Implementing the null-stream veto in the network analysis of gravitational-wave bursts

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Null-stream veto

- Time series data from n detectors can be linearly-combined to construct a *null-stream*, which contains no trace of the GWs [Gürsel and Tinto, 1989].
- The null-stream can be used to distinguish between actual GW events and spurious events in a triggered search.
- If the coincident triggers correspond to ..
  - An actual GW burst: Burst signal will cancel out in the null-stream; null-stream will contain only noise.
  - Spurious instrumental bursts: Burst signals will not cancel out in the null-stream; null-stream will contain some excess power.
- This can be tested with a given false-dismissal probability.

Constructing the null-stream

- In the case of widely-separated and misaligned detectors, null-stream is a function of the antenna patterns:

Time-delays

$$\begin{aligned} n(t;\theta,\phi) &= A_{32}(\theta,\phi)\,h_1(t) \\ &+ A_{13}(\theta,\phi)\,h_2(t+\tau_{12}) \\ &+ A_{21}(\theta,\phi)\,h_3(t+\tau_{13}) \end{aligned}$$

Functions of antenna responses

- In the case of two co-located detectors, null-stream is particularly simple:

$$n(t) = h_1(t) - h_2(t)$$

Calibrated data streams

Testing the null-stream hypothesis

- Let $N_k(\theta,\phi) = \mathcal{F}[n(t;\theta,\phi)]\,,\ \ x_k = \mathrm{Re}[N_k]\,, y_k = \mathrm{Im}[N_k]$
- The statistic:

$$S(\theta,\phi) = \sum_{k=m}^{m+M} P_k(\theta,\phi)$$

$$P_k = \left(\frac{x_k - \bar{x_k}}{\sigma_{x_k}}\right)^2 + \left(\frac{y_k - \bar{y_k}}{\sigma_{y_k}}\right)^2$$

will follow a Γ distribution in the case of real GW triggers. Parameters of the expected distribution can be estimated from the noise.

- Test this hypothesis (in the Frequentist sense) after choosing a false-dismissal probability.

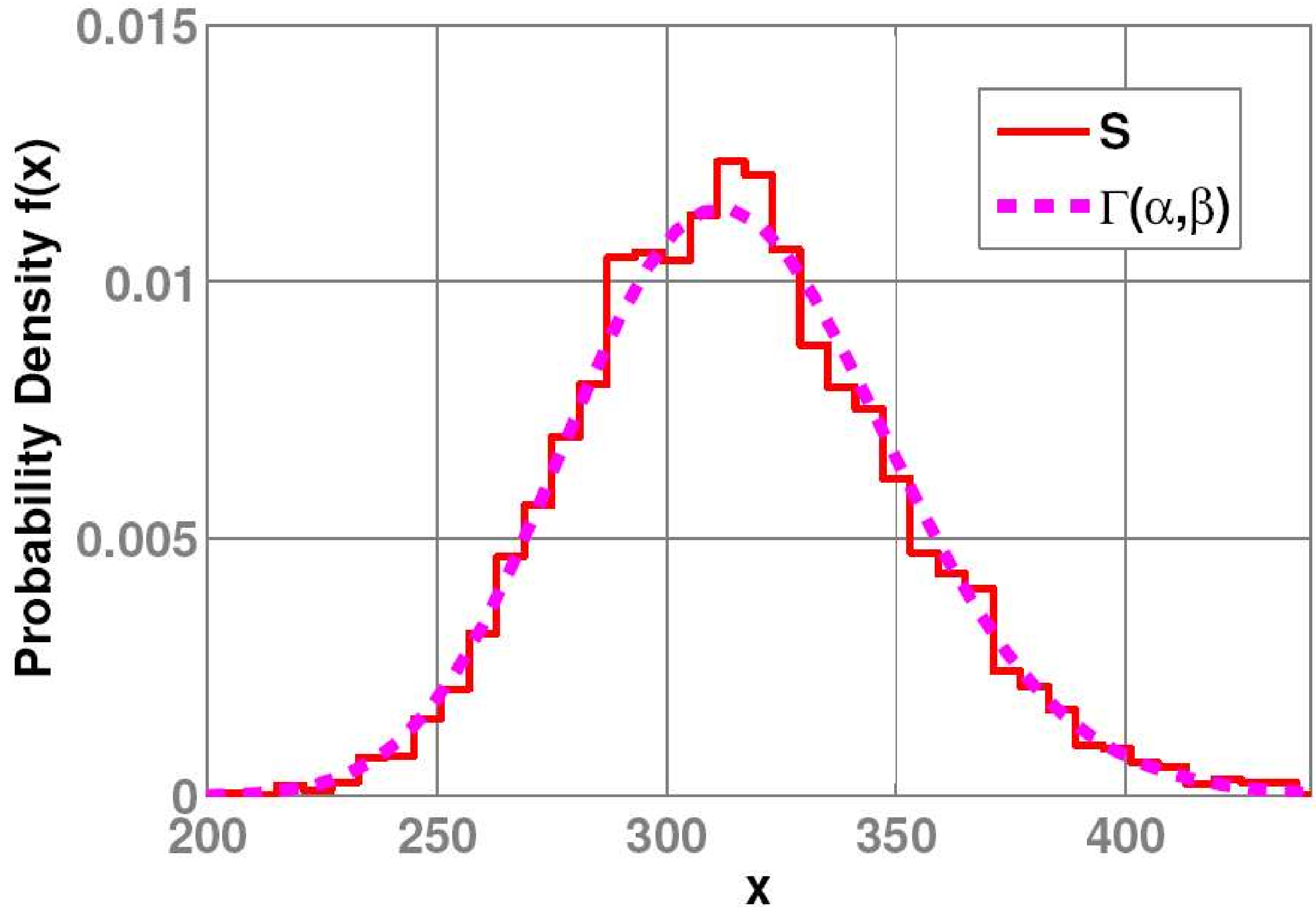
Assumption The frequency components (FFT) of the null stream, N_k, are normally distributed in the absence of any bursts. This is true if the frequency components of the individual data streams are normally distributed.

I. Two Co-located Detectors

The simplest network.

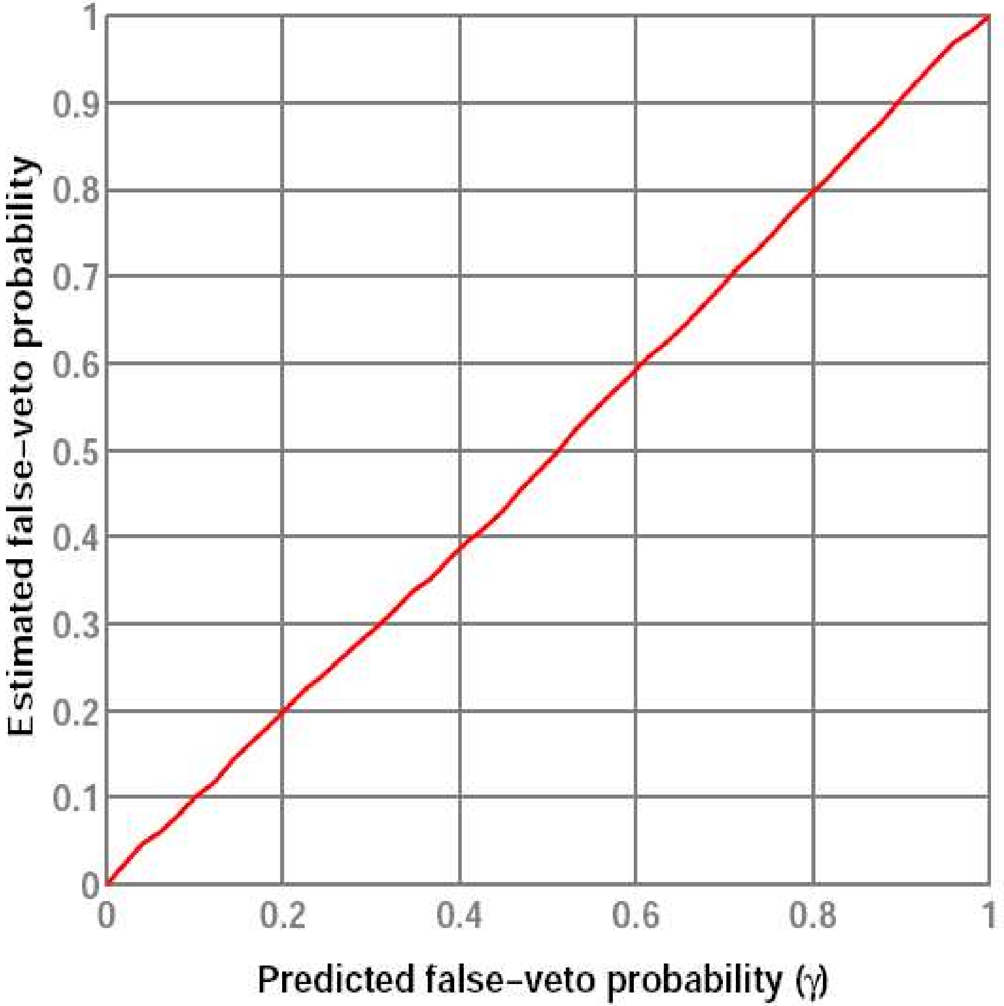
Null-stream is independent of the source position

Distribution of the null-stream statistic:
Expected Vs. Estimated



Estimating the false-veto probability

- Probability of an actual GW event being accidentally vetoed.
  - Two identical sine-Gaussian waveforms are injected into two streams of Gaussian white noise.



Fraction of vetoed events is in good agreement with the prediction.

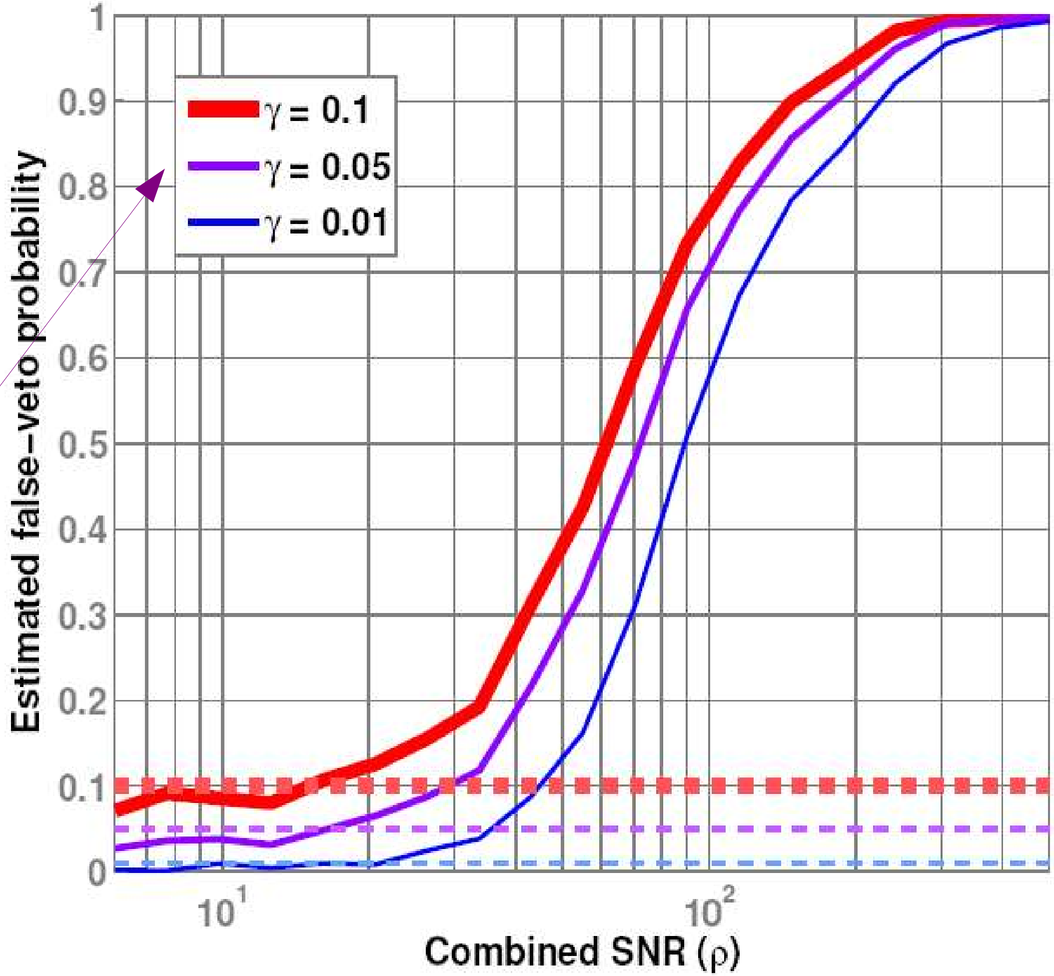
Caveat: Calibration errors

- In the presence of calibration errors, the null-stream at the time of GW triggers can contain some residual signal.

➔ The constructed statistic will deviate from the expected distribution.

➔ A different false-veto probability than predicted by the hypothesis test.

Predicted false-veto probability



A more general null-stream

- Let

$$\begin{aligned} h_1(t) &= n_1(t) + \hat{h}(t) \\ h_2(t) &= n_2(t) + (1+\epsilon)\,\hat{h}(t) \end{aligned}$$

GW signal

noise

Rel. cal. error

- We construct the combination

$$\begin{aligned} n'(t) &= h_1(t) - (1+\delta)\,h_2(t) \\ &= n(t) + (\epsilon + \delta + \delta\,\epsilon)\,\hat{h}(t) \end{aligned}$$

'Perfect' null-stream

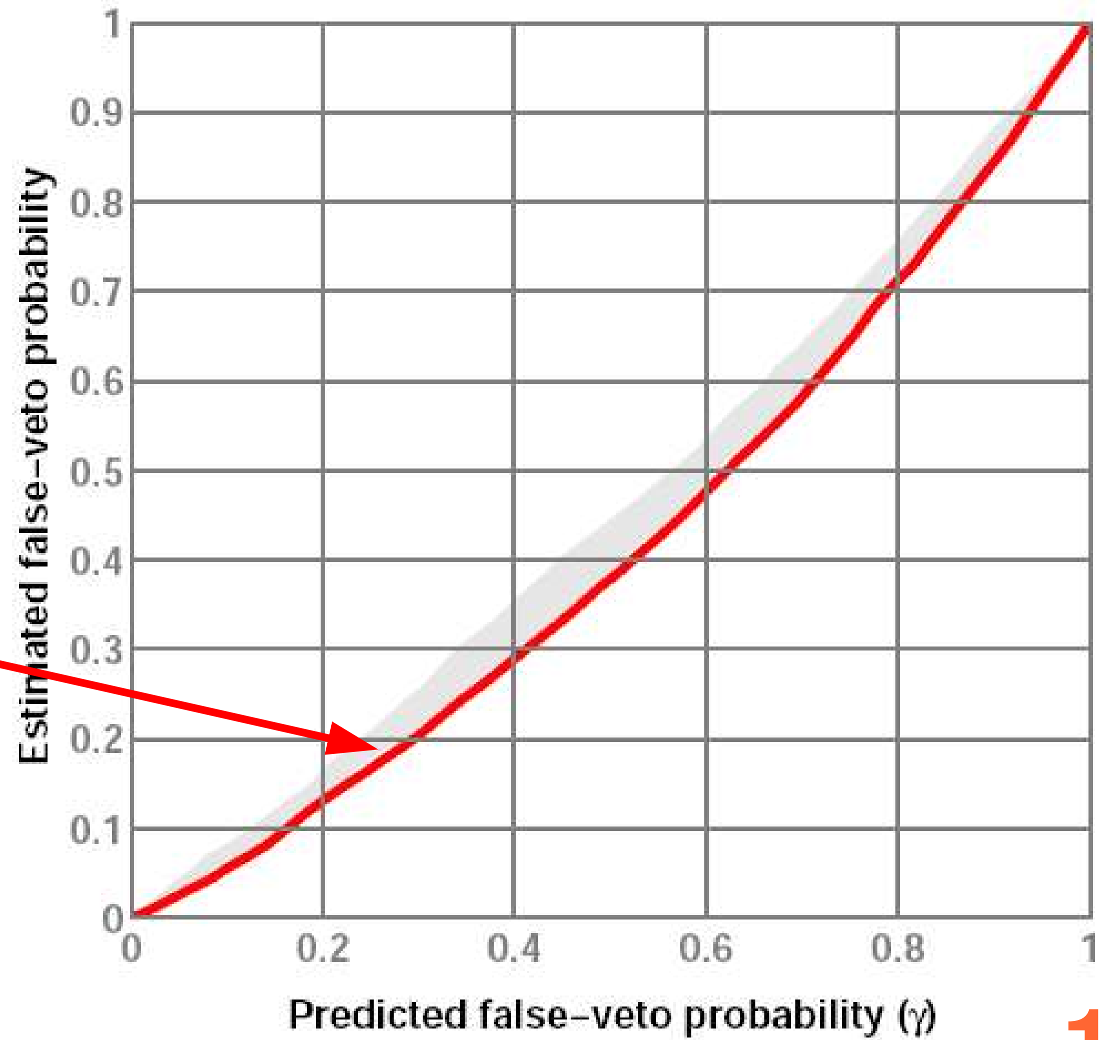
& minimize the power N'^2 by varying δ over $\left(-\frac{\epsilon_{\mathrm{max}}}{1+\epsilon_{\mathrm{max}}}, \frac{\epsilon_{\mathrm{max}}}{1+\epsilon_{\mathrm{max}}}\right)$

- For $\delta = \frac{-\epsilon}{1+\epsilon}$, the 'perfect null-stream' power is retrieved. In some cases, the minimum power is less than this (because of the 'cross terms').

Re-estimating the false-veto probability

- Inject the signals with a relative calibration error $-0.1 < \epsilon < 0.1$ and estimate the false-veto probability with the modified veto pipeline (assuming $\epsilon_{max} = 0.1$).

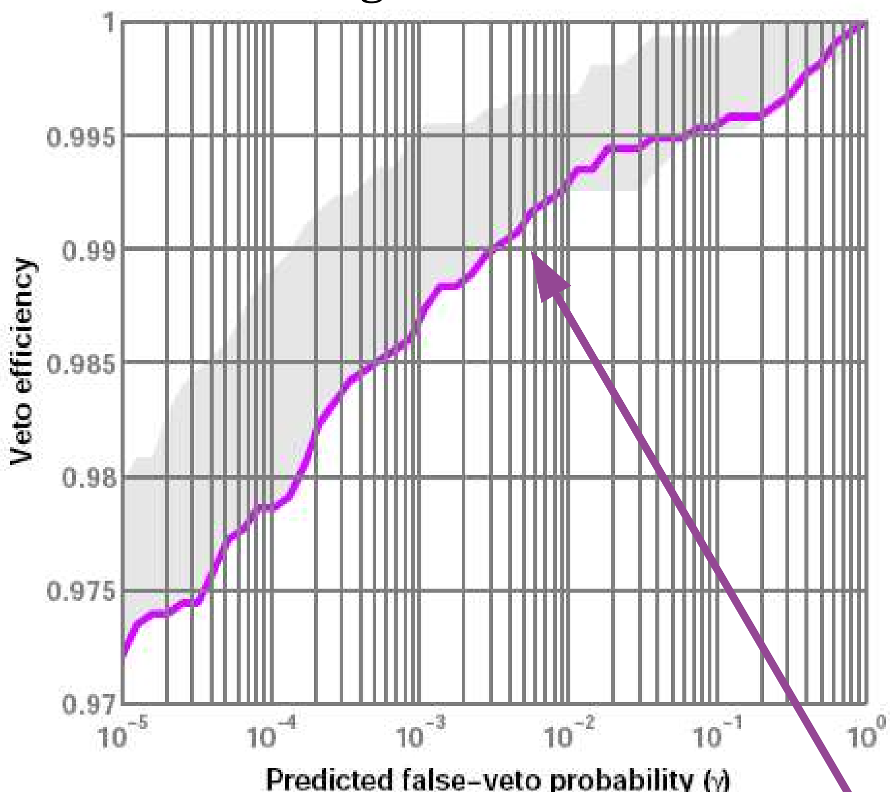
Corresponding to zero calibration error



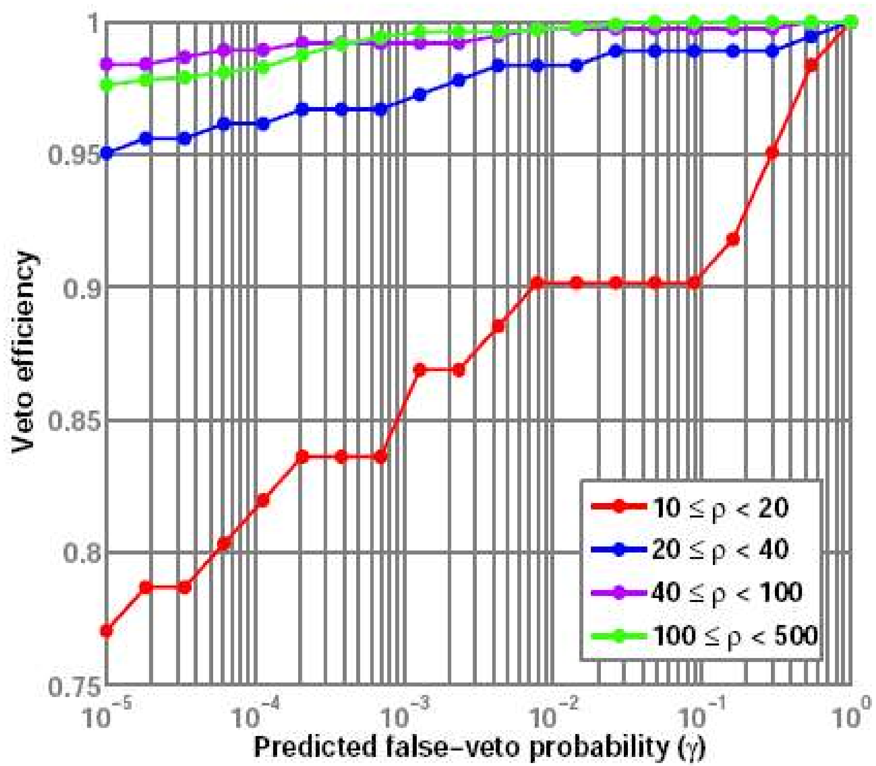
Estimating the veto-efficiency

• A plausible estimation: By injecting random waveforms with random parameters to the two data streams.

Averaged over all SNRs



Different SNR-ranges



Corresponding to
zero calibration error

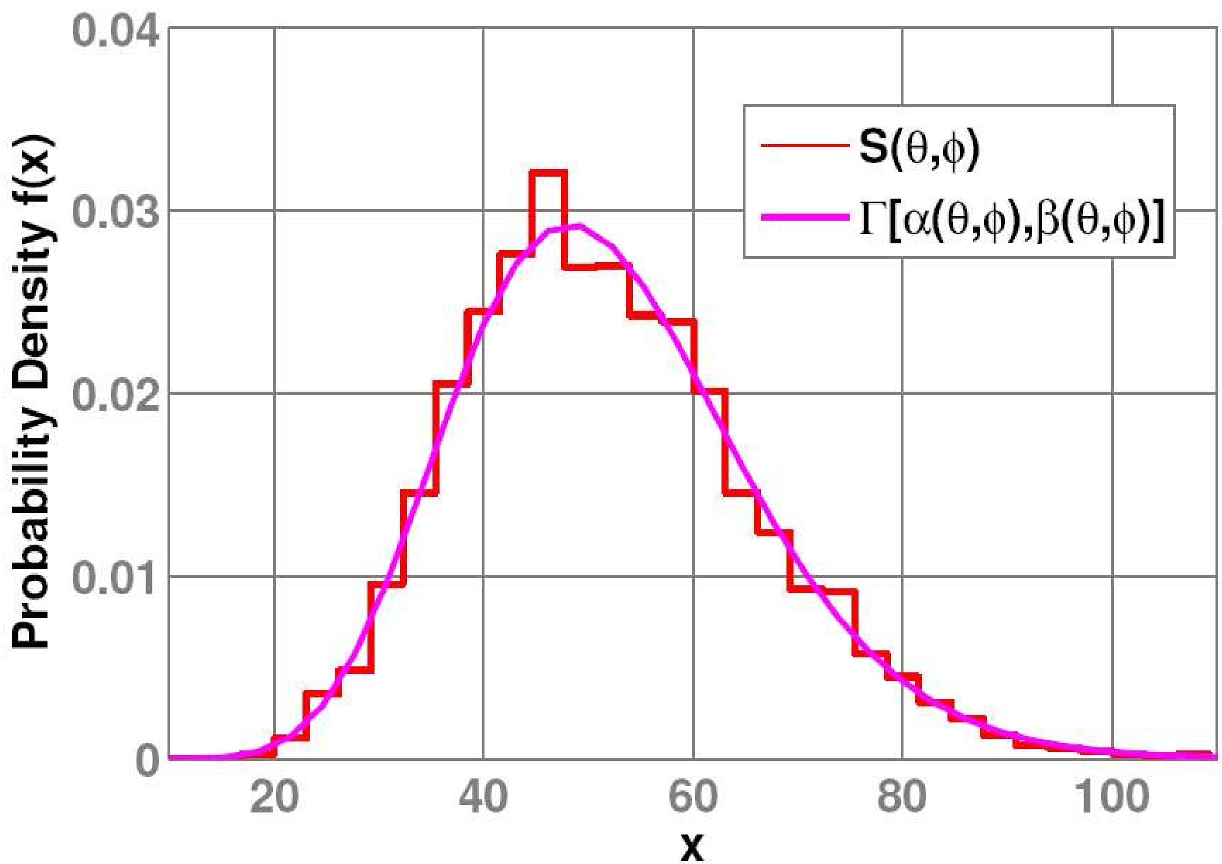
II. Three Misaligned and Widely-separated Detectors

Null-stream is a function of the source position

Distribution of the null-stream statistic:

Expected Vs. Estimated

- Simulate bursts at a specific sky-location, construct the null-stream statistic assuming prior knowledge of the source-location.



Reconstructing the source-locations

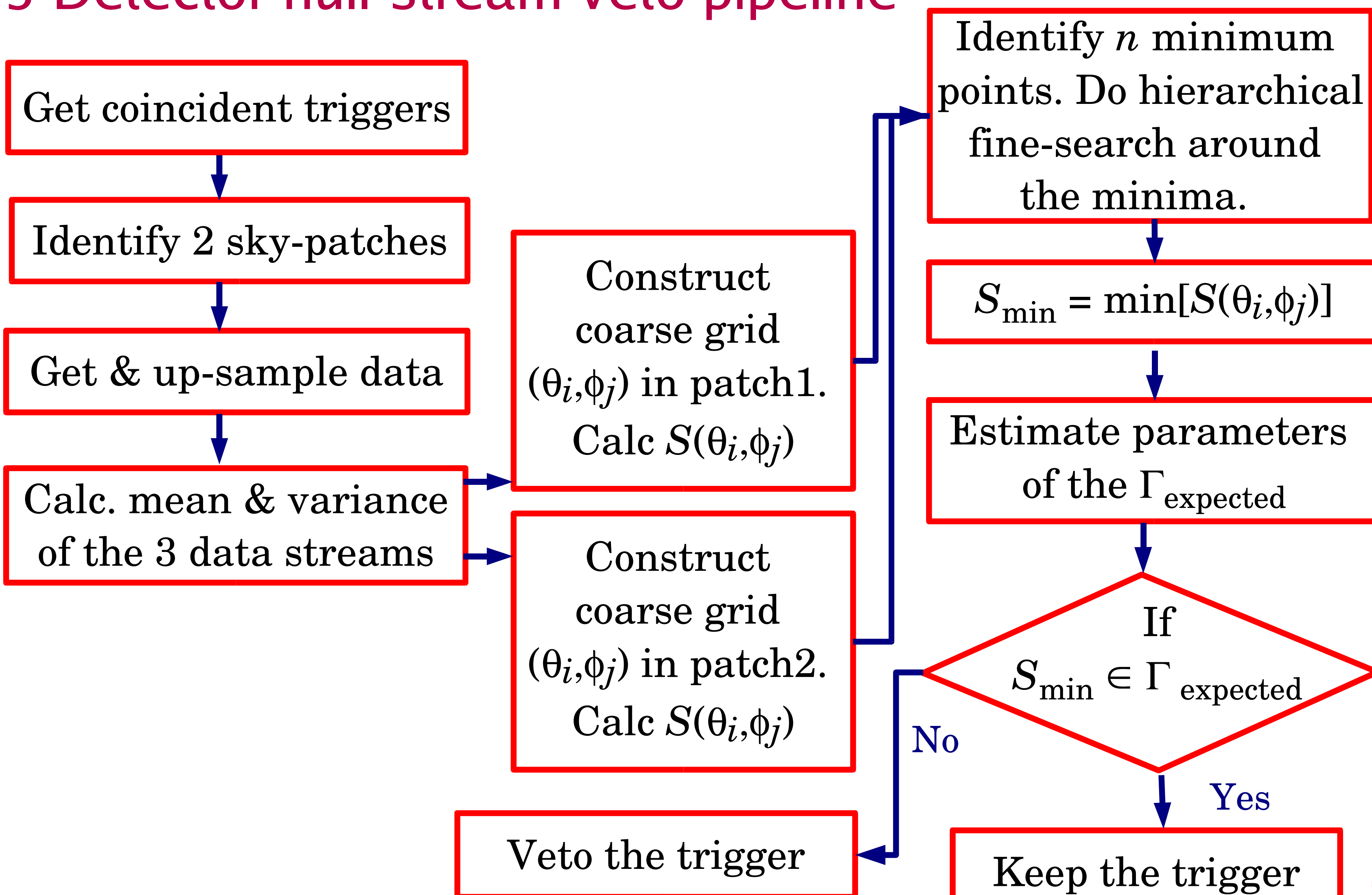
- From the time-delay between coincident triggers, the source position can be re-constructed. Three detectors → Two independent time-delays → Two possible sky-positions.

$$\{\tau_{12}, \tau_{23}\} \longrightarrow \{(\theta_1, \phi_1), (\theta_2, \phi_2)\}$$

- Time-delays inferred from the triggers are not perfect (because of the noise) → But one can identify two 'sky-patches' .

$$\{\tau_{12} \pm \Delta\tau_{12}, \tau_{23} \pm \Delta\tau_{23}\} \longrightarrow$$
$$\{(\theta_1 \pm \Delta\theta_1, \phi_1 \pm \Delta\phi_1), (\theta_2 \pm \Delta\theta_2, \phi_2 \pm \Delta\phi_2)\}$$

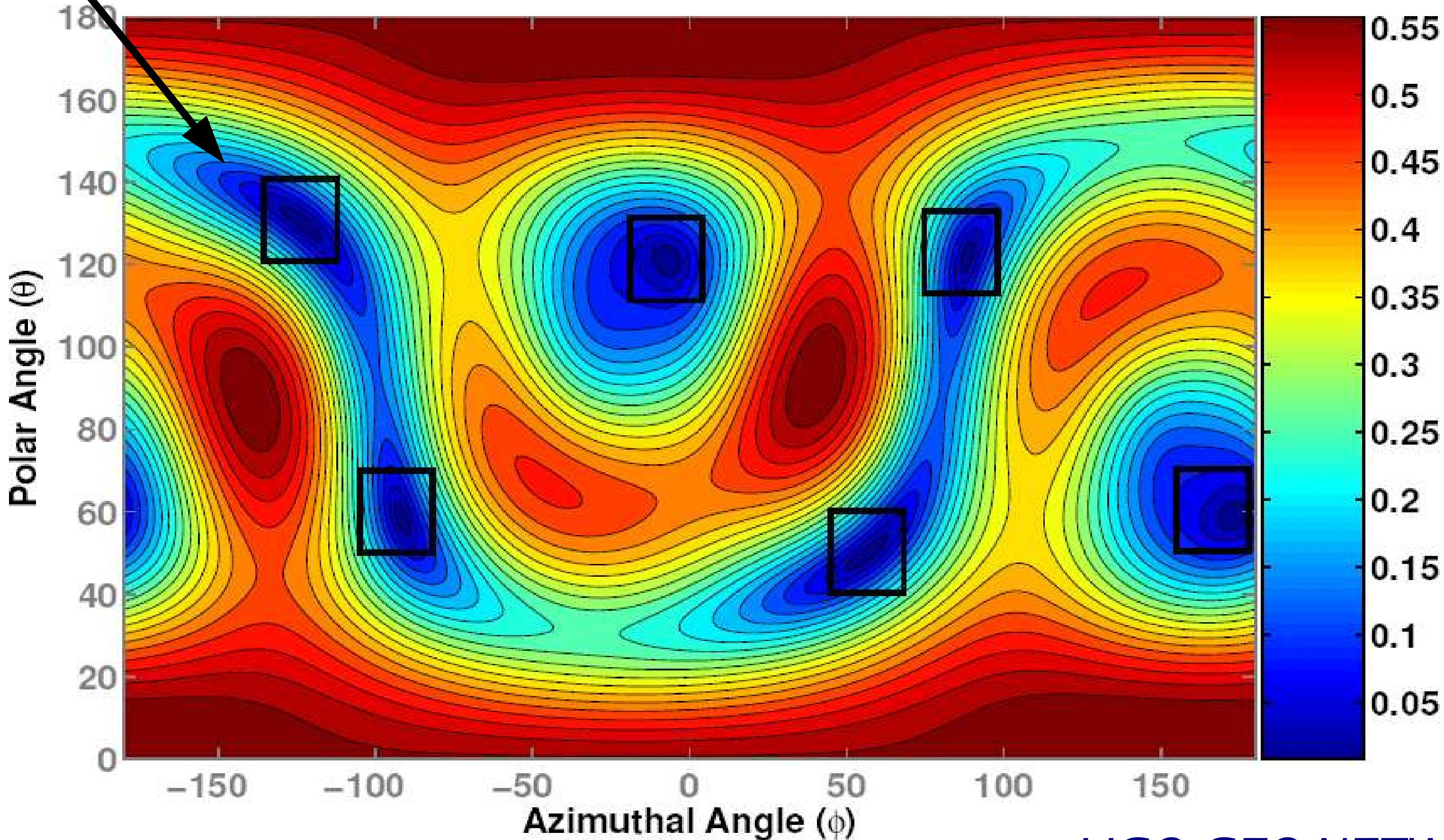
3 Detector null-stream veto pipeline



Pitfalls in the sky: *Global-nulls*

- Positions in the sky where all null-stream coefficients are zero.

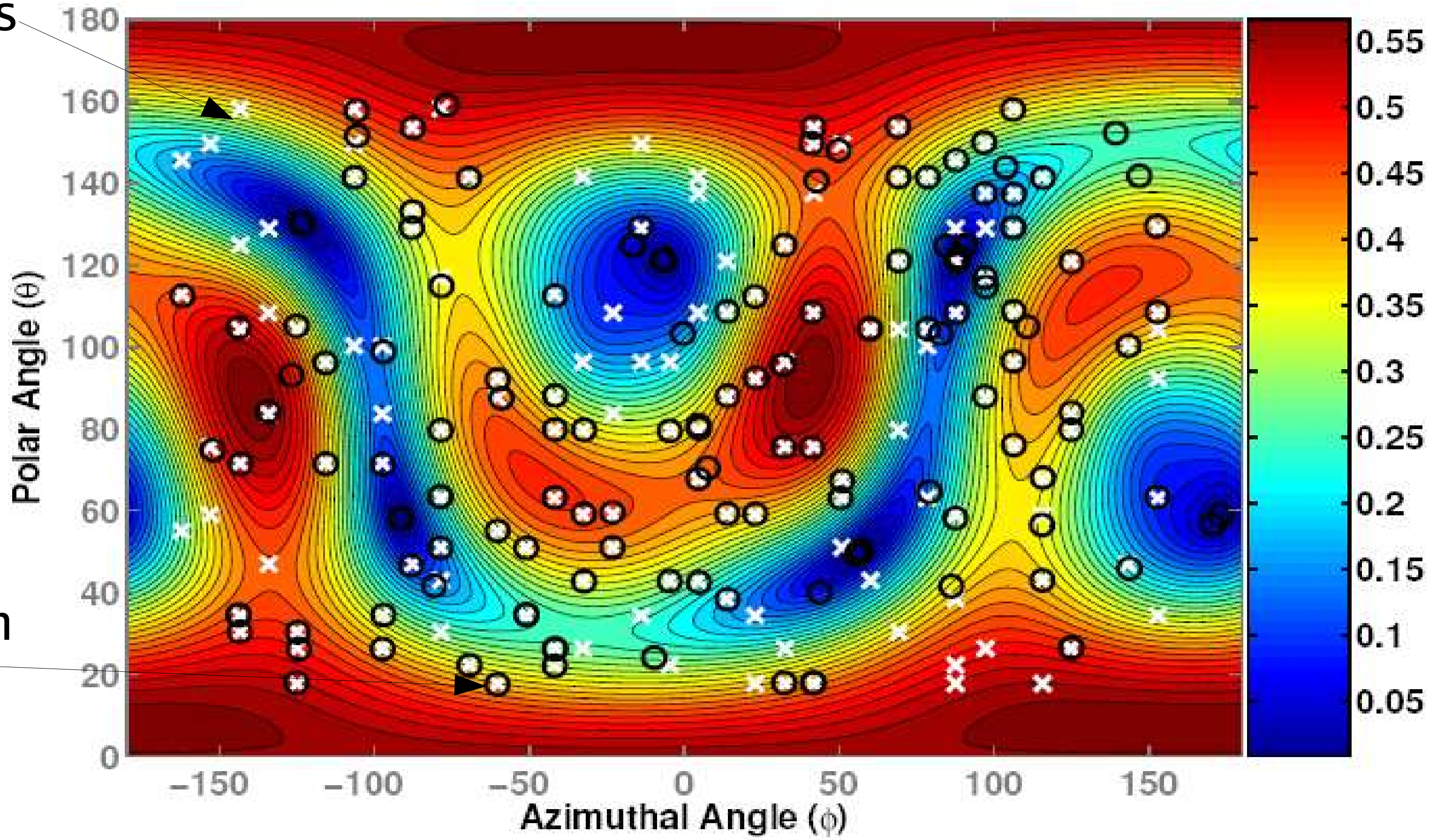
Sky-map of $A(\theta, \phi) = \sqrt{A_{12}^2(\theta, \phi) + A_{23}^2(\theta, \phi) + A_{31}^2(\theta, \phi)}$



Pitfalls in the sky: *Global-nulls*

Source-position estimation breaks in many cases
(even for noise-free detectors!)

Injections



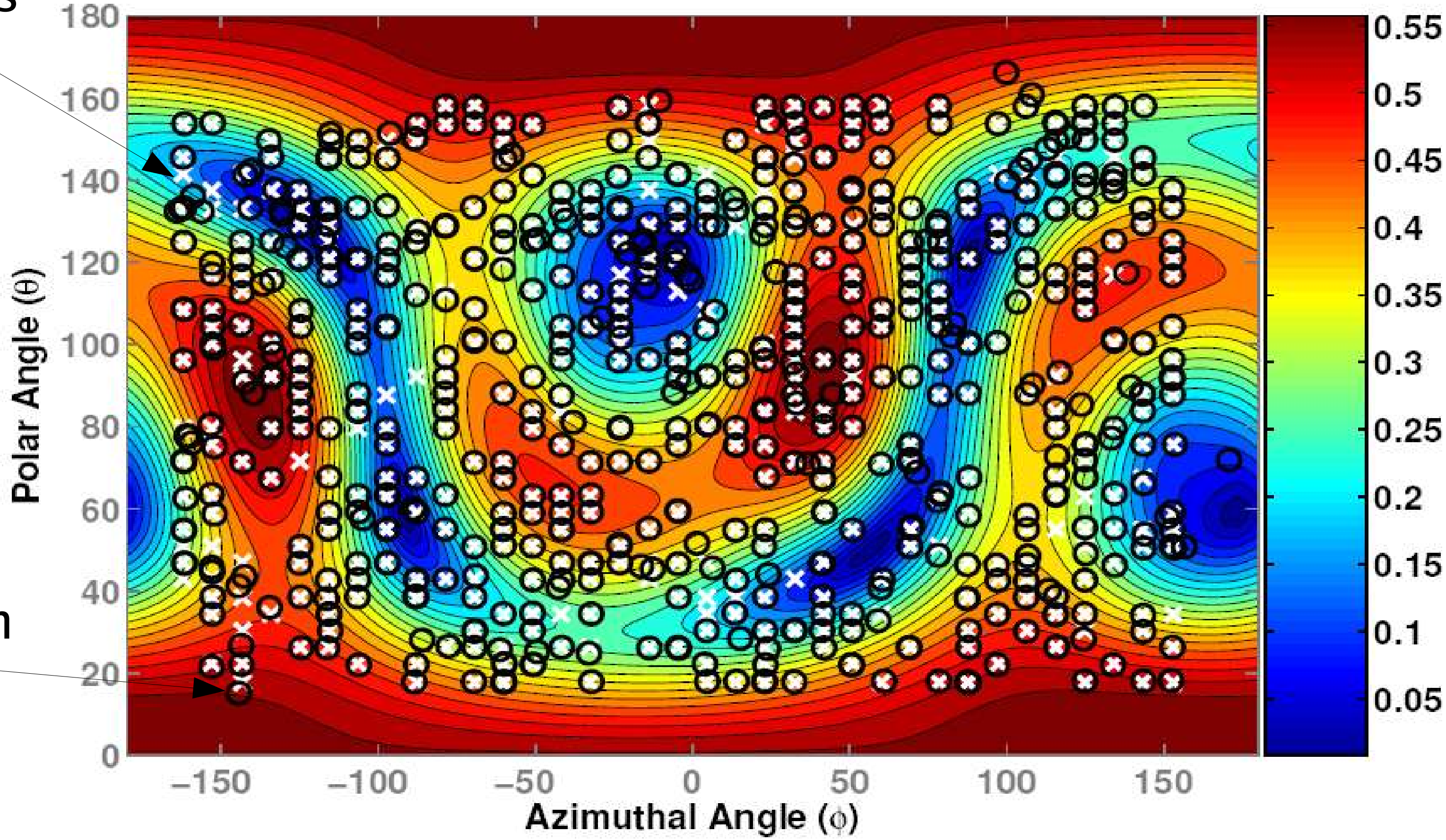
Null-stream
minima

Pitfalls in the sky: *Global-nulls*

- **Possible** way out: Minimize $S'(\theta,\phi) \propto \dfrac{S(\theta,\phi)}{A^2(\theta,\phi)}$.

Noise-free simulations

Injections



Null-stream
minima

More challenges: *Cross-terms*

- Constructed null-stream at an arbitrary position

$$n'(t;\theta_i,\phi_j) \quad = \quad n(t;\theta_i,\phi_j) + \alpha(\theta_i,\phi_j)\,\hat{h}(t)$$

noise

signal

- When we minimize the null-stream power, $N'^2_{\rm min}$ can be < N^2, depends on the SNR.
- **Possible way out:** additional constraints based on SNR and the sky-positions.

Summary

- A veto strategy is formulated using the null-stream constructed from a network of detectors.
- The method is demonstrated (in realistic circumstances) in the network of two co-located detectors. The estimated false-veto probability is in good agreement with the prediction.
- A three-detector veto algorithm is proposed. Preliminary results using the pipeline are presented.

Plans:

- Robust algorithm to distinguish between actual source position and spurious null-stream minima.
- Effect of calibration uncertainties in the 3-detector network.
- Applications to H1-H2, LIGO-GEO and simulated LIGO-Virgo data.