text{eff}}(f) \propto \begin{cases} (f/f_{\text{merg}})^{-7/6} & \text{if } f < f_{\text{merg}} \\ (f/f_{\text{merg}})^{-2/3} & \text{if } f_{\text{merg}} \leq f < f_{\text{ring}} \\ w \mathcal{L}(f, f_{\text{ring}}, \sigma) & \text{if } f_{\text{ring}} \leq f < f_{\text{cut}} \end{cases}$$

$$\Psi_{\text{eff}}(f) = 2\pi f t_0 + \phi_0 + \sum_{k=0}^9 \psi_k f^{(k-5)/3}$$

These phenomenological waveforms, which have very good overlaps with the hybrid waveforms (see Fig 4) can be reparametrised in terms of the physical parameters (M, η) of the

binary (see Fig 3) as

$$\mathbf{A} = (\mathbf{a} \eta + \mathbf{b}) / \pi M, \quad \mathbf{P} = (\mathbf{c} \eta + \mathbf{d}) / (\pi M)^{(5-k)/3}$$

Amplitude parameters

Polynomial coefficients (see Fig 3)

Phase parameters

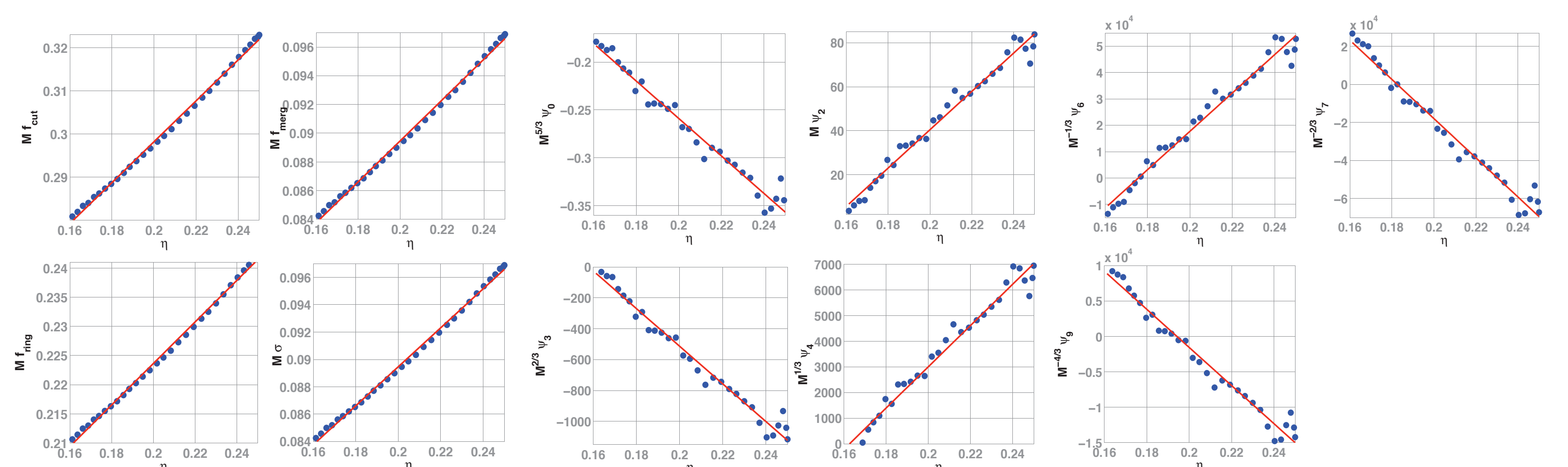


Fig 3 Best-matched phenomenological parameters plotted against the physical parameters of the binary. Horizontal axis shows the symmetric mass ratio, and the parameters are scaled with the total mass. Linear polynomial fits are also shown.

The template bank

Since the phenomenological waveforms can be reparametrised in terms of the physical parameters of the binary, the template bank is, at the end, two-dimensional. A metric can be defined on this manifold, which allows us to lay down templates allowing a certain mismatch.

A preliminary comparison of the “coherent search” with other existing searches (inspiral & ring down) is shown in Fig 5. The figure shows the distance to optimally-oriented, equal mass binaries that can produce an optimal SNR of 8 at Initial LIGO noise spectrum. PN inspiral templates are truncated at Schwarzschild ISCO, and the ring down templates assume that $\epsilon = 0.7\%$ of the BH mass is radiated in the ring down stage.

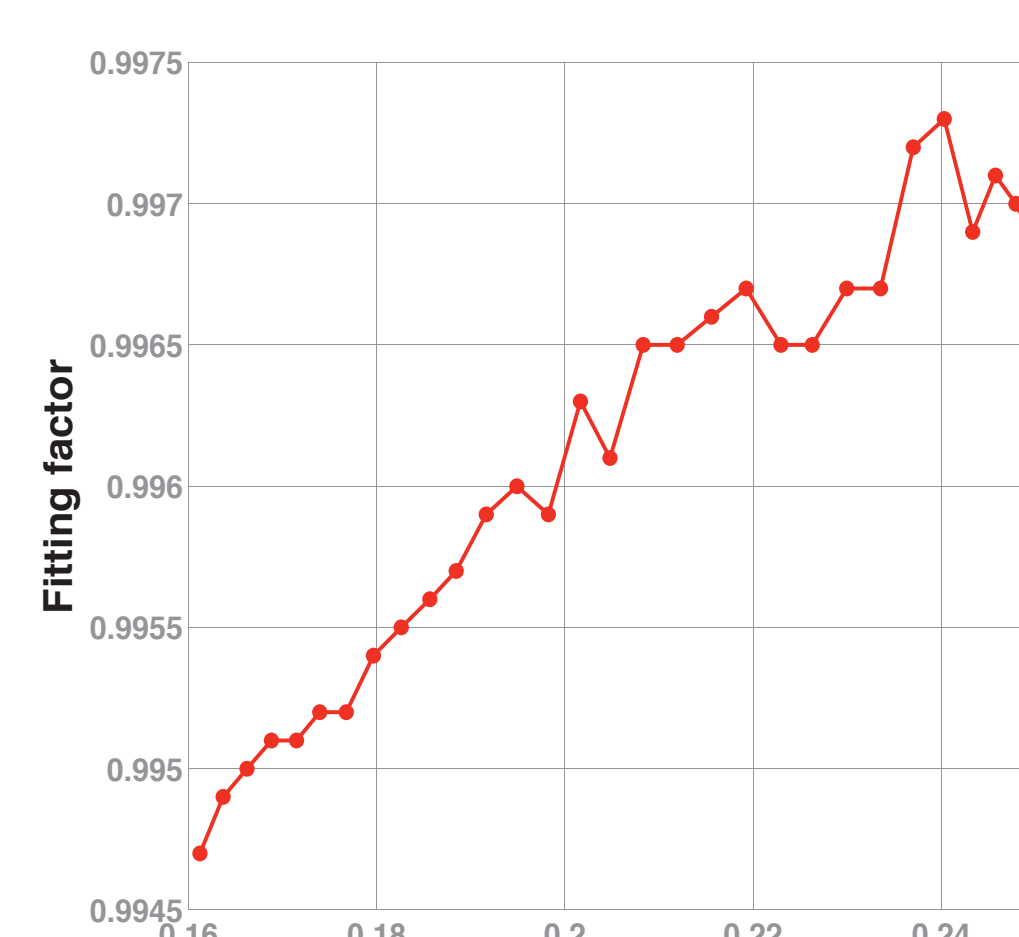


Fig 4 Fitting factors of the phenomenological waveforms with the hybrid waveforms.

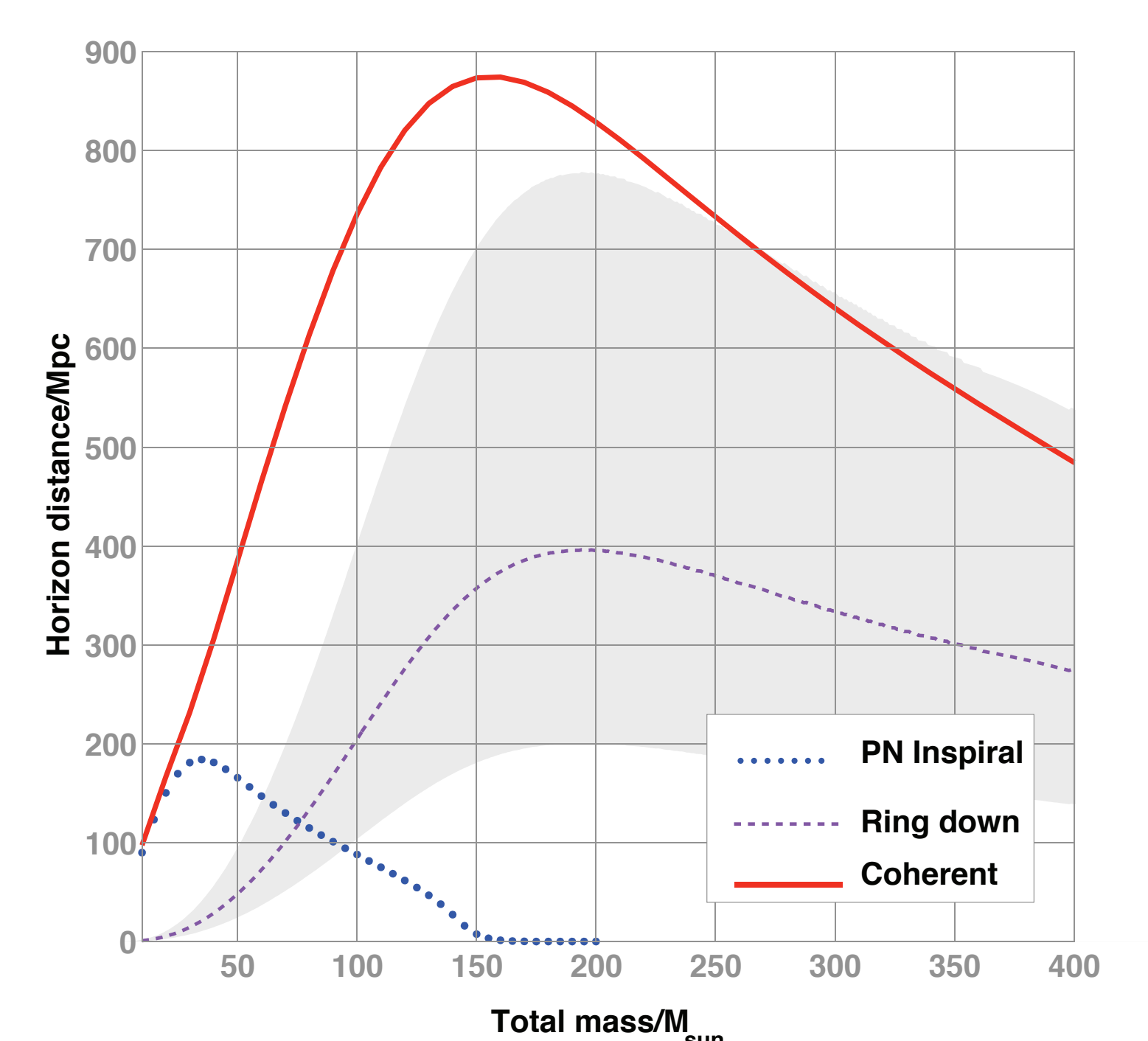


Fig 5 Dotted line: search using PN templates. Dashed line: ring down search (Shaded region correspond to $0.18\% < \epsilon < 2.7\%$). Solid line: ‘coherent’ search.